Chapter 3

3.1.1. $V = 10\ m^3$, $W = 60$ kN.
Specific weight, $\gamma = W/V = 60\ kN/10\ m^3 = 6\ kN/m^3 = 6000\ N/m^3$
Density, $\rho = \gamma/g = 6000\ N/m^3\ /9.81\ kN/m^3 = 611.62\ kg/m^3$
Specific gravity, s.g. $= \gamma/\gamma_w = 6\ kN/m^3\ /9.81\ kN/m^3 = 0.612$.

3.1.2. s.g. = 0.7.
Using SI units: ρ_{oil} = s.g. $\rho_w = 0.7(1000\ kg/m^3) = 700\ kg/m^3$ and
γ_{oil} = s.g. $\gamma_w = 0.7(9810\ N/m^3) = 6867\ N/m^3$.

Using English (FPS) units: $\rho_{oil} = 0.7(1.94\ slugs/ft^3) = 1.36\ slugs/ft^3$ and
$\gamma_{oil} = 0.7(62.4\ lb/ft^3) = 43.68\ lb/ft^3$.

3.1.3. From Newton's second law,
F = ma. Thus, unit of force = unit of mass x unit of acceleration = kg x m/s^2.
This is unit is called newton, denoted by N, and it is equivalent to kg x m/s^2.

3.1.4. In English units, unit of mass = the unit of Force divided by the unit of acceleration due to gravity = $lb/(ft/s^2) = lb\text{-}s^2/ft$.
This unit is called slug, and it is thus equivalent to $lb\text{-}s^2/ft$.

3.1.5. $W_{earth} = 9810$ N. $g_{moon} = (1/6)g_{earth} = (1/6)(9.81\ m/s^2) = 1.635\ m/s^2$.
$M_{earth} = M_{moon} = W_{earth}/g_{earth} = 9810\ N/9.81\ m/s^2 = 100$ kg.
$W_{moon} = M_{moon}\ g_{moon} = 100\ kg\ (1.635\ m/s^2) = 163.5$ N.

3.1.6. T = 110 ^oF. Using the relationship $^oC = (5/9)(^oF – 32)$:
T in $^oC = (5/9)(110 – 32) = 43.33\ ^oC$.

3.1.7. At 20 ^oC, 50 ^oC and 80 ^oC, the densities of water are 998.2 kg/m^3, 988.1 kg/m^3 and 971.8 kg/m^3, respectively (Table 3.1.2). Thus, from
(i) 20 ^oC to 50 ^oC, percent change in ρ = (998.2 – 988.1)(100)/998.2 = 1 %;
(ii) 20 ^oC to 80 ^oC percent change in ρ = (998.2 – 971.8)(100)/998.2 = 2.6%.

3.1.8. $\nu = 4 \times 10^{-4}\ m^2/s$, s.g. = 1.5.
ρ_{liquid} = s.g. $\rho_w = 1.5(1000\ kg/m^3) = 1500\ kg/m^3$.
$\mu = \nu\rho = 4 \times 10^{-4}\ (1500\ kg/m^3) = 0.6\ N\text{-}s/m^2$ (Pa-s)

3.1.9. $\mu = 2.09 \times 10^{-5}\ lb\text{-}s/ft^2$.
In $N\text{-}s/m^2$, m = $[2.09 \times 10^{-5}\ lb\text{-}s/ft^2] \times [4.448\ N/lb] \times [1\ ft/0.3048\ m]^2 = 1 \times 10^{-3}$ $N\text{-}s/m^2$ (kg/m.s)
1 poise = 1 g/cm.s, and 1 centipoise = 0.01 g/cm.s = 0.001 kg/m.s.
Thus, $2.09 \times 10^{-5}\ lb\text{-}s/ft^2$ = 1 centipoise.

3.1.10. 1 stoke = 1 $cm^2/s = 10^{-4}\ m^2/s$.
1 centistoke = 0.01 stoke = $10^{-6}\ m^2/s$.

Thus, 800 centistokes = 800 x 10^{-6} m^2/s = 8 x 10^{-4} m^2/s.
In ft^2/s, 8 x 10^{-4} m^2/s (1 ft/0.3048 m)^2 = 8.61 x 10^{-3} ft^2/s.

3.1.11. 1 stoke = 1 cm^2/s = 10^{-4} m^2/s = 10^{-4} m^2/s (3.281 ft/m)^2 = 1.076 x 10^{-3} ft^2/s.

3.1.12. ρ = 700 kg/m^3, μ = 0.0042 kg/m.s.
$\nu = \mu/\rho$ = 0.0042/700 = 6 x 10^{-6} m^2/s.
In ft^2/s unit, ν = 6 x 10^{-6} m^2/s (3.281 ft/m)^2 = 6.46 x 10^{-5} ft^2/s.
In stoke unit, ν = 6 x 10^{-6} m^2/s (100 cm/m)^2 = 0.06 stokes.
In centistoke unit, ν = 0.01(0.06) = 6 x 10^{-4} centistokes.

3.1.13. V = 10 ft^3, T = 50 ^oC, Δp = 200 psi.

$$E = -\frac{dp}{dV/V}, \quad dV = -\frac{V.dp}{E}$$

From Table (2.1.1), E at 50 ^oF = 305000 psi.
Thus, dV = -10(200)/305000 = -6.56 x 10^{-3} ft^3.

3.1.14. dV/V = 0.05 % = 0.0005, Δp = 160 psi.

$$E = -\frac{dp}{dV/V} = 160/0.0005 = 320000 \text{ psi.}$$

3.1.15. V_i = 3000 cm^3, V_f = 2750 cm^3, thus, dV/V = (2750 – 3000)/3000 = -250/3000.
p_i = 2 Mpa, p_f = 3 Mpa, thus, Δp = 1 Mpa.

$$E = -\frac{dp}{dV/V} = -1/(-250/3000) = 12 \text{ Mpa.}$$

3.1.16. ρ = 1025 kg/m^3, E = 234 x 10^7 Pa. $\Delta\rho$ = 1.5 kg/m^3.

$$E = \frac{-\Delta p}{\left(\frac{\Delta V}{V}\right)} = \frac{-\Delta p}{\left(\frac{\Delta \rho}{\rho}\right)}$$

$$\text{Hence, } -\Delta p = E\left(\frac{\Delta\rho}{\rho}\right) = 234 \times 10^7 \left(\frac{1.5}{1025}\right) = 3424390 \text{ Pa}$$

Assuming the pressure at the surface to be zero (atmospheric), the pressure at a depth where the density is 1026.5 kg/m^3 = 3424390 Pa ≈ 3424.4 kPa.

3.1.17. dV/V = 1% = -0.01.
At 50 ^oF, E = 305000 psi and at 100 ^oF, E = 327000 psi. Hence,
(i) at 50 ^oF, dp = -E(dV/V) = -305000(-0.01) = 3050 psi.
(ii) at 100 ^oF, dp = -327000(-0.01) = 3270 psi.
Thus, the change in the pressure at the higher temperature is higher.
Percent change in dp = (3270 – 3050)(100)/3050 = 7.2 %.

3.1.18. p_{gage} = 10 kPa, p_{atm} = 101.3 kPa.
$p_{abs} = p_{atm} + p_{gage}$ = 101.3 kPa + 10 kPa = 111.3 kPa.

3.1.19. y = 15 ft, p = 30 psi = 4320 lb/ft^2.
Using p = γy, γ = p/y = 4320/15 = 288 lb/ft^3.
s.g. = γ_{liquid}/γ_w = 288/62.4 = 4.615.

3.1.20. s.g. = 0.8, p_{abs} = 200 kPa, p_{atm} is taken as 101.3 kPa.
$p_{abs} = p_{atm} + \gamma y$; thus, $y = (p_{abs} - p_{atm})/\gamma$ = (200 – 101.3)/(0.8 x 9.81) = 12.58 m.

3.1.21. The pressure is 5 psi vacuum and 14.7 – 5 = 9.7 psia (absolute).

3.1.22. p_{atm} = 14.7 psi.
Since $p_{atm} = \gamma_w y$, 14.7 psi (144 lb/ft^2)/psi = 62.4y_w.
y_w = 2116.8/62.4 = 33.92 ft.
Similarly, 2116.8 = 13.6(62.4)y_m.
y_m = 2116.8/(13.6x62.4) = 2.49 ft = 29.9 in.

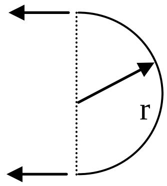
3.1.23. p_{gage} = -10 in mercury = -(10/12)(13.6)(62.4) = -707.2 lb/ft^2 = -4.9 psi.
p_{atm} = 14.3 psia.
p_{abs} = 14.3 – 4.9 = 9.4 psia.

3.1.24. p_{atm} = 100 kPa. T = 10 ^oC, 50 ^oC and 100 ^oC.
γ_w = 9.803 kN/m^3, 9.697 kN/m^3, and 9.438 kN/m^3 at 10 ^oC, 50 ^oC and 100 ^oC, respectively.
Thus, at 10 ^oC, y = 100/9.803 = 10.2 m.
At 50 ^oC, y = 100/9.697 = 10.31 m.
At 100 ^oC, y = 100/9.438 = 10.60 m.

3.1.25. Let the depth of the water behind the dams be H
$F = pA = \gamma y_c A = \gamma(H/2)(H)(1) = \gamma H^2/2$ (considering 1 meter width of the dam)
where y_c is the depth to the centroid of the face of the dam.
(i) For the fresh water,
F = 9.81(1000)H^2/2 = 4905 H^2 N
(ii) For the salty water,
F = 9.81(1030)H^2/2 = 5052.15 H^2 N
Percent change in the pressure force due to salt is = (5052.15H^2 – 4905H^2)(100)/4905H^2 = 3 %.

3.1.26. Take the summation of forces in the vertical direction: $\sum F_y = 0$. Thus,
$F_\sigma - F_w = 0$
$(2\pi r\sigma)\cos\theta - \gamma\pi r^2 h = 0$
$$h = \frac{2\sigma\cos\theta}{\gamma r}$$

3.1.27. Consider the droplet cut into half as shown in the following Figure.
F_σ

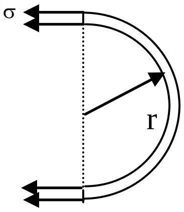


The force due to surface tension, $F_\sigma = 2\pi r\sigma$
The force due to pressure inside the droplet = $\pi r^2 p$
Since $\sum F_x = 0$,
$2\pi r\sigma = \pi r^2 p$

$$p = \frac{2\sigma}{r}$$

Note that the pressure inside the droplet is greater than the pressure outside due to the surface tension.

3.1.28. The surface of the bubble is made of a thin film of liquid of some thickness, the free body diagram of which can be given as follows. Note that the surface tension force acts on both the inside and outside surfaces that make the bubble, as shown.



The force due to surface tension, $F_\sigma = 2(2\pi r\sigma)$
(assuming the r to be the same for both surfaces)
The force due to pressure inside the droplet = $\pi r^2 p$
Since $\sum F_x = 0$,
$2(2\pi r\sigma) = \pi r^2 p$

$$p = \frac{4\sigma}{r}$$

3.1.29. Droplet diameter = 0.5 mm; thus, r = 0.25 mm = 2.5×10^{-4} m. At 25 ^oC, σ = 0.0726 N/m (for water from Table 3.1.2).
$p = 2\sigma/r = 2(0.0726)/2.5 \times 10^{-4} = 580.8$ N/m^2 (Pa) = 0.58 kPa.
In absolute terms, the pressure inside the droplet is 101.3 + 0.58 = 101.88 kPa.

3.1.30. Tube diameter = 0.02 in; thus, r = 0.01 in. At 70 ^oF, σ = 0.005 lb/ft and γ = 62.3 lb/ft^3 (for water from Table 3.1.1).

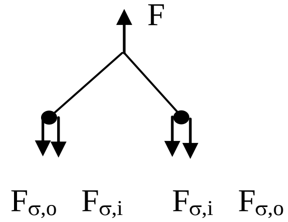
$$h = \frac{2\sigma\cos\theta}{\gamma r} = \frac{2(0.005)(\cos 0)}{62.3(0.01/12)} = 0.0193 \text{ ft} = 2.31 \text{ in.}$$

3.1.31. Tube diameter = 0.03 in; thus, r = 0.015 in. σ_m = 0.03562 lb/ft, s.g = 13.57, γ_w = 62.3 lb/ft^3.

$$h = \frac{2\sigma\cos\theta}{\gamma r} = \frac{2(0.03562)(\cos 140)}{13.57(62.3)(0.015/12)} = -0.052 \text{ ft} = -0.62 \text{ in.}$$

Hence, the depression is about 0.62 inches.

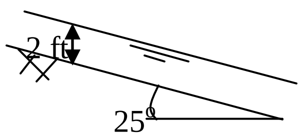
3.1.32. Consider the following free body diagram.
At 10 ^oC, s = 0.0748 (for water from Table 3.1.2).
The ring has inside and outside diameters. However, it is assumed that both are practically equal. Thus, D = 10 mm = 0.01 m.
$F = F_{\sigma,i} + F_{\sigma,o} = 2\sigma\pi D = 2\pi\,(0.0748)(0.01) = 4.7 \times 10^{-3}$ N.



3.1.33. $D = 6$ in = 0.5 ft, $V = 7$ ft/s, $\nu = 0.00444$ ft^2/s.
$R_e = VD/\nu = 7(0.5)/0.00444 = 788$.

3.1.34. $D = 20$ cm = 0.2 m, $Q = 0.02$ m^3/s.
$V = Q/A$; $A = \pi D^2/4 = \pi(0.2)^2/4 = 0.0314$ m^2.
Thus, $V = 0.02/0.0314 = 0.637$ m/s.
At 15 ^{o}C, $\rho = 999.1$ kg/m^3 and $\mu = 1.14 \times 10^{-3}$ Pa.s (for water from Table 3.1.2)
$R_e = VD\rho/\mu = 0.637(0.2)(999.1)/1.14 \times 10^{-3} = 1.12 \times 10^5$
Mass flow rate = $\rho Q = 999.1(0.02) = 19.982$ kg/m^3

3.1.35. $V = 3$ ft/s, vertical depth of flow = 2 ft, bottom slope of channel, $\theta = 25^o$; normal depth of flow = 2 ft (cos25^o) = 1.813 ft. Width of flow = 1 ft. Hence, the area of cross-section of flow, $A = 1.813(1) = 1.813$ ft^2. $Q = VA = 3(1.813) = 5.439$ ft^3/s.



3.1.36. $D = 10$ cm = 0.1 m, mass flow rate, $\dot{m} = 500$ kg/min, at 20 ^{o}C, $\rho = 998.2$ kg/m^3 (for water from Table 3.1.2)
$Q = \dot{m}/\rho = 500/998.2 = 0.501$ m^3/min = 8.35×10^{-3} m^3/s.
$V = Q/A$, $A = \pi D^2/4 = \pi(0.1)^2/4 = 7.85 \times 10^{-3}$ m^2.
$V = 8.35 \times 10^{-3}/7.85 \times 10^{-3} = 1.064$ m/s.

3.1.37. $t = 30$ min = 30(60) = 1800 s, $Q = 1.5$ ft^3/s, at 50 ^{o}F, $\rho = 1.94$ slugs/ft^3 (for water from Table 3.1.1)
$\dot{m} = \rho Q = 1.94(1.5) = 2.91$ slugs/s.
Increase in weight = $\dot{m}gt = 2.91(32.2)(1800) = 168663.6$ lb.

3.1.38. $v = 0.3y$, vertical depth of channel = 2 m, channel width = 4 m.
Since the velocity equation is a linear function of the depth, maximum velocity occurs where y is Maximum, that is, at the surface.
$v_{max} = 0.3(2) = 0.6$ m/s.
At the channel bottom, $v = 0$.
Hence, mean velocity, $v_{mean} = (0 + 0.6)/2 = 0.3$ m.
The discharge can be obtained by direct integration or using the mean velocity. Both give the same result.
$Q = v_{mean}A = 0.3(2)(4) = 2.4$ m^3/s.

3.1.39. D = 10 mm = 0.01 m, at 20 ^oC, $\nu = 1.007 \times 10^{-6}$ m^2/s (for water from Table 3.1.2).
For flow to be laminar, $R_e \le 2000$. To get the maximum discharge for which laminar flow may be expected, R_e may be set to 2000.
Thus, $R_e = V_{max}D/\nu = 2000$.
$V_{max} = 2000\nu/D = 2000(1.007 \times 10^{-6})/0.01 = 0.201$ m/s.
$Q_{max} = V_{max}A$, then $A = \pi D^2/4 = \pi(0.01)^2/4 = 7.85 \times 10^{-5}$ m^2.
$Q_{max} = 0.201(7.85 \times 10^{-5}) = 1.58 \times 10^{-5}$ m^3/s = 0.0158 liters/s.

3.3.1. $D_1 = 8$ in

$$V_1 = Q/A_1 = \frac{Q}{(\pi/4)D_1^2} = \frac{4Q}{\pi D_1^2}; \qquad \text{Similarly, } V_2 = \frac{4Q}{\pi D_2^2}$$

From the requirement that $V_2 \le 2V_1$,

$$\frac{4Q/\pi D_2^2}{4Q/\pi D_1^2} \le 2, \text{ that is, } \frac{D_1^2}{D_2^2} \le 2 \text{ or } D_2 \ge \frac{D_1}{\sqrt{2}} = \frac{8}{\sqrt{2}}$$

$D_2 \ge 5.66$ in. so a pipe of 5 ¾ in diameter is appropriate.

3.4.1. $D_1 = 3$ in = 0.25 ft.

$$\text{Velocity head in the given pipe} = \frac{V_1^2}{2g}, \qquad V_1 = Q/A_1, \qquad A_1 = \frac{\pi D_1^2}{4},$$

$$V_1 = \frac{4Q}{\pi D_1^2}, \text{ thus, } \frac{V_1^2}{2g} = \left(\frac{4Q}{\pi D_1^2}\right)^2 \left(\frac{1}{2g}\right)$$

$$\text{Similarly, velocity head in the second pipe} = \frac{V_2^2}{2g} = \left(\frac{4Q}{\pi D_2^2}\right)^2 \left(\frac{1}{2g}\right)$$

$$\text{Since } \frac{V_2^2}{2g} = 4\frac{V_1^2}{2g}, \left(\frac{4Q}{\pi D_2^2}\right)^2 \left(\frac{1}{2g}\right) = 4\left(\frac{4Q}{\pi D_1^2}\right)^2 \left(\frac{1}{2g}\right)$$

$$\frac{1}{D_2^4} = \frac{4}{D_1^4} \text{ or } D_2 = 0.707D_1 = 0.707(3) = 2.12 \text{ in.}$$

3.5.1. Using Bernoulli's equation:

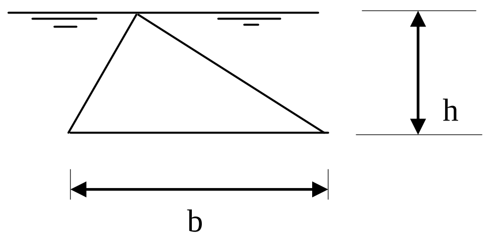
$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + \frac{V_2^2}{2g}, \text{ or } \frac{p_1 - p_2}{\gamma} = \frac{V_2^2 - V_1^2}{2g}$$

$$V_1 = \frac{4Q}{\pi(8/12)^2} = (9/\pi)Q, \; V_2 = \frac{4Q}{\pi(5.75/12)^2} = (9216/529\pi)Q$$

$$\text{Hence, } \frac{p_1 - p_2}{\gamma} = \frac{1}{2(32.2)}\left[\left(\frac{9216}{529\pi}Q\right)^2 - \left(\frac{9}{\pi}Q\right)^2\right]$$

$$\frac{p_1 - p_2}{\gamma} = 0.35Q^2$$

3.6.1. This problem can be solved using either a direct integration approach or using equation (3.6.8). The latter is used here.

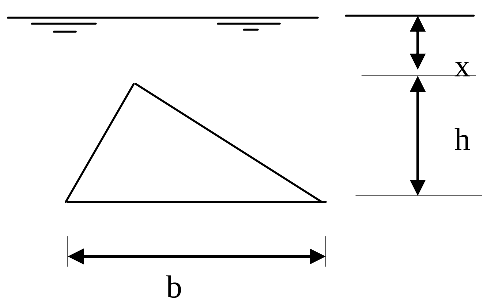


$$y_{cp} = y_c + \frac{\bar{I}}{y_c A}$$

$$y_c = \frac{2}{3}h\,,\ \bar{I} = \frac{bh^3}{36} \quad \text{(for triangular areas)}$$

$$\text{Hence,}\ y_{cp} = \frac{2}{3}h + \frac{bh^3/36}{(2/3)h(bh/2)} = \frac{2}{3}h + \frac{h}{12} = \frac{3}{4}h$$

3.6.2. This problem is solved in a similar way as problem 3.6.1 above.



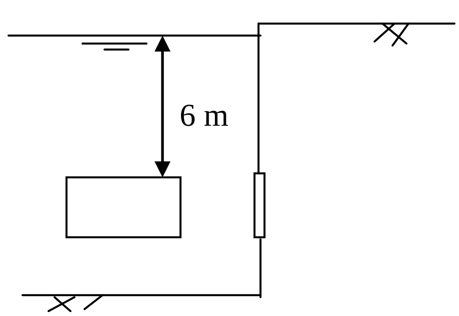
$$y_{cp} = y_c + \frac{\bar{I}}{y_c A}$$

$$y_{cp} = \left(x + \frac{2}{3}h\right) + \frac{bh^3/36}{[x + (2/3)h](bh/2)}$$

$$y_{cp} = \frac{3x + 2h}{3} + \frac{h^2}{6(3x + 2h)}$$

It may be noted that if x = 0 in the above expression, the formula for y_{cp} reduces to the solution obtained for problem 3.6.1.

3.6.3.



$F = \gamma h_c A$, where $h_c = 6 + 2/2 = 7$ m.
$F = 9.79(7)(4 \text{ x } 2) = 548.24$ kN.

$$y_{cp} = y_c + \frac{\bar{I}}{y_c A} = 7 + \frac{4(2)^3/12}{7(4)(2)} = 7.048 \text{ m.}$$

3.6.4. $F_{water} = pA = \gamma_w y_c A = 9.81(4/2)(4)(1)$
$= 78.48$ N (per meter width)
Similarly, the pressure force due to the oil is
$F_{oil} = \gamma_{oil} y_c A = 0.86(9.81)(H/2)(H)(1)$
$= 4.2183\ H^2$ (per meter width)
At equilibrium $F_{water} – F_{oil} = 0$ so $78.48 – 4.2183H^2 = 0$ and solving
H = 4.313 m.
Moment due to $F_{water} = F_{water}$ (1/3)(4) = 78.48(4/3) = 104.64 N.m (for 1 meter width)
Moment due to $F_{oil} = F_{oil}$ (1/3)(4.313) = 78.48(4.313/3) = 112.83 N.m (for 1 meter width)
Thus, the two moments are not equal in magnitude. The reason is that although the pressure forces are equal in magnitude, their moment arms about the base of the plate are different. Net moment due to due to the pressure forces acts on the plate.

3.6.5. Considering the free body diagram shown in the problem,
$F_y = F_z$
F_y = Weight of the wood plus weight of the steel
$= V_{wood}\gamma_{wood} + V_{steel}\gamma_{steel}$
$= V_{wood}(0.8)(9.81) + V_{steel}(7.8)(9.81)$
$= 7.848V_{wood} + 76.518V_{steel}$
(a)
F_z = Weight of the water displaced
$= (V_{wood} + V_{steel})\gamma = 9.81(V_{wood} + V_{steel})$
(b)
Combining (a) and (b):
$7.848V_{wood} + 76.518V_{steel} = 9.81V_{wood} + 9.81V_{steel}$
$66.708V_{steel} = 1.962V_{wood}$
$V_{steel}/V_{wood} = 1/34$

3.6.6. $F = \gamma h_c A$; $h_c = 3 + ½\ (4\cos 60^o) = 4$ m
F = 62.4(4)(4)(6) = 5990.4 lb

$$y_{cp} = y_c + \frac{\bar{I}}{y_c A};\ y_c = \frac{3}{\cos 60} + \frac{4}{2} = 8\,\text{m};\ \bar{I} = \frac{bh^3}{12} = \frac{6(4)^3}{12} = 32\,\text{ft}^4$$

$$y_{cp} = 8 + \frac{32}{8(4)(6)} = 8.17 \text{ m}$$

$h_{cp} = y_{cp}\sin 30^o = 4.08$ m

3.7.1. By definition, $\alpha = \frac{1}{AV^3}\int_A v^3 dA$

For discrete subsections as the ones shown in Figure P3.7.1, this equation can be approximated as $\alpha = \frac{\sum\left(v_i^3 A_i\right)}{V^3 \sum A_i}$

The mean velocity can be calculated as

$$V = \frac{\sum(v_i A_i)}{\sum A_i}$$

Also, $V = \frac{Q_i}{A_i}$

$$\text{Hence, } \alpha = \frac{\sum\left[\left(\frac{Q_i}{A_i}\right)^3 A_i\right]}{\left[\frac{\sum\left(\frac{Q_i}{A_i}\right)A_i}{\sum A_i}\right]^3 \sum A_i} = \frac{\sum\left(\frac{Q_i^3}{A_i^2}\right)\left(\sum A_i\right)^2}{\left(\sum Q_i\right)^3}$$

$$\sum\left(\frac{Q_i^3}{A_i^2}\right) = 2\left(\frac{1073^3}{1050^2}\right) + \frac{8854^3}{3000^2} = 79362.647 \text{ ft}^5/\text{s}^3$$

$$\left(\sum A_i\right)^2 = [2(1050) + 3000]^2 = 2.601 \text{ x } 10^7 \text{ ft}^4$$

$$\left(\sum Q_i\right)^3 = [2(1073) + 8854]^3 = 1.331 \text{ x } 10^{12} \text{ ft}^9/\text{s}^3$$

$$\alpha = \frac{79362.647(2.601 \text{ x } 10^7)}{1.331 \text{ x } 10^{12}} = 1.55$$

3.7.2. By definition, $\beta = \frac{1}{AV^2}\int_A v^2 dA$

In a similar fashion as the solution for problem 3.7.1 above, for discrete subsections,

$$\beta = \frac{\sum v_i^2 A_i}{V^2 \sum A_i}$$

$$\beta = \frac{\sum\left(\frac{Q_i^2}{A_i}\right)\left(\sum A_i\right)}{\left(\sum Q_i\right)^2}$$

where

$$\sum\left(\frac{Q_i^2}{A_i}\right) = 2\left(\frac{1073^2}{1050}\right) + \frac{8854^2}{3000} = 28324.113\,\text{ft}^4/\text{s}^2$$

$$\left(\sum A_i\right) = 2(1050) + 3000 = 5100\,\text{ft}^2$$

$$(\sum Q_i)^2 = [2(1073) + 8854]^2 = 1.21\text{x}10^8\ \text{ft}^6/\text{s}^2$$

$$\beta = 28324(5100)/1.21\text{x}10^8 = 1.19$$