Chapter 2

1) Consider a batch manufacturing process where a machine processes jobs in batches of 3 units. The process starts only when there are 3 or more jobs in the buffer in front of the machine. Otherwise, the machine stays idle until the batch is completed. Assume that job inter-arrival times are equally likely between 2 and 8 hours. Batch service times are equally likely between 5 and 15 hours.

Simulate the system manually for 3 batch service completions and calculate the following statistics:

- Average number of jobs in the buffer (excluding the batch being served)
- Probability distribution of number of jobs in the buffer (excluding the batch being served)
- Machine utilization
- Average job waiting time (time in buffer)
- Average job system time (total time in the system, including processing time)
- System throughput (number of departing jobs per unit time)

Approach: After each arrival, schedule the next inter-arrival time, and when each batch goes into service, schedule its service completion time. To obtain the requisite values, use your calculator to generate a sequence of random numbers (these are equally likely between 0 and 1, and statistically independent of each other). Then transform these numbers as follows:

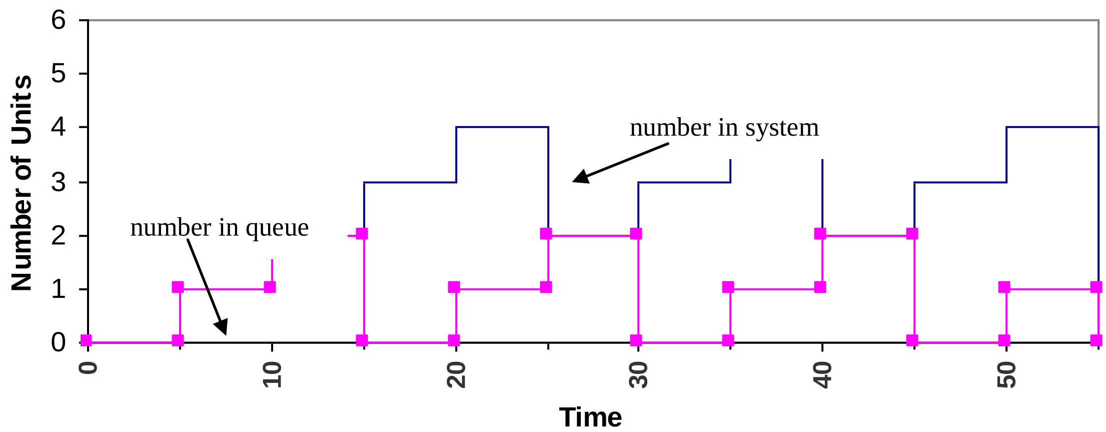
(a) To generate the next random inter-arrival time, $\boldsymbol{A}$, generate the next random number $\boldsymbol{U}$ from your calculator, and set $\boldsymbol{A} = 2 + 6\boldsymbol{U}$.

(b) To generate the next random batch service time, $\boldsymbol{B}$, generate the next random number $\boldsymbol{U}$ from your calculator, and set $\boldsymbol{B} = 5 + 10\boldsymbol{U}$.

If you cannot use random numbers from your calculator, use instead deterministic inter-arrival times, $\boldsymbol{A} = 5$ (the average of 2 and 8), and deterministic service times $\boldsymbol{B} = 10$ (the average of 5 and 15).

Answers:

In this exercise, we used the deterministic approach discussed in the exercise. That is, we simply used the mean of the distributions when we need to generate events. The graph of the number of units in the system as well as in the queue is given below. Notice that the server is busy in time slots (15,25), (30,40), and (45,55), which correspond to batch service times.



The average number of jobs in the queue:

Area under the “number in queue” graph = [(10-5)*1+(15-10)*2]*3 + (55-50)*1=50
Average of “number in the queue” graph = Area / Total time = 50/55 = 0.91

Probability distribution of the number in the queue :
Pr{0 in the queue} = 20/55, Pr{1 in the queue} = 20/55, Pr(2 in the queue) = 15/55

Machine utilization = 30/55 = 0.545 (3 batch services are completed)

Average delay in the queue:
A total of 11 jobs arrived, and the queue and system times are given below: Included in the statistics are the last 2 jobs that were in the system when simulation terminated.

Jobs	Queue time	System time
1	10	20
2	5	15
3	0	10
4	10	20
5	5	15
6	0	10
7	10	20
8	5	15
9	0	10
10	5	5
11	0	0

Average queue time = 50/11 hrs/unit
Average system time = 140/11 hrs/unit
Output Rate:
9 jobs departed in 55 hours, so the output rate is: 9/55 = 0.16 units/hr.

2) A manufacturing facility has a repair shop with two repairmen who repair failed machines on First-Fail-First-Serve basis. They work together on the machine if there is one machine down (the repair still takes the same time), and otherwise, each works on a separate machine. Thus, if there are more than 2 machines down, new failures simply wait for their turn to be repaired. Assume that machine failures arrive in a combined failure stream, so that we do not need to maintain machine identity. More specifically, assume that the times between machine failures are equally likely between 10 and 20 hours, and that repair times are equally likely between 5 and 55 hours for each machine.

To simulate the manufacturing facility, use the approach of items (a) and (b) of Exercise **1)** utilizing the following sequence of uniform random numbers (between 0 and 1), $\boldsymbol{U} = \{0.2, 0.5, 0.9, 0.7, 0.8, 0.1, 0.5, 0.2, 0.7, 0.4, 0.3\}$, to generate times to failure and repair times. Simulate the manufacturing facility manually for 5 machine repair completions, and calculate the following statistics:

- Average fraction of time a machine is down
- Average number of down machines waiting to be repaired.
- Fraction of time each repairman is busy (repairman utilization)
- Fraction of time the repair facility is idle
- Average time a failed machine waits until its repair starts
- Throughput of the repair facility (average number of repair completions per hour)

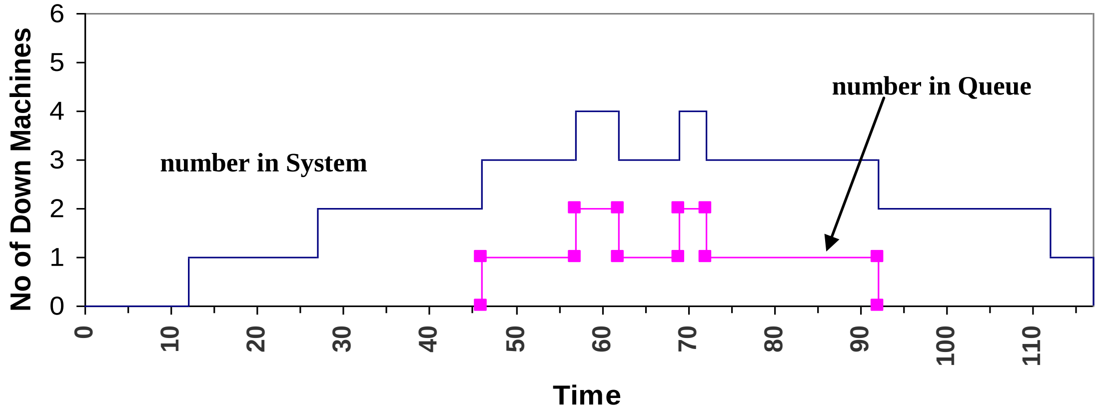
Note: These manual procedures will simulate the system for a short period of time. The tedium involved in simulating the system should make you realize that one needs a computer program for long simulations or of even moderate complexity.

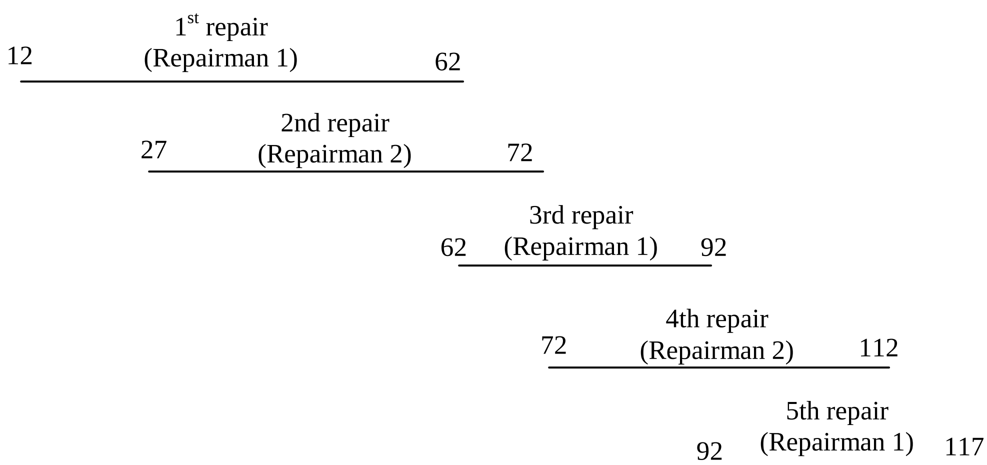
Answers:

The random numbers are: 0.2 0.5 0.9 0.7 0.8 0.1 0.5 0.2 0.7 0.4 0.3 (add more numbers if you need them in yours simulation).
The resultant sequences of times to failure and repair times are given in the table below.

No	**Time to Failure**	**Repair Time**
1	0.2*(20-10)+10=12	0.9*(55-5)+5=50
2	0.5*(20-10)+10=15	0.8*(55-5)+5=45
3	0.9*(20-10)+10=19	0.5*(55-5)+5=30
4	0.1*(20-10)+10=11	0.7*(55-5)+5=40
5	0.2*(20-10)+10=12	0.4*(55-5)+5=25

The corresponding sample path of the number of down machines is given in the graph below.





Fraction of time a machine is down:

Total time each machine spends in the repair facility is given by
S1=50, S2=45, S3=16+30=46, S4=15+40=55, S5=23+25=48 hours.

Average time per machine = 244/5=48.8 hours

Pr{ a machine is down} = 48.8/117=0.417

Average number of down machines waiting to be repaired:

Area under the "number in queue" graph divided by simulation time, that is,

[11+7+10)*1+(5+3)*2]/117 = 0.376

Fraction of time each repairman is busy (repairman utilization):

Total time repairman 1 worked = (50+30+25)/117 = 0.897
Total time repairman 2worked = (15+45+40+5)/117 = 0.897
Keep in mind that both repairmen are busy between 12 and 117.

Fraction of time the repair facility is idle:

Pr{Facility is idle} = 12/117.

Average time a failed machine waits until its repair starts:

Waiting times at each machine are:

$W1=0, \quad W2=0, \quad W3=16, \quad W4=15, \quad W5=23$

Thus average delay is 54/5=10.2 hours.

Throughput of the repair facility (average number of repair completions per hour):

There were 5 repair completions in 117 hours. Thus, the throughput is 5/117 = 0.043 repair completions per hour.