CHAPTER 2

EXERCISE 1 a)

Y	50,000	10,000	1,000	10	0
P(y)	1/150,000	5/150,000	25/150,000	1,000/150,000	148,969/150,000

b) mean= (1/150000) * 50000 + (5/150000) * 10000 + (25/150000) * 1000 + (1000/150000) * 10 + (148,969/150,000)*0 = $0.90

c) no, expected value is only $0.90 so there would be an expected (average over many tickets) loss of $0.10 per ticket.

d) σ^2= (50000-.9)²*(1/150000) + (10000-.9)²*(5/150000) +

(1000-.9)²*(25/150000) + (10-.9)²*(1000/150000) +

(0-.9)²*(148969/150000) = 20166.5233

σ = $142.01 This distribution is very skewed, so the standard deviation does not reflect the very large (but low probability) prizes, even if you use an interval of mean +3 standard deviations.

EXERCISE 2

a) 0.5, using area of rectangle with base from 0 to 1.

b) using formulas for uniform distribution in section 2.4, $\mu = 1$,

Variance = 4/12 = 1/3.

EXERCISE 3 a) mean=(0*.3277)+ (1*.4096) + (2*.2048) + (3*.0512) + (4*.0064) + (5*.0003) = 0.9999

Variance= 0.7997

b) $\mu= np = (5)(.2) = 1$ $\sigma^2 = np(1-p) = (5)(.2)(.8)= .8$

Yes, they agree except for small rounding error.

EXERCISE 4

The system will fail if at least one component fails (since they are in series). Hence, they will fail if they do not all work.

Probability of failing = 1 – probability all work = $1 – (0.999)^{10} = 0.00996$

Arrangement 1: probability of failure =

        prob [(A1 fails and A2 fails) OR (B1 fails and B2 fails)] $= (0.01)^2 + (0.01)^2 - (0.01)^4 = 0.0001999$

 Arrangement 2: probability of failure =

        Prob[(A1 fails or B1 fails) AND (A2 fails or B2 fails)] =

        $(0.01 + 0.01 - 0.01^2)^2 = 0.000396$

Arrangement 1 is most reliable.

If you use a scientific calculator rather than table A.1, your values may differ slightly.

a) 0.1587

b) 0.8413

c) $0.5 - 0.1587 = 0.3413$

d) $1 - 0.9332 - 0.0668$

e) $0.9808 - 0.1635 = 0.8173$

f) look for 0.05 in the column for probability, A is between 1.64 and 1.65 (or use Table A.1A to find it is 1.645, or an inverse normal function on a scientific calculator or software).

g) look for 0.025 in the column for probability, C is 1.96

   a) $P(Y > 15) = P(Z > 1) = 0.1587$ on TI-84, normcdf(15,1E99,10,5)

b) $P(8 < Y < 12) = P(-2/5 < Z < 2/5) = .6554 - .3446 = 0.3108$

      on TI-84, normcdf(8,12,10,5) = 0.3108

c) $P(Z < c) = .90$ $P(Z > c) = .1$ $c = 1.282 = (Y - 10)/5$

            $Y = 16.41$

  On TI-84 calculator, invnorm(0.90,10,5) = 16.41

Treating grades as a continuous random variable.

a) Prob $(Y < 60) = P(Z < (60-76)/14) = P(Z < -1.14) = 0.1265$

b) $(Y - 76)/14 = 1.28$, so lowest score for an A is 93.92, or 94.

c) Adding points changes the mean, not the standard deviation. If only 5% are to fall at or below 60, then the new mean must be such that

$(60 - \mu)/14 = -1.645$, or $\mu = 60+1.645*14 = 83.03$. Since the original mean was 76, we need to add 7 points.

The sample proportion favoring the increase has mean = p = 0.20,
and variance = $p(1-p)/n$ = (0.2 * 0.8)/250, so std dev= 0.025298
Z= (0.24 - 0.2)/0.025298 = 1.581
P(Y > 1.581) = 0.0571 (or on TI-84, 0.0569)

EXERCISE 10

a) μ = 0(0.368)+1(0.368)+2(0.184)+3(0.061)+4(0.015)+5(0.003)+6(0.001)= 1.00

Variance = $(0-1)^2(0.368)+(1-1)^2(0.368)+ \ldots +(6-1)^2(0.001) = 1.004$

b) the mean is correct, but the variance is off a little bit. Perhaps it is rounding error.

EXERCISE 11 Let O be the number of operators and Y = number of calls arriving. If number of calls exceeds number of operators, get a busy signal. Want to choose O so P(Y > O) = 0.05.
Z = 1.645 = (O-40)/(sqrt(40)) → O = 50.4, so need 50 or 51 operators.

EXERCISE 12 We set $\mu = np$ = 200*0.015 = 3. The probability of 0 cases is $e^{-3}*3^0/0! = 0.0498$.

EXERCISE 13   standard error of sample mean = 25/sqrt(100) = 2.5
Z1 = (138-140)/2.5 = -0.8   Z2 = (142-140)/2.5 = 0.8
P(138 < Y < 142) = P(-.8 < y < .8) = .7881 - .2119 = 0.5762
On TI-84 calculator, normcdf(138,142,140,2.5) = 0.5762

EXERCISE 14

We replace products with lifetime less than A, where P(X < A) = 0.10. The z-score that corresponds is -1.28, so (A – 1000)/150 = -1.28, hence A = 808. So the manufacturer should guarantee the product for 808 days.

EXERCISE 15 P(Z < z1) = .1   and   P(Z > z2) = .1

Z1= -1.282 and Z2= 1.282

Z1 = -1.282 = (60 - μ)/σ and   Z2 = 1.282 = (90 - μ)/σ

Solving two equations for two unknowns yields:

σ = 11.70   σ² = 136.9   μ = 75

EXERCISE 16

a) If treatment not effective, probability of survival still 0.3. Probability of at least 2 out of 3 using binomial is 0.216.

b) Probability of at least 4 out of 6 using binomial is 0.070.

c) In both cases, at least 2/3 of victims survived, but the chance of this happening in a sample of 6 if the treatment is NOT effective is much smaller, and less likely to be due to chance.

EXERCISE 17 For a normal distribution, the quartiles are about 0.67σ above and below the mean, so the IQR is about 1.34σ. The upper fence starts at $0.67\sigma + 1.5*(1.34\sigma) = 2.68\sigma$. The probability beyond this is 0.0037. Counting the lower tail as well, there is about 0.0074 probability (0.74%) that a value will be an outlier.

EXERCISE 18 The upper limit for the control chart is `0.05 + 3 * `$\sqrt{.05 * .95 / 100}$` = 0.115.` The process was in control throughout the first shift. During the second half of the second shift an upward trend began in the fraction of nonconforming. No single shift was listed as 'out of control' until the third shift, but the persistent high nonconforming rates during the second shift could be viewed as a warning.
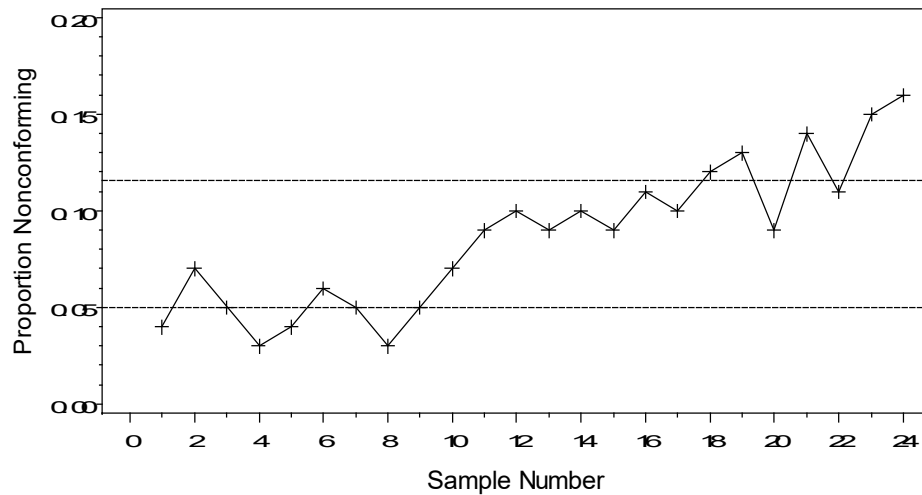
EXERCISE 19 The number of different tickets is 53*52*51*50*49*48/6! = 22,957,480

a) There is only one choice of numbers that will win the grand prize, probability = 1/22957480 = 4.356*10^{-8}

b) 1.233*10^{-5} There are 6 choices for a number to get wrong, and 47 numbers to insert in its place.

c) 0.01484

EXERCISE 20

The control limits are $4 \pm 3\sqrt{0.25 / 5} = (3.33, 4.67)$ The process was initially producing windshields that were too thin, shifted to ones that were too thick, and then back to windshields that were too thin. Perhaps the production control is over-compensating in its attempt to correct the process.
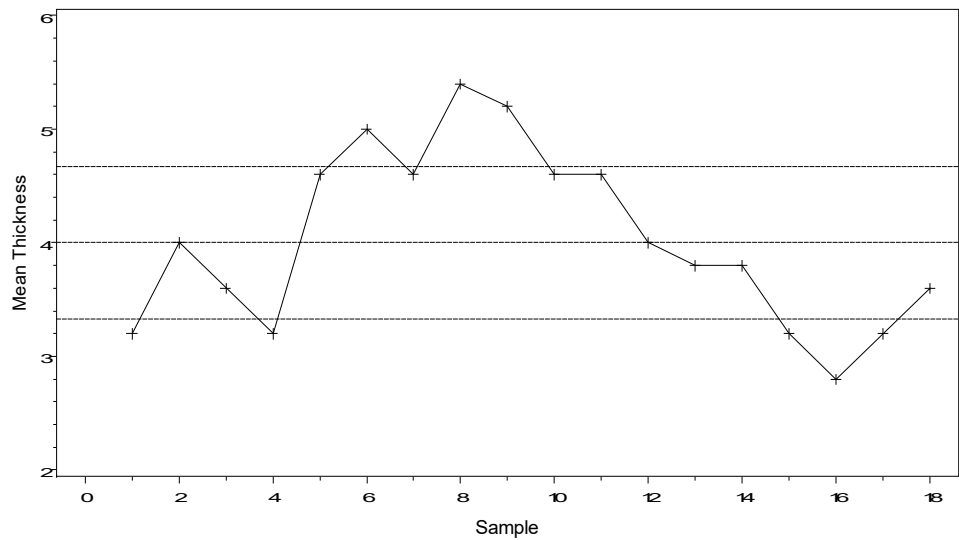
EXERCISE 21 **a)** Use binomial with $p = 0.05$. A: 0.939, B: 0.921

b) Use binomial with $p = 0.15$. A: 0.322, B: 0.097

c) Plan B is more expensive but has a smaller probability of missing an increase in the true error rate.

EXERCISE 22

a) All people who have recently filed for unemployment benefits in your county.

b) If the true proportion p is 10%, then the number in our sample should be a binomial with $n = 68$ and $p = 0.10$. Denote the number of those with these feelings in the sample as X. If you are using a calculator or program that has the binomial cdf, then

$\text{Prob}(X \geq 12) = 1 - \text{Prob}(X \leq 11) = 1 - 0.9638 = 0.0362$

c) If the true proportion in the population is 10%, it would be unusual to find 12 or more in a sample of 68. I would conclude that the proportion of unemployed people who have had feelings of sadness recently is most likely more than 10%.

d) The sample proportion would be nearly normally distributed with a mean of 0.10 and a standard deviation of $\sqrt{\dfrac{.1 * .9}{68}} = 0.0364$. Using the normal distribution, the probability that the sample proportion will be less than 0.05 or more than 0.15 is 0.0848+0.0848 = 0.1696.

a) 119.2

b) 0.4972 (TI84: 0.4950)

c) standard error = 15/sqrt(20) = 3.354, probability = 0.3734 (TI84: 0.3711)

a) If $n = 50$ and $p = 0.03$, then the expected number of birth defects is 50(0.03) = 1.5. Using the binomial distribution, the probability of $X > 3$ (note 'more than twice') is the same as

$1 – \text{Prob } (X \leq 3) = 1 – 0.9372 = 0.0628$.

b) If n = 150 and p = 0.03, then the expected number of birth defects is 150(0.03) = 4.5. Using the binomial distribution, the probability of

$X > 9 = 1 – \text{prob } (X \leq 9) = 1 – 0.9845 = 0.0155$.

Though the decision rules are very similar, our chances of a 'false' spike in the observed proportion of birth defects is much smaller when we have a large sample.

Inspecting Table A.4 with 5 and 5 df, $P(F > 5)$ = about 5%.

On TI84 calculator, get 0.051 = Fcdf(5,1E99,5,5).