Forecasting
Solutions To Problems From Chapter 2

2.1 Trend
Seasonality
Cycles
Randomness

2.2 Cycles have repeating patterns that vary in length and magnitude.

2.3 a) Time Series
b) Regression or Causal Model
c) Delphi Method

2.4 Marketing: New sales and existing sales forecasts. Causal models relating advertising to sales

Accounting Interest rate forecasts; cost components, bad debts.

Finance: Changes in stock market, forecast return on investment return from specific projects.

Production: Forecast product demand (aggregate and individual), availability of resources, labor.

2.5 a) Aggregate forecasts deal with item groups or families.

b) Short term forecasts are generally next day or month; Long term forecasts may be for many months or years into the future.

c) Causal models are based on relationship between predictor variables and other variables. Naive models are based on the past history of series only

2.6 The Delphi Method is a technique for achieving convergence of group opinion. The method has several potential advantages over the Jury of Executive Opinion method depending upon how that method is implemented. If the executives are allowed to reach a consensus as a group, strong personalities may dominate. If the executives are interviewed, the biases of the interviewer could affect the results.

2.7 Some of the issues that a graduating senior might want to consider when choosing a college to attend include: a) how well have graduates fared on the job market, b) what are the student attrition rates, c) what will the costs of the college education be and d) what part-time job opportunities might be available in the region. Sources of data might be college catalogues, surveys on salaries of graduating seniors, surveys on numbers of graduating seniors going on to graduate or professional schools, etc.

2.8 The manager should have been prepared for the consequences of forecast error.

2.9 It is unlikely that such long term forecasts are accurate.

2.10 This type of criteria would be closest to MAPE, since the errors measured are relative not absolute. It makes more sense to use a relative measure of error in golf. For example, an error of 10 yards for a 200 yard shot would be fine for most golfers, but a similar error for a 20 yard shot would not.

2.11 a) (26)(.1) + (21)(.1) + (38)(.2) + (32)(.2) + (41)(.4) = 35.1

b) (23)(.1) + (28)(.1) + (33)(.2) + (26)(.2) + (21)(.4) = 25.3

2.12 a) and b)

	Forecast	Period	Actual	e_t
(86 + 75)/2 =	80.5	3	72	+8.5
(75 + 72)/2 =	73.5	4	83	-9.5
etc	77.5	5	132	-54.5
	107.5	6	65	42.5
	98.5	7	110	-11.5
	87.5	8	90	-2.5
	100.0	9	67	+33.0
	78.5	10	92	-13.5
	79.5	11	98	-18.5
	95.0	12	73	+22.0

c) MAD = (216)/10 = 21.6
MSE = (7175)/10 = 717.5

$$\text{MAPE} = 100\left(\frac{1}{n}\sum\left|\frac{e_i}{D_i}\right|\right) = 25.61$$

2.13

Fcst 1	Fcst 2	Demand	Err 1	Err 2	Er1^2	Er2^2	|Err1|
223	210	256	33	46	1089	2116	33
289	320	340	51	20	2601	400	51
430	390	375	-55	-15	3025	225	55
134	112	110	-24	-2	576	4	24
190	150	225	35	75	1225	5625	35
550	490	525	-25	35	625	1225	25
					1523.5	1599.166	37.16666
					(MSE1	(MSE2)	(MAD1)

|Err2|	|e1/D|*100	|e2/D|*100
46	12.89062	17.96875
20	15.0000	5.88253
15	14.66667	4.00000
2	21.81818	1.81818
75	15.55556	33.33333
35	4.761905	6.66667
32.16666	14.11549	11.61155
(MAD2)	(MAPE1)	(MAPE2)

2.14 It means that $E(e_i) \neq 0$. This will show up by considering

$$\sum_{i=1}^{n} e_i$$

A bias is indicated when this sum deviates too far from zero.

2.15 Using the MAD: 1.25 MAD = (1.25)(21.6) = 27.0
(Using s, the sample standard deviation, one obtains 28.23)

2.16
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MA (3)  forecast:    258.33
MA (6)  forecast:    249.33
MA (12) forecast:    205.33
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2.17, 2.18, and 2.19.

Month	One-step-ahead Forecast	Two-step-ahead Forecast	Demand	e_1	e_2
July	205.50	149.75	223	-17.50	-73.25
August	225.25	205.50	286	-60.75	-80.50
September	241.50	225.25	212	29.50	13.25
October	250.25	241.50	275	-24.75	-33.50
November	249.00	250.25	188	61.00	62.25
December	240.25	249.00	312	-71.75	-63.00
			MAD =	44.2	54.3

The one step ahead forecasts gave better results (and should have according to the theory).

2.20

Month	Demand	MA(3)	MA(6)
July	223	226.00	161.33
August	286	226.67	183.67
September	212	263.00	221.83
October	275	240.33	233.17
November	188	257.67	242.17
December	312	225.00	244.00

MA (6) Forecasts exhibit less variation from period to period.

2.21 An MA(1) forecast means that the forecast for next period is simply the current period's demand.

Month Demand MA(4) MA(1) Error

Month	Demand	MA(4)	MA(1)	Error
July	223	205.50	280	57
August	286	225.25	223	-63
September	212	241.50	286	74
October	275	250.25	212	-63
November	188	249.00	275	87
December	312	240.25	188	-124

MAD = 78.0

(Much worse than MA(4))

2.22 $F_t = \alpha D_{t-1} + (1-\alpha)F_{t-1}$

a) $F_{Feb} = (.15)(23.3) + (.85)(25) = 24.745$

$F_{March} = (.15)(72.3) + (.85)(24.745) = 31.88$

$F_{Apr} = (.15)(30.3) + (.85)(31.88) = 31.64$

$F_{May} = (.15)(15.5) + (.85)(31.63) = 29.22$

b) $F_{Feb} = (.40)(23.3) + (.60)(25) = 24.32$

$F_{March} = 43.47$

$F_{Apr} = 38.20$

$F_{May} = 29.12$

c) Compute MSE for February through April:

Month	Error (a) (α = .15)	Error (b) (α = .40)
Feb	47.45	47.88
Mar	1.56	13.17
Apr	16.13	22.70
MSE =	838.04	993.74

α = .15 gave a better forecast

2.23 Small α implies little weight is given to the current forecast and virtually all weight is given to past history of demand. This means that the forecast will be stable but not responsive.

Large α implies that a great deal of weight is applied to current observation of demand. This means that the forecast will adjust quickly to changes in the demand pattern but will vary considerably from period to period.

2.24 a)

Week	MA(3) Forecast
4	17.67
5	20.33
6	28.67
7	22.67
8	21.67

b) and c

Week	ES(.15)	Demand	MA(3)	|err|	|err|
4	17.67	22	17.67	4.33	4.33
5	18.32	34	20.33	15.68	13.67
6	20.67	12	28.67	8.67	16.67
7	19.37	19	22.67	0.37	3.67
8	19.32	23	21.67	3.68	1.33
				6.547540	7.934
				MAD-ES	MAD-MA

Based on these results, ES(.15) had a lower MAD over the five weeks

d) It is the same as the exponential smoothing forecast made in week 6 for the demand in week 7, which is 19.37 from part c).

2.25 a) $\alpha = \dfrac{2}{N+1} \propto \alpha = \dfrac{2}{7} = .286$

b) $N = \dfrac{2-\alpha}{\alpha} \propto N = \dfrac{2-.05}{.05} = 39$

c) From Appendix 2-A $\sigma_e^{\,2} = \dfrac{\sigma^2\ 2}{2-\alpha} = 1.1\sigma^2$

Hence $\dfrac{2}{2-\alpha} = 1.1$ Solving gives $\alpha = .1818$

2.26 It is the same as the one step ahead forecast made at the end of March which is 31.64.

2.27 The average demand from Jan to June is 161.33. Assume this is the forecast for July.

a)

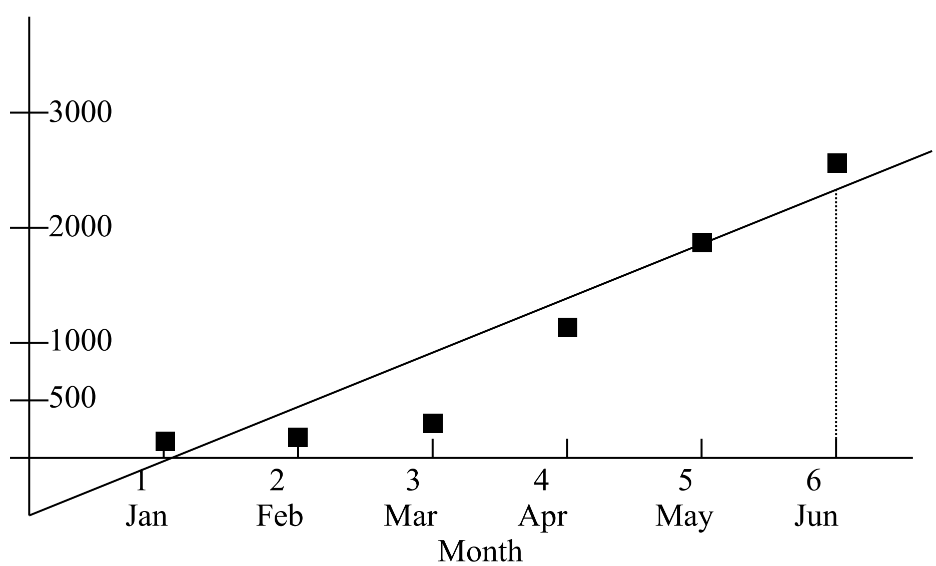
Month	Forecast	
Aug	173.7	[.2(223) + (.8)(161.33)]
Sept	196.2	etc.
Oct	199.4	
Nov	214.5	
Dec	209.2	

b)

Month	Demand	ES(.2)	(Error)	MA(6)	(Error)
Aug	286	173.7	112.3	183.7	102.3
Sept	212	196.2	15.8	221.8	9.8
Oct	275	199.4	75.6	233.2	41.8
Nov	188	214.5	26.5	242.2	54.2
Dec	312	209.2	102.8	244	68.0
		MAD	66.6		55.2

MA(6) gave more accurate forecasts.

c) For $\alpha = .2$ the consistent value of N is $(2-\alpha)/\alpha = 9$. Hence MA(6) will be somewhat more responsive. Also the ES method may suffer from not being able to flush out "bad" data in the past.



a) "Eyeball" estimates: slope = 2750/6 = 458.33, intercept = -500.

b) Regression solution obtained is

S_{xy} = (6)(28,594) - (21)(5667) = 52,557

S_{xx} = (6)(91) - $(21)^2$ = 105

$$b = \frac{S_{xy}}{S_{xx}} = \frac{52,577}{105} = 500.54$$

$$a = \overline{D} - b\,(n+1)/2 = -.807.4$$

c) Regression equation

$$\hat{D}_t = -807.4 + (500.54)t$$

Month	Forecasted Usage
July (t = 7)	2696
Aug (t = 8)	3197
Sept (t = 9)	3698
Oct (t = 10)	4198
Nov (t = 11)	4699
Dec (t = 12)	5199

d) One would think that peak usage would be in the summer months and the increasing trend would not continue indefinitely.

2.29 a)

Month	Forecast	Month	Forecast
Jan	5700	July	8703
Feb	6200	Aug	9203
Mar	6700	Sept	9704
Apr	7201	Oct	10,204
May	7702	Nov	10,705
June	8202	Dec	11,206

(note that these are obtained from the regression equation
$\hat{D}_t$ = 807.4 + 500.54 t with t = 13, 14,. . . .)

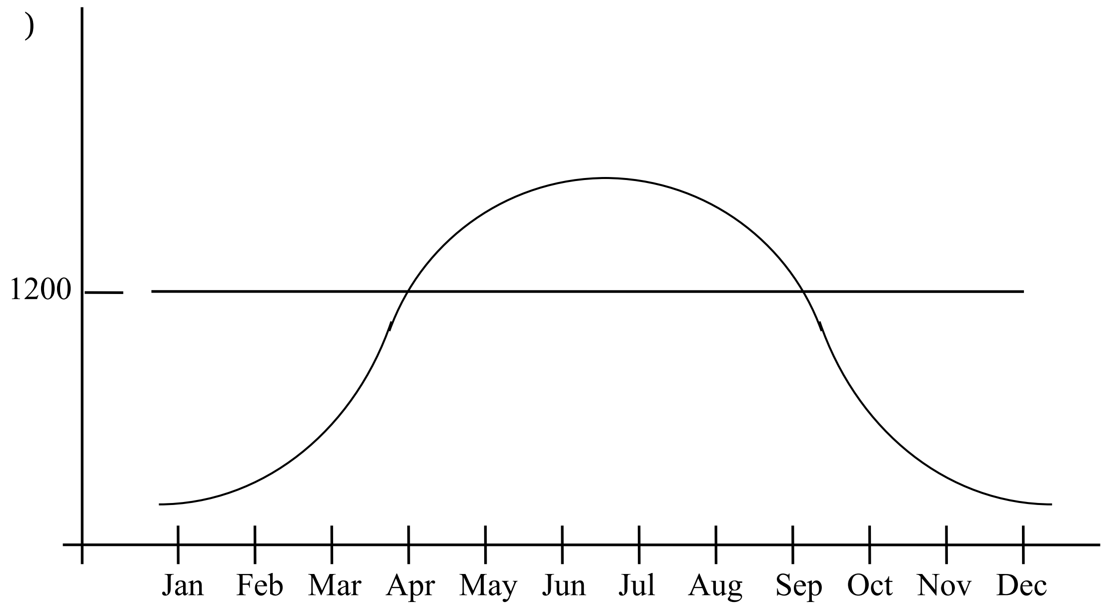
The total usage is obtained by summing forecasted monthly usage.

Total forecasted usage for 1994 = 101,431

b) Moving average forecast made in June = 944.5/mo.

Since this moving average is used for both one-step-ahead and multiple-step-ahead forecasts, the total forecast for 1994 is (944.5)(12) = 11,334.)

c)



The monthly average is about 1200 based on a usage graph of this shape. This graph assumes peak usage in summer months. The yearly usage is (1200)(12) = 14,400 which is much closer to (b), since the moving average method does not project trend indefinitely.

2.30 From the solution of problem 24,

a) slope = 500.54
value of regression in June = -807.4 + (500.54)(6) = 2196

S_0 = 2196 α = .15
G_0 = 500.54 β = .10

$S_1 = \alpha D_1 + (1-\alpha)(S_0 + G_0)$
$= (.15)(2150) + (.85)(2196 + 500.54)$
$= 2615$

$G_1 = (.1)[2615 - 2196] + (.9)(500.54) = 492.4$
$S_2 = (.15)(2660) + (.85)(2615 + 492.4) = 3040$
$G_2 = .1\ [3040 - 2615] + (.9)\ (492.4) = 485.7$

b) One-step-ahead forecast made in Aug. for Sept. is
$S_2 + G_2 = 3525.7$

Two-step-ahead forecast made in Aug for Oct is
$S_2 + G_2 = 3040 + 2(485.7) = 4011.4$

c) $S_1 + 5(G_1) = 2615 + 5(492.4) = 5077.$

2.31 This observation would lower future forecasts. Since it is probably an "outlier" (non-representative observation) one should not include it in forecast calculations.

2.32 Both regression and Holt's method are based on the assumption of constant linear trend. It is likely in many cases that the trend will not continue indefinitely or that the observed trend is just part of a cycle. If that were the case, significant forecast errors could result.

2.33

Month	Yr 1	Yr 2	Dem1/Mean	Dem2/Mean	Avg (factor)"
1	12	16	0.20	0.27	0.24
2	18	14	0.31	0.24	0.27
3	36	46	0.61	0.78	0.70
4	53	48	0.90	0.81	0.86
5	79	88	1.34	1.49	1.42
6	134	160	2.27	2.71	2.49
7	112	130	1.90	2.20	2.05
8	90	83	1.53	1.41	1.47
9	66	52	1.12	0.88	1.00
10	45	49	0.76	0.83	0.80

11	23	14	0.39	0.24	0.31
12	21	26	0.36	0.44	0.40
Totals	689	726			12

We used the Quick and Dirty Method here. The average demand for the two years was (689 + 726)/2 = 707.5.

2.34 a)

Quarter	(1) Demand	MA	Centered MA	(2) Centered MA on periods	Ratio (1)/(2)
1	12			42.440	0.2828
2	25		41.25	42.440	0.5891
3	76		42.25	41.750	1.8204
4	52	41.25	44.00	43.125	1.2058
5	16	42.25	42.75	43.375	0.3689
6	32	44.00	45.25	44.000	0.7272
7	71	42.75	44.75	45.000	1.5778
8	62	45.25	48.00	46.375	1.3369
9	14	44.75	51.25	49.625	0.2821
10	45	48.00	47.50	49.375	0.9114
11	84	51.25		49.500	1.6970
12	47	47.50		49.500	0.9494

The four seasonal factors are obtained by averaging the appropriate quarters (1, 5, 9 for first; 2, 6, 10 for the second, etc.)

One obtains the following seasonal factors

0.3112
0.7458
1.6984
1.1641

The sum is 3.9163. Norming the factors by multiplying each by

$$\frac{4}{3,9163} = 1.0214$$

we finally obtain the factors:

0.318
0.758
1.735
1.189

b)

Quarter	Demand	Factor	Deseasonalized Series
1	12	0.318	37.74
2	25	0.758	32.98
3	76	1.735	43.80
4	52	1.189	43.73
5	16	0.318	50.31
6	32	0.758	42.22
7	71	1.735	40.92
8	62	1.189	52.14
9	14	0.318	44.03
10	45	0.758	59.37
11	84	1.735	48.41
12	47	1.189	39.53

c) 47.40

d) Must "re-seasonalize" the forecast from part (c)

(47.40)(.318) = 15.07

2.35 a) $V_1 = (16 + 32 + 71 + 62)/4 = 45.25$

$V_2 = (14 + 45 + 84 + 47)/4 = 47.5$

1. $G_0 = (V_2 - V_1)/N = 0.5625$

2. $S_0 = V_2 + G_0 (N-1/2) = 47.5 + (0.5625)(3/2) = 48.34$

3. $c_t = \dfrac{D_t}{V_i[N + 1/2 - j]G_0}$ $\quad -2N+1 = \leq t \leq 0$

$$c_{-7} = \frac{16}{45.25 - (5/2 - 1)(..56)} = 0.36$$

$$c_{-6} = \frac{32}{45.25 - (5/2 - 2)(.56)} = 0.71$$

$$c_{-5} = \frac{71}{43.25 - (5/2 - 3)(.56)} = 1.56$$

$$c_{-4} = \frac{62}{45.25 - (5/2 - 4)(.56)} = 1.35$$

$$c_{-3} = \frac{14}{47.5 - (5/2 - 1)(.56)} = 0.30$$

$$c_{-2} = \frac{45}{47.5 - (5/2 - 2)(.56)} = 0.95$$

$$c_{-1} = \frac{84}{47.5 - (5/2 - 3)(.56)} = 1.76$$

$$c_{0} = \frac{47}{47.5 - (5/2 - 4)(.56)} = 0.97$$

$(c_7 + c_3)/2 = .33$

$(c_6 + c_2)/2 = .83$

$(c_5 + c_1)/2 = 1.66$

$(c_4 + c_0)/2 = \underline{1.16}$

Sum = 3.98

Norming factor = 4/3.9 = 1.01
Hence the initial seasonal factors are:

$c_{-3} = .33$ $c_{-1} = 1.67$

$c_{-2} = .83$ $c_{-0} = 1.17$

b) $\alpha = 0.2$, $\beta = 0.15$, $\gamma = 0.1$, $D_1 = 18$

$S_1 = \alpha(D_1/c_{-3}) + (1-\alpha)(S_0 + G_0) = 0.2(18/0.33)$
$+ 0.8(48.34 + 0.56) = 50.03$

$G_1 = \gamma(S_1 - S_0) + (1 - \gamma) = G_0 = 0.1(50.03 - 48.34)$
$+ 0.9(0.56) = 0.70$

$c_1 = \beta(D_1/S_1) + (1-\beta)c_3 = 0.15(18/50.03) + 0.85(0.33)$

= .3345

c) Forecasts for 2nd, 3rd and 4th quarters of 1993

$F_{1,2} = [S_1 + G_1]c_2 = (50 + .70)0.83 = 42.08$

$F_{1,3} = [S_1 + 2G_1]c_3 = (50 + 2(.70))1.67 = 85.84$

$F_{1,4} = [S_1 + 3G_1]c_4 = (50 + 3(.70))1.17 = 60.96$

2.36

Period	D_t	Forecast from 30(d)	$\lvert e_t \rvert$	Forecast from 31(c)	$\lvert e_t \rvert$
1					
2	51	35.8	15.2	42.08	8.92
3	86	82.4	3.6	85.84	0.16
4	66	56.5	9.5	60.96	5.04

MAD = 9.43 MAD = 4.71
MSE = 111.42 MSE = 35.00

Hence we conclude that Winter's method is more accurate.

2.37 $S_1 = 50.03$ $\alpha = 0.2$ $\beta = 0.15$ $\gamma = 0.1$ $D_1 = 18$
$G_1 = 0.67$ $D_2 = 51$
$D_3 = 85$
$D_4 = 66$

$S_2 = 0.2(51/0.83) + 0.8(50.03 + 0.70) = 52.87$
$G_2 = 0.1(52.87 - 50.03) + 0.9(0.70) = 0.914$

$S_3 = 0.2(86/1.67) + 0.8(52.87 + 0.914) = 53.33$
$G_3 = 0.1(53.33 - 52.85) + 0.9(0.885) = 0.8445$

$S_4 = 0.2(66/1.17) + 0.8(53.33 + 0.8445) = 54.62$
$G_4 = 0.1(54.62 - 53.33) + 0.9(0.8445) = 0.8891$

$c_1 = (.15)[18/50] + (0.85)(.33) = .3345 \approx .34$
$c_2 = (.15)[51/52.85] + 0.85(0.83) = .8502 \approx .85$
$c_3 = (.15)(86/53.29) + 0.85(1.67) = 1.6616 \approx 1.66$
$c_4 = (.15)(66/54.59) + 0.85(1.17) = 1.1758 \approx 1.18$

The sum of the factors is 4.02. Norming each of the factors by multiplying by 4/4.02 = .995 gives the final factors as:

$c_1 = .34$

$c_2 = .84$
$c_3 = 1.65$
$c_4 = 1.17$

The forecasts for all of 1995 made at the end of 1993 are:

$F_{4,9} = [S_4 + 5G_4]c_1 = [54.62 + 5(0.89)]0.34 = 20.08$

$F_{4,10} = [S_4 + 6G_4]c_2 = [54.62 + 6(0.89)]0.84 = 50.37$

$F_{4,11} = [S_4 + 7G_4]c_3 = [54.62 + 7(0.89)]1.65 = 100.40$

$F_{4,12} = [S_4 + 8G_4]c_4 = [54.62 + 8(0.89)]1.17 = 72.24$

2.42. ARIMA(2,1,1) means 2 autoregressive terms, one level of differencing, and 1 moving average term. The model may be written $u_t = a_0 + a_1 u_{t-1} + a_2 u_{t-2} + \varepsilon_t - b_1 \varepsilon_{t-1}$ where $u_t = D_t - D_{t-1}$. Since $u_t = (1-B)D_t$, we have

a) $(1-B)D_t = a_0 + (a_1 B + a_2 B^2)(1-B)D_t + (1-b_1 B)\varepsilon_t$

b) $\nabla D_t = a_0 + (a_1 B + a_2 B^2)\nabla D_t + (1-b_1 B)\varepsilon_t$

c) $D_t - D_{t-1} = a_0 + a_1(D_{t-1} - D_{t-2}) + a_2(D_{t-2} - D_{t-3}) + \varepsilon_t - b_1 \varepsilon_{t-1}$ or

$$D_t = a_0 + (1+a_1)D_{t-1} - a_1 D_{t-2} + a_2(D_{t-2} - D_{t-3}) + \varepsilon_t - b_1 \varepsilon_{t-1}$$

2.43. ARIMA(0,2,2) means no autoregressive terms, 2 levels of differencing, and 2 moving average terms. The model may be written as

$$w_t = b_0 + \varepsilon_t - b_1 \varepsilon_{t-1} - b_2 \varepsilon_{t-2}$$

Where $w_t = u_t - u_{t-1}$ and $u_t = D_t - D_{t-1}$. Using backshift notation, we may also write $w_t = (1-B)^2 D_t$, so that we have for part a)

a) $(1-B)^2 D_t = b_0 + (1 - b_1 B - b_2 B^2)\varepsilon_t$

b) $\nabla^2 D_t = b_0 + (1 - b_1 B - b_2 B^2)\varepsilon_t$

c) $D_t - 2D_{t-1} + D_{t-2} = b_0 + \varepsilon_t - b_1 \varepsilon_{t-1} - b_2 \varepsilon_{t-2}$ or $D_t = 2D_{t-1} - D_{t-2} + b_0 + \varepsilon_t - b_1 \varepsilon_{t-1} - b_2 \varepsilon_{t-2}$

2.44. The ARMA(1,1) model may be written $D_t = a_0 + a_1 D_{t-1} - b_1 \varepsilon_{t-1} + \varepsilon_t$. If we substitute for $D_{t-1}, D_{t-2}, \ldots$ one can easily see this reduces to a polynomial in $(\varepsilon_t, \varepsilon_{t-1}, \ldots)$ and if we substitute for $\varepsilon_t, \varepsilon_{t-1}, \ldots$ we see that this reduces to a polynomial in $D_{t-1}, D_{t-2}, \ldots$.

2.45 a) $1400 - 1200 = 200$
$200/5 = 40$ Change = -40 (He should decrease the forecast by 40.)

b) $(0.2)(0.8)^4 = 0.08192$
$200(0.08192) = 16.384$ Change = -16.384 (He should decrease the forecast by 16.384)

2.46 From Example 2.2 we have the following:

Quarter	Failures	Forecast (ES(.1))	Observed Error (e_t)
2	250	200	-50
3	175	205	+30
4	186	202	+16
5	225	201	-24
6	285	203	-82
7	305	211	-94
8	190	220	+30

Using $MAD_t = \alpha |e_t| + (1 - \alpha)MAD_{t-1}$, we would obtain the following values:

$MAD_1 = 50$ (given)

$MAD_2 = (.1)(50) + (.9)(50) = 50.0$
$MAD_3 = (.1)(30) + (.9)(50) = 48.0$
$MAD_4 = (.1)(16) + (.9)(48) = 44.8$
$MAD_5 = (.1)(24) + (.9)(44.8) = 42.7$
$MAD_6 = (.1)(82) + (.9)(42.7) = 46.6$
$MAD_7 = (.1)(94) + (.9)(46.6) = 51.3$
$MAD_8 = (.1)(30) + (.9)(51.3) = 49.2$

The MAD obtained from direct computation is 46.6, so this method gives a pretty good approximation after eight periods. It has the important advantage of not requiring the user to save past error values in computing the MAD.

2.47 $c_1 = 0.7$
$c_2 = 0.8$
$c_3 = 1.0$
$c_4 = 1.5$

2.48

Dept	yr 1	yr 2	yr 3	ratio 1	ratio 2	ratio 3	average
Management	835	956	774	1.20	1.37	1.11	1.23
Marketing	620	540	575	0.89	0.78	0.83	0.83
Accounting	440	490	525	0.63	0.70	0.75	0.70
Production	695	680	624	1.00	0.98	0.90	0.96
Finance	380	425	410	0.55	0.61	0.59	0.58
Economics	1220	1040	1312	1.75	1.49	1.88	1.71
							6

Mean pages over all fields and years = 696.72.

The multiplicative factors in the final column give the percentages above or below the grand mean when multiplied by 100.

2.49 a) and b)

Month	Sales	MA(3	Error	Abs Err	Sq Err	Per Err
1	238					
2	220					
3	195					
4	245	217.67	-27.33	27.33	747.11	11.16
5	345	220.00	-125.00	125.00	15625.00	36.23
6	380	261.67	-118.33	118.33	14002.78	31.14
7	270	323.33	53.33	53.33	2844.44	19.75
8	220	331.67	111.67	111.67	12469.44	50.76
9	280	290.00	10.00	10.00	100.00	3.57
10	120	256.67	136.67	136.67	18677.78	113.89
11	110	206.67	96.67	96.67	9344.44	87.88
12	85	170.00	85.00	85.00	7225.00	100.00
13	135	105.00	-30.00	30.00	900.00	22.22
14	145	110.00	-35.00	35.00	1225.00	24.14
15	185	121.67	-63.33	63.33	4011.11	34.23
16	219	155.00	-64.00	64.00	4096.00	29.22
17	240	183.00	-57.00	57.00	3249.00	23.75
18	420	214.67	-205.33	205.33	42161.78	48.89
19	520	293.00	-227.00	227.00	51529.00	43.65
20	410	393.33	-16.67	16.67	277.78	4.07
21	380	450.00	70.00	70.00	4900.00	18.42
22	320	436.67	116.67	116.67	13611.11	36.46
23	290	370.00	80.00	80.00	6400.00	27.59
24	240	330.00	90.00	90.00	8100.00	37.50
				86.62	10547.47	38.31
				MAD	MSE	MAPE

2.49 c)

Month	Sales	MA(6	Error	Abs Err	Sq Err	Per Err
1	238					
2	220					
3	195					
4	245					
5	345					
6	380					
7	270	270.50	0.50	0.50	0.25	0.19
8	220	275.83	55.83	55.83	3117.36	25.38
9	280	275.83	-4.17	4.17	17.36	1.49
10	120	290.00	170.00	170.00	28900.00	141.67
11	110	269.17	159.17	159.17	25334.03	144.70
12	85	230.00	145.00	145.00	21025.00	170.59
13	135	180.83	45.83	45.83	2100.69	33.95
14	145	158.33	13.33	13.33	177.78	9.20
15	185	145.83	-39.17	39.17	1534.03	21.17
16	219	130.00	-89.00	89.00	7921.00	40.64
17	240	146.50	-93.50	93.50	8742.25	38.96
18	420	168.17	-251.83	251.83	63420.03	59.96
19	520	224.00	-296.00	296.00	87616.00	56.92
20	410	288.17	-121.83	121.83	14843.36	29.72
21	380	332.33	-47.67	47.67	2272.11	12.54
22	320	364.83	44.83	44.83	2010.03	14.01
23	290	381.67	91.67	91.67	8402.78	31.61
24	240	390.00	150.00	150.00	22500.00	62.50
				86.63	14282.57	42.63
				MAD	MSE	MAPE

MA(6) has about the same MAD and higher MSE and MAPE.

2.50

Month	Sales	ES(.1)	Error	Abs Err	Sq Err	Per Err	Alpha
1	238	225	-13.00	13.00	169.00	5.46	0.1
2	220	226.30	6.30	6.30	39.69	2.86	
3	195	225.67	30.67	30.67	940.65	15.73	
4	245	222.60	-22.40	22.40	501.63	9.14	
5	345	224.84	-120.16	120.16	14437.78	34.83	
6	380	236.86	-143.14	143.14	20489.51	37.67	
7	270	251.17	-18.83	18.83	354.47	6.97	
8	220	253.06	33.06	33.06	1092.65	15.03	
9	280	249.75	-30.25	30.25	915.07	10.80	
10	120	252.77	132.77	132.77	17629.15	110.65	
11	110	239.50	129.50	129.50	16769.56	117.72	
12	85	226.55	141.55	141.55	20035.72	166.53	
13	135	212.39	77.39	77.39	5989.65	57.33	

14	145	204.65	59.65	59.65	3558.55	41.14
15	185	198.69	13.69	13.69	187.37	7.40
16	219	197.32	-21.68	21.68	470.05	9.90
17	240	199.49	-40.51	40.51	1641.27	16.88
18	420	203.54	-216.46	216.46	46855.50	51.54
19	520	225.18	-294.82	294.82	86915.99	56.70
20	410	254.67	-155.33	155.33	24128.54	37.89
21	380	270.20	-109.80	109.80	12056.10	28.89
22	320	281.18	-38.82	38.82	1507.01	12.13
23	290	285.06	-4.94	4.94	24.39	1.70
24	240	285.56	45.56	45.56	2075.31	18.98
				79.18	11616.03	36.41
				MAD	MSE	MAPE

The error turns out to be a declining function of α for this data. Hence, $\alpha = 1$ gives the lowest error.

2.51 a)

Year	(Y_i) Sales ($100,000)	(X_i) Births Preceding Year
1		
2	6.4	2.9
3	8.3	3.4
4	8.8	3.5
5	5.1	3.1
6	9.2	3.8
7	7.3	2.8
8	12.5	4.2

Obtain $\sum X_i$ - 23.7, $\sum Y_i = 57.6$, $\sum X_iY_i = 201.29$

$\sum X_i^2 = 81.75$, $\sum Y_i^2 = 507.48$

$S_{xx} = 10.56$ $\quad S_{xy} = 43.91$

$$b = \frac{S_{XY}}{S_{XX}} = 4.158$$

$$a = \bar{y} - b\,\bar{x} = -5.8$$

Hence $Y_t = -5.8 + 4.158X_{t-1}$ is the resulting regression equation.

b) $Y_{10} = -5.8 + (4.158)(3.3) = 7.9214$ (that is, $792,140) c)

Year	US Births (in 1,000,000) (X_i)	Forecasted Births Using ES(.15)
1	2.9	
2	3.4	
3	3.5	
4	3.1	
5	3.8	3.2
6	2.8	3.3
7	4.2	3.2
8	3.7	3.4
9		3.4
10		3.4

Hence, forecasted births for years 9 and 10 is 3.4 million.

d) $Y_t = -5.8 + 4.158\ X_{t-1}$

$X_{t-1} = 3.4$ million in years 8 and 9.

Substituting gives $Y_t = -5.8 + (4.158)(3.4) = 8.3372$ for sales in each of years 9 and 10. Hence the forecast of total aggregate sales in these years is (8.3372)(2) = 16.6744 or $1,667,440.

2.52 a)

Month	Ice cream Sales	Park Attendees
1	325	880
2	335	976
3	172	440
4	645	1823
5	770	1885
6	950	2436

	X_i Month	Y_i Ice Cream Sales	X_iY_i
	1	325	325
	2	335	670
	3	172	516
	4	645	2580
	5	770	3850
	6	950	5700
Sum =	21	3197.	13641
Avg =	3.5	532.8	

$S_{xx} = 105$
$S_{xy} = 14709$

$b = S_{xy}/S_{xx} = 140.1$

$a = \bar{Y} - b\ \bar{X} = 42.5$

$Y_{30} = 42.5 + (30)(140) = \$4245.1$

We would not be very confident about this answer since it assumes the trend observed over the first six months continues into month 30 which is very unlikely.

b)

	X_i Park attendees	Y_i Ice Cream Sales	X_iY_i
	880	325	286000
	976	335	326960
	440	172	75680
	1823	645	1175835
	1885	770	1451450
	2436	950	2314200
Sum =	8440	3197	5630125
Avg =	1406.666	532.8333	

$S_{xx} = 17,153,756$
$S_{xy} = 6,798,070$

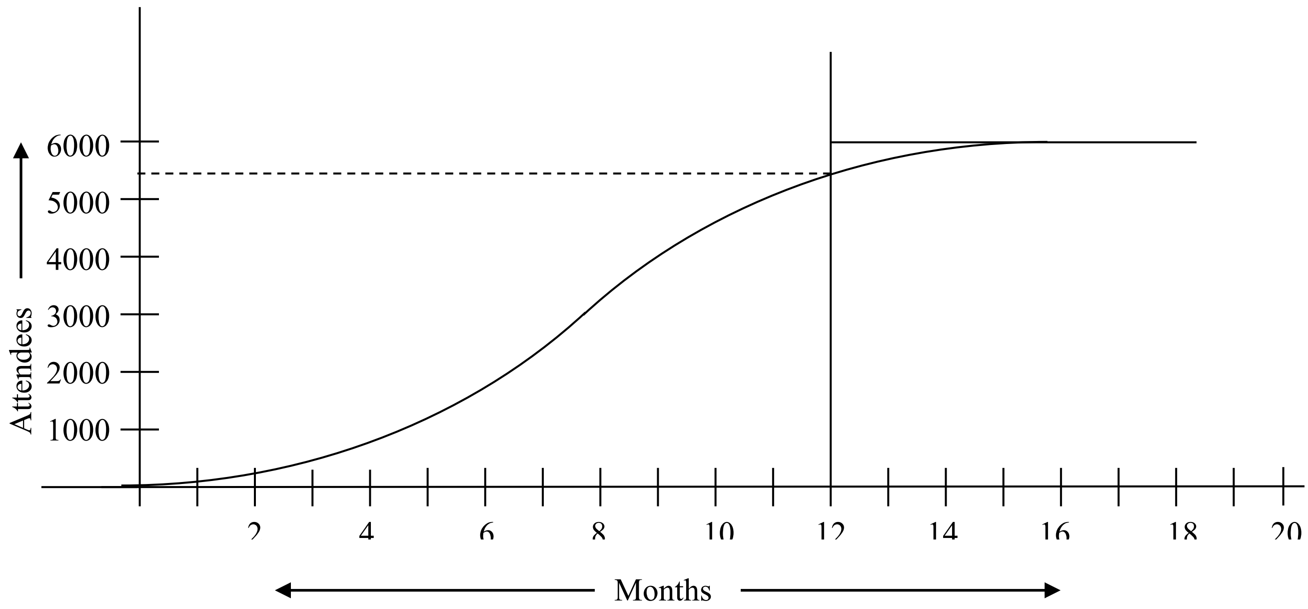
$b = S_{xy}/S_{xx} = 0.396302$

$a = \bar{Y} - b\ \bar{X} = 24.6316$

Hence the resulting regression equation is:

$Y_i = -24.63 + 0.4X_i$

c)



Readng the values from the curve:
$X_{12} \approx 5100$
$X_{13} \approx 5350$
$X_{14} \approx 5600$
$X_{15} \approx 5800$
$X_{16} \approx 5900$
$X_{17} \approx 5950$
$X_{18} \approx 5980$

Using the regression equation $Y_i = -24.63 + 0.4X_i$ derived in part (b) we obtain the ice cream sales predictions below.

Month	Attendees	Predicted Ice Cream Sales
12	5100	2015.37
13	5350	2115.37
14	5600	2215.37
15	5800	2295.37
16	5900	2335.37
17	5950	2355.37
18	5980	2367.37

2.53 The method assumes that the "best" α based on a past sequence of specific demands will be the "best" α for future demands, which may not be true. Furthermore, the best value of the smoothing constant based on a retrospective fit of the data may be either larger or smaller than is desirable on the basis of stability and responsiveness of forecasts.

2.54

Year	Demand	S sub t	G sub t	Forecast	alpha	beta	|error|	error^2
		0	8		0.2	0.2		
1981	0.2	6.44	7.69	8.00			7.80	60.84
1982	4.3	12.16	7.29	14.13			9.83	96.59
1983	8.8	17.33	6.87	19.46			10.66	113.58
1984	18.6	23.08	6.64	24.19			5.59	31.30
1985	34.5	30.68	6.84	29.72			4.78	22.85
1986	68.2	43.65	8.06	37.51			30.69	941.74
1987	85.0	58.37	9.39	51.71			33.29	1108.00
1988	58.0	65.81	9.00	67.77			9.77	95.37
							14.05	308.78
							MAD	MSE

The forecast error appears to decrease with decreasing values of α and β. That is, $\alpha = \beta = 0$ appears to give the lowest value of the forecast error.

2.55 a) We are given in problem 22 that the forecast for January was 25.
Hence $e_1 = 25\text{-}23.3 = 1.7 = E_1$ and $M_1 = |e_1| = 1.7$ as well. Hence $\alpha_1 = 1$.

$F_{Feb} = (1)(23.3) + (0)(25) = 23.3$

$e_2 = 23.3 - 72.2 = -48.9$

$E_2 = (.1)(-48.9)(.9)(1.7) = -3.36$
$M_2 = (.1)(48.9) + (.9)(1.7) = 6.42$

$\alpha_2 = 3.36/6.42 = .5234$

$F_{March} = (.5234)(72.2) + (.4766)(23.3) = 48.73$

$e_3 = 48.73 - 30.3 = 18.43$

$E_3 = (.1)(18.43) + (.9)(-3.36) = -3.024$
$M_3 = (.1)(18.43) + (.9)(6.42) = 7.621$

$\alpha_3 = 3.024/7.621 = .396 \sim .40$

$F_{Apr} = (.40)(30.3) + (.60)(48.73) = 41.358$

Comparison of Methods

Month	Demand	ES(.15)	|Error|	Trigg-Leach	|Error|
Feb	72.2	24.745	47.5	23.3	48.9
March	30.3	31.87	1.6	48.7	18.4
April	15.5	31.63	16.1	41.4	25.9

Obviously Trigg-Leach performed much worse for this 3-month period than did ES(.12). (The respective MAD's are 21.7 for ES and 31.1 for Trigg-Leach.)

b) Consider only the period July to December as in problem 36. As in part (a) $\alpha_7 = 1$. Use $E_6 = 567.1 - 480 = 87$.

$F_7 = 480$

$e_7 = 480 - 500 = -20$

$E_7 = (.2)(-20) + (.8)(87) = 65.6$

$M_7 = (.2)(20) + (.8)(87) = 73.6$

$\alpha_7 = 65.6/73.6 = .89$

$F_8 = (.89)(500) + (.11)(480) = 498$

$e_8 = 498 - 950 = -452$

$E_8 = (.2)(-452) + (.8)(65.6) = -37.9$

$M_8 = (.2)(452) + (.8)(73.6) = 149.3$

$\alpha_8 = 37.9/149.3 = .25$

$F_9 = (.25)(950) + (.75)(498) = 611$

$e_9 = 611 - 350 = 261$

$E_9 = (.2)(261) + (.8)(-37.9) = 21.9$

$M_9 = (.2)(261) + (.8)(149.3) = 171.6$

$\alpha_9 = 21.9/171.6 = .13$

$F_{10} = (.13)(350) + (.87)(620) = 584.9$

$e_{10} = 584.9 - 600 = -15.1$

$E_{10} = (.2)(-15.1) + (.8)(21.9) = 14.5$

$M_{10} = (.2)(17.8) + (.8)(171.6) = 140.8$

$\alpha_{10} = 14.5/140.8 = .10$

$F_{11} = (.10)(600) + (.90)(584.9) = 586.4$

$e_{11} = 586.4 - 870 = -283.6$

$E_{11} = (.2)(-283.6) + (.8)(14.5) = -45.1$

$M_{11} = (.2)(283.6) + (.8)(140.8) = 169.4$

$\alpha_{11} = 45.1/169.4 = .27$

$F_{12} = (.27)(870) + (.73)(586.4) = 663.0$

Performance Comparison

Month	Demand	Trigg-Leach Forecast	|Error|
7	500	480	20
8	950	498	452
9	350	611	261
10	600	585	15
11	870	586	284
12	740	663	77

MAD = 185

The MAD for ES(.2) from problem 36 was 194.5.
Hence Trigg-Leach was slightly better for this problem.

c) Trigg-Leach will out-perform simple exponential smoothing when there is a trend in the data or a sudden shift in the series to a new level, since α will be adjusted upward in these cases and the forecast will be more responsive. However, if the changes are due to random fluctuations, as in part (a), Trigg-Leach will give poor performance as the forecast tries to "chase" the series.

2.56 Given information:

$\alpha = .2, \quad \beta = 0.2, \quad \text{and } \gamma = 0.2$

$S_{10} = 120, \quad G_{10} = 14$

$c_{10} = 1.2$
$c_9 = 1.1$
$c_8 = 0.8$
$c_7 = 0.9$

a) $F_{11} = (S_{10} + G_{10})c_7 = (120 + 14)(0.9) = 120.6$

b) $D_{11} = 128$

$S_{11} = \alpha(D_{11}/c_7) + (1 - \alpha)(S_{10} + G_{10}) = 135.6$
$G_{11} = \gamma(S_{11} - S_{10}) + (1 - \gamma)G_{10} = 14.3$

$c_{11} = \beta(D_{11}/S_{11}) + (1-\beta)c_7 = .909$

$$\sum_{t=8}^{11} C_t = 4.009$$

The factors are normed by multiplying each by 1/4.009 = .9978
They will not change appreciably.

$F_{11,13} = (S_{11} + 2G_{11})C_9 = (135.6 + (2)(14.3))1.1 = 180.6$

2.57 a)

	X_i	Y_i	X_iY_i
	1	649.8	649.8
	2	705.1	1410.2
	3	772.0	2316.0
	4	816.4	3265.6
	5	892.7	4463.5
	6	963.9	5783.4
	7	1015.5	7108.5
	8	1102.7	8821.6
	9	1212.8	10915.2
	10	1359.3	13593.0
	11	1472.8	16200.8
Sum =	66	10,963.0	74,527.6
Avg =	6	996.64	

$$S_{xy} = n \sum_{i=1} i\, D_i - \frac{(n)(n+1)}{2} \sum D_i$$

$$= (11)(74,527.6) = \frac{(11)(12)}{2} (10,963.0) = 96,245.6$$

$$S_{XX} = \frac{n^2(n+1)(2n+1)}{6} - \frac{n^2(n+1)^2}{4} = \frac{(11^2)(12)(23)}{6} - \frac{((11)^2(12)^2)}{4} = 1210$$

$$b = \frac{S_{xy}}{S_{xx}} = \frac{96,245.6}{1210} = 79.54$$

$$a = \bar{Y} - b\bar{X} = \frac{10,963.0}{11} - (79.54)\frac{66}{11} = 519.4$$

<u>Initialization for Holt's Method</u>

S_0 = regression line in year 11 (1974)
= 519.4 + (11)(79.54) = 1394.34

<u>Updating Equations</u>

G_0 = slope of regression line = 79.54

$S_i = \alpha D_i + (1 - \alpha)(S_{i-1} + G_{i+1})$

$G_i = \beta (S_i - S_{i-1}) + (1 - \beta)G_{i-1}$

Obs	Yr	D_i	S_i	G_I		|Error|	$\|Error\|^2$
1	1975	1598.4	1498.78	82.03	$F_{0,1} = S_0 + G_0 =$ 1473.88	124.52	15505.23
2	1976	1782.8	1621.21	86.07	$F_{1,2} = S_1 + G_1 =$ 1580.81	201.99	40798.18
3	1977	1990.9	1764.01	91.74	$F_{2,3} = S_2 + G_2 =$ 1707.28	283.62	80439.38
4	1978	2249.7	1934.54	99.62	$F_{3,4} = S_3 + G_3 =$ 1855.75	393.95	155198.35
5	1979	2508.2	2128.97	109.10	$F_{4,5} = S_4 + G_4 =$ 2034.16	474.04	224714.16
6	1980	2732.0	2336.86	118.98	$F_{5,6} = S_5 + G_5 =$ 2238.07	493.93	243966.72
7	1981	3052.6	2575.20	130.92	$F_{6,7} = S_6 + G_6 =$ 2455.84	596.76	356126.04
8	1982	3166.0	2798.08	140.11	$F_{7,8} = S_7 + G_7 =$ 2706.11	459.89	211502.67
9	1983	3401.6	3030.88	149.38	$F_{8,9} = S_8 + G_8 =$ 2938.20	463.40	214740.75
10	1984	3774.7	3299.15	161.27	$F_{9,10} = S_9 + G_9 =$ 3180.26	594.44	353357.64
					Totals	4086.54	1896349.11

MAD = 408.6,   MSE = 189,634.9

b)

Year	GNP	%	MA(6)	Forecast GNP	|Error|	ES(.2)	Forecast GNP	|Error|
1964	649.8							
1965	705.1	8.51%						
1966	772.0	9.49%						
1967	816.4	5.75%						
1968	892.7	9.35%						
1969	963.9	7.98%						
1970	1015.5	5.35%						
1971	1102.7	8.59%						
1972	1212.8	9.98%						
1973	1359.3	12.08%						
1974	1472.8	8.35%						
1975	1598.4	8.53%	8.72%	1601.3	2.9	8.54%	1598.6	0.2
1976	1782.8	11.54%	8.81%	1739.3	43.5	8.54%	1734.9	47.9
1977	1990.9	11.67%	9.84%	1958.3	32.6	9.14%	1945.7	45.2

1978	2249.7	13.00%	10.36%	2197.1	52.6	9.65%	2182.9	66.8
1979	2508.2	11.49%	10.86%	2494.0	14.2	10.32%	2481.8	26.4
1980	2732.0	8.92%	10.76%	2778.2	46.2	10.55%	2772.8	40.8
1981	3052.6	11.73%	10.86%	3028.6	24.0	10.23%	3011.4	41.2
1982	3166.0	3.71%	10.98%	3387.9	221.9	10.53%	3374.0	208.0
1983	3401.6	7.44%	10.30%	3492.0	90.4	9.16%	3456.2	54.6
1984	3774.7	10.97%	9.71%	3731.9	42.8	8.82%	3701.6	73.1
			*MAD	=	57.1	*MAD	=	60.4

The moving average and exponential smoothing forecasts based on percentage increases are more accurate than Holt's method.

c) One would expect that a causal model might be more accurate. Large-scale econometric models for predicting GNP and other fundamental economic time series are common.