Chapter 2 Review of Basic Algebra

Exercise 2.1

A. 1. $\boxed{19a}$

2. $\boxed{3m}$

3. $\boxed{-a-10}$

4. $\boxed{-3a-14}$

5. $\boxed{-2x-4y}$

6. $\boxed{3p+q}$

7. $\boxed{14f-4v}$

8. $\boxed{2c-3d}$

9. $\boxed{0.8x}$

10. $\boxed{1.06x}$

11. $\boxed{1.4x}$

12. $\boxed{0.98x}$

13. $\boxed{2.79x}$

14. $\boxed{4.05y}$

15. $\boxed{-x^2-x-8}$

16. $\boxed{-ax+x-2}$

17. $2x-3y-x-4y=\boxed{x-7y}$

18. $-4+5a+2-3a=\boxed{2a-2}$

19. $12b+4c+9+8-8b-2c-15=\boxed{4b+2c+2}$

20. $a^2-ab+b^2-3a^2-5ab+4b^2=\boxed{-2a^2-6ab+5b^2}$

21. $-3m^2+4m+5-4+2m+2m^2=\boxed{-m^2+6m+1}$

22. $6-4x+3y-1-5x-2y+9=\boxed{14-9x+y}$

23. $7a-5b+3a-4b-5b=\boxed{10a-14b}$

24. $3f-f^2+fg-f+3f^2+2fg=\boxed{2f+2f^2+3fg}$

B. 1. $\boxed{-12x}$

2. $\boxed{-56a}$

3. $\boxed{-10ax}$

4. $\boxed{27ab}$

5. $\boxed{-2x^2}$

6. $\boxed{24m^2}$

7. $\boxed{60xy}$

8. $\boxed{-24abc}$

9. $\boxed{-2x+4y}$

10. $\boxed{10x-20}$

11. $\boxed{2ax^2-3ax-a}$

12. $\boxed{-24x+12bx+6b^2x}$

13. $20x-24-6+15x=\boxed{35x-30}$

14. $-24a+3b+14a-18b=\boxed{-10a-15b}$

15. $-15ax+3a+5a-2ax-3ax-3a=\boxed{-20ax+5a}$

16. $24y-32-4y+2-1+y=\boxed{21y-31}$

17. $3x^2-x+6x-2=\boxed{3x^2+5x-2}$

18. $5m^2-2mn-15mn+6n^2=\boxed{5m^2-17mn+6n^2}$

19. $x^3-x^2y+xy^2+x^2y-xy^2+y^3=\boxed{x^3+y^3}$

20. $a^3-2a^2+a-a^2+2a-1=\boxed{a^3-3a^2+3a-1}$

21. $10x^2-8x-5x+4-3x^2+21x-5x+35=\boxed{7x^2+3x+39}$

22. $2(2a^2-2a-3a+3)-3(3a^2-2a+3a-2)$

$=4a^2-10a+6-9a^2-3a+6$

$=\boxed{-5a^2-13a+12}$

23. $\boxed{4ab}$

24. $\boxed{-5y}$

25. $\boxed{4x}$

26. $\boxed{-6}$

27. $\boxed{10m-4}$

28. $\boxed{-2x+3}$

29. $\boxed{-2x^2+3x+6}$

30. $\boxed{a^2+4a+3}$

C. 1. $3x-2y-3=3(-4)-2(-5)-3=-12+10-3=\boxed{-5}$

2. $\frac{1}{2}(3x^2-x-1)-\frac{1}{4}(5-2x-x^2)$

$=\frac{1}{2}[3(-3)^2-(-3)-1]-\frac{1}{4}[5-2(-3)-(-3)^2]$

$=\frac{1}{2}(27+3-1)-\frac{1}{4}(5+6-9)$

$=\frac{1}{2}(29)-\frac{1}{4}(2)$

$=14.5-0.5$

$=\boxed{14}$

3. $(pq-vq)-f=(p-v)q-f=(12-7)2000-4500=10\ 000-4500=\boxed{5500}$

4. $F/C=13\ 000/0.65=\boxed{20\ 000}$

5. $(1-d1)(1-d2)(1-d3)=(1-0.35)(1-0.08)(1-0.02)=(0.65)(0.92)(0.98)=\boxed{0.58604}$

6. $C+0.38C=0.24C=(1+0.38+0.24)C=1.62C=1.62(\$25.00)=\boxed{\$40.50}$

7. $\frac{RP(n+1)}{2\mathrm{N}}=\frac{0.21\times\$1200\times(77+1)}{2\times26}=\boxed{\$378}$

8. $\dfrac{\text{I}}{\text{Pt}} = \dfrac{63}{840 \times \frac{219}{365}} = \dfrac{63}{840 \times 0.60} = \boxed{0.125}$

9. $\dfrac{\text{I}}{\text{rt}} = \dfrac{\$198}{0.165 \times \frac{146}{365}} = \dfrac{\$198}{0.165 \times 0.40} = \boxed{\$3000}$

10. $\dfrac{2\text{NC}}{\text{P}(n+1)} = \dfrac{2 \times 52 \times 60}{1800(25+1)} = \dfrac{2 \times 52 \times 60}{1800 \times 26} = \boxed{0.1\dot{3}}$

11. $\text{P}(1+rt) = \$880\left(1 + 0.12 \times \dfrac{76}{365}\right)$

$= \$880(1 + 0.024986) = \$880(1.024986) = \boxed{\$901.99}$

12. $\text{FV}(1-rt) = \$1200\left(1 - 0.175 \times \dfrac{256}{365}\right)$

$= \$1200(1 - 0.122740) = \$1200(0.877260) = \boxed{\$1052.71}$

13. $\dfrac{\text{P}}{1-dt} = \dfrac{\$1253}{1 - 0.135 \times \frac{284}{365}} = \dfrac{\$1253}{1 - 0.083219} = \dfrac{\$1253}{0.916781} = \boxed{\$1400.06}$

14. $\dfrac{\text{S}}{1+rt} = \dfrac{\$1752}{1 + 0.152 \times \frac{228}{365}} = \dfrac{\$1752}{1 + 0.094948} = \dfrac{\$1752}{1.094948} = \boxed{\$1600.08}$

Exercise 2.2

A. 1. $\boxed{81}$

2. $\boxed{1}$

3. $\boxed{16}$

4. $\boxed{1}$

5. $\boxed{\dfrac{16}{81}}$

6. $\boxed{\dfrac{625}{1296}}$

7. $\boxed{-\dfrac{1}{64}}$

8. $\boxed{-\frac{8}{27}}$

9. $\boxed{0.25}$

10. $\boxed{113.379904}$

11. $\boxed{-0.001}$

12. $\boxed{-335.544320}$

13. $\boxed{1}$

14. $\boxed{1}$

15. $\boxed{\frac{1}{9}}$

16. $\boxed{512}$

17. $\boxed{-\frac{1}{125}}$

18. $\boxed{\frac{1}{167.9616}}$

19. $\boxed{125}$

20. $\boxed{\frac{81}{16}}$

21. $\boxed{\frac{1}{1.01}}$

22. $\boxed{1}$

B. 1. $2^5 \times 2^3 = 2^{5+3} = \boxed{2^8}$

2. $(-4)^3 \times (-4) = (-4)^{3+1} = \boxed{(-4)^4}$

3. $4^7 \div 4^4 = 4^{7-4} = \boxed{4^3}$

4. $(-3)^9 \div (-3)^7 = (-3)^{9-7} = \boxed{(-3)^2}$

5. $(2^3)^5 = 2^{3\times 5} = \boxed{2^{15}}$

6. $\left[(-4)^3\right]^6 = (-4)^{3\times 6} = \boxed{(-4)^{18}}$

7. $a^4 \times a^{10} = a^{4+10} = \boxed{a^{14}}$

8. $m^{12} \div m^7 = m^{12-7} = \boxed{m^5}$

9. $3^4 \times 3^6 \times 3 = 3^{4+6+1} = \boxed{3^{11}}$

10. $(-1)^3(-1)^7(-1)^5 = (-1)^{3+7+5} = \boxed{(-1)^{15}}$

11. $\dfrac{6^7 \times 6^3}{6^9} = 6^{7+3-9} = \boxed{6}$

12. $\dfrac{(x^4)(x^5)}{x^7} = x^{4+5-7} = \boxed{x^2}$

13. $\left(\dfrac{3}{5}\right)^4\left(\dfrac{3}{5}\right)^7 = \left(\dfrac{3}{5}\right)^{4+7} = \boxed{\dfrac{3^{11}}{5^{11}}}$

14. $\left(\dfrac{1}{6}\right)^5 \div \left(\dfrac{1}{6}\right)^3 = \left(\dfrac{1}{6}\right)^{5-3} = \boxed{\dfrac{1}{6^2}}$

15. $\left(-\dfrac{3}{2}\right)\left(-\dfrac{3}{2}\right)^6\left(-\dfrac{3}{2}\right)^4 = \left(-\dfrac{3}{2}\right)^{1+6+4} = \boxed{\dfrac{(-3)^{11}}{2^{11}}}$

16. $\left(-\dfrac{3}{4}\right)^8 \div \left(-\dfrac{3}{4}\right)^7 = \left(-\dfrac{3}{4}\right)^{8-7} = \boxed{-\dfrac{3}{4}}$

17. $(1.025)^{80}(1.025)^{70} = (1.025)^{80+70} = \boxed{1.025^{150}}$

18. $1.005^{240} \div 1.005^{150} = 1.005^{240-150} = \boxed{1.005^{90}}$

19. $\left[1.04^{20}\right]^4 = 1.04^{20\times 4} = \boxed{1.04^{80}}$

20. $\left[\left(-\dfrac{3}{7}\right)^5\right]^3 = \left(-\dfrac{3}{7}\right)^{5\times 3} = \boxed{\dfrac{-3^{15}}{7^{15}}}$

21. $(1+i)^{100}(1+i)^{100} = (1+i)^{100+100} = \boxed{(1+i)^{200}}$

22. $(1-r)^2(1-r)^2(1-r)^2 = (1-r)^{2+2+2} = \boxed{(1-r)^6}$

23. $\left[(1+i)^{80}\right]^2 = (1+i)^{80\times 2} = \boxed{(1+i)^{160}}$

24. $\left[(1-r)^{40}\right]^3 = (1-r)^{40\times 3} = \boxed{(1-r)^{120}}$

25. $(ab)^5 = \boxed{a^5b^5}$

26. $(2xy)^4 = \boxed{16x^4y^4}$

27. $(m^3n)^8 = \boxed{m^{24}n^8}$

28. $\left(\frac{a^3b^2}{x}\right)^4 = \boxed{\frac{a^{12}b^8}{x^4}}$

29. $2^3 \times 2^5 \times 2^{-4} = 2^{3+5-4} = \boxed{2^4}$

30. $5^2 \div 5^{-3} = 5^{2-(-3)} = \boxed{5^5}$

31. $\left(\frac{a}{b}\right)^{-8} = \boxed{\frac{b^8}{a^8}}$

32. $\left(\frac{1+i}{i}\right)^{-n} = \boxed{\frac{i^n}{(1+i)^n}}$

***Exercise* 2.3**

A. 1. $\sqrt{5184} = \boxed{72.0000}$

2. $\sqrt{205.9225} = \boxed{14.3500}$

3. $\sqrt[7]{2187} = \boxed{3.0000}$

4. $\sqrt[10]{1.1046221} = \boxed{1.0100}$

5. $\sqrt[20]{4.3184} = 1.075886 = \boxed{1.0759}$

6. $\sqrt[16]{0.00001526} = 0.500002 = \boxed{0.5000}$

7. $\sqrt[6]{1.0825} = \boxed{1.0133}$

8. $\sqrt[12]{1.15} = 1.011715 = \boxed{1.0117}$

B. 1. $3025^{\frac{1}{2}} = \boxed{55}$

2. $2401^{\frac{1}{4}} = \boxed{7}$

3. $525.21875^{\frac{2}{5}} = \boxed{12.25}$

4. $21.6^{\frac{4}{3}} = \boxed{60.154991}$

5. $\sqrt[12]{1.125^7} = \boxed{1.071122}$

6. $\sqrt[6]{1.095} = \boxed{1.015241}$

7. $4^{\left(-\frac{1}{3}\right)} = \dfrac{1}{4^{\frac{1}{3}}} = \dfrac{1}{1.587401} = \boxed{0.629961}$

8. $1.06^{\left(-\frac{1}{12}\right)} = \dfrac{1}{1.06^{\frac{1}{12}}} = \dfrac{1}{1.004868} = \boxed{0.995156}$

9. $\dfrac{1.03^{60}-1}{0.03} = \dfrac{5.891603-1}{0.03} = \boxed{163.053437}$

10. $\dfrac{1-1.05^{-36}}{0.05} = \dfrac{1-0.172657}{0.05} = \boxed{16.546852}$

11. $\boxed{2.158925}$

12. $\boxed{0.589664}$

13. $26.50(1.043)\left(\dfrac{3.536138-1}{0.043}\right) = 26.50(1.043)(58.979962) = \boxed{1630.176673}$

14. $350.00(1.05)\left(\dfrac{2.653298-1}{0.05}\right) = 350.00(1.05)(33.065954) = \boxed{12\,151.73813}$

15. $133.00\left(\dfrac{1-0.520035}{0.056}\right) = 133.00\,(8.570795) = \boxed{1139.915716}$

16. $270.00\left(\dfrac{1-0.759412}{0.035}\right) = 270.00\,(6.873956) = \boxed{1855.967995}$

17. $5000.00\,(0.581251) + 137.50\left(\dfrac{1-0.581251}{0.0275}\right)$

$= 2906.252832 + 137.50\,(15.227252) = 2906.252832 + 2093.747168 = \boxed{5000.00}$

18. $1000.00\,(0.623167) + 300.00\left(\dfrac{1-0.623167}{0.03}\right)$

$= 623.166939 + 300.00\,(12.561102) = 623.166939 + 3768.330608 = \boxed{4391.497547}$

19. $112.55 = 100.00(1+i)^4$

$(1+i)^4 = 1.1255$

$(1+i) = 1.1255^{0.25}$

$(1+i) = 1.029998$

$i = \boxed{0.029998}$

20. $380.47 = 300.00(1+i)^{12}$

$(1+i)^{12} = 1.268233$

$(1+i) = 1.268233^{0.08\dot{3}}$

$(1+i) = 1.019999$

$i = \boxed{0.019999}$

21. $3036.77 = 2400.00(1+i)^6$

$(1+i)^6 = 1.265321$

$(1+i) = 1.265321^{0.16}$

$(1+i) = 1.04$

$i = \boxed{0.04}$

22. $1453.36 = 800.00(1+i)^{60}$

$(1+i)^{60} = 1.8167$

$(1+i) = 1.8167^{0.01\dot{6}}$

$(1+i) = 1.01$

$i = \boxed{0.01}$

***Exercise* 2.4**

A. 1. $2^9 = 512$

$\boxed{9 = \log_2 512}$

2. $3^7 = 2187$

$\boxed{7 = \log_3 2187}$

3. $5^{-3} = \dfrac{1}{125}$

   $\boxed{-3 = \log_5 \dfrac{1}{125}}$

4. $10^{-5} = 0.00001$

   $\boxed{-5 = \log_{10} 0.00001}$

5. $e^{2j} = 18$

   $2j = \log_e 18$

   or $\boxed{2j = \ln 18}$

6. $e^{-3x} = 12$

   $-3x = \log_e 12$

   or $\boxed{-3x = \ln 12}$

B. 1. $\log_2 32 = 5$

   $\boxed{2^5 = 32}$

2. $\log_3 \dfrac{1}{81} = -4$

   $\boxed{3^{-4} = \dfrac{1}{81}}$

3. $\log_{10} 10 = 1$

   $\boxed{10^1 = 10}$

4. $\ln e^2 = 2$

   $\boxed{e^2 = e^2}$

C. 1. $\ln 2 = \boxed{0.693147}$

2. $\ln 200 = \boxed{5.298317}$

3. $\ln 0.105 = \boxed{-2.253795}$

4. $\ln\left[300(1.10^{15})\right] = \ln 300 + \ln 1.10^{15}$

$= \ln 300 + 15(\ln 1.10)$

$= 5.703782 + 15(0.095310)$

$= 5.703782 + 1.429653$

$= \boxed{7.133435}$

5. $\ln\left[\dfrac{2000}{1.09^{9}}\right] = \ln 2000 - \ln 1.09^{9}$

$= \ln 2000 - 9(\ln 1.09)$

$= 7.600902 - 9(0.086178)$

$= 7.600902 - 0.775599$

$= \boxed{6.825303}$

6. $\ln 850\left[\dfrac{1.01^{-120}}{0.01}\right] = \ln 850 + \ln 1.01^{-120} - \ln 0.01$

$= \ln 850 - 120(\ln 1.01) - \ln 0.01$

$= 6.745236 - 120(0.009950) - (-4.605170)$

$= 6.745236 - 1.194040 + 4.605170$

$= \boxed{10.156367}$

Business Math News Box

1. The IOC distributes 90% of broadcast revenue to other organizations in the Olympic Movement. It retains 10% to cover operational and administrative costs.

   IOC retains 0.10(1 339 000 000) = $\boxed{\$133\ 900\ 000\ (C\$133.9\ million)}$

2. IOC distribution (C$ millions) = $\boxed{C\$133.9 \times 10^{6}}$

3. Estimate for total Winter Games revenue = Revenue from broadcast partnerships / 0.50 = 1 339 000 000 / 0.50 = $2 678 000 000

   Answer expressed in exponent form = $\boxed{C\$2678 \times 10^{6}\ million}$

4. Annual growth rate in broadcast revenue since Lake Placid = $(1.339\ billion/21.7\ million)^{1/30} - 1 = 0.147301 =$ $\boxed{14.73\%\ per\ year}$

Exercise 2.5

A. 1. $15x = 45$

$\boxed{x = 3}$

2. $-7x = 35$

$\boxed{x = -5}$

3. $0.9x = 72$

$\boxed{x = 80}$

4. $0.02x = 13$

$\boxed{x = 650}$

5. $\frac{1}{6}x = 3$

$\boxed{x = 18}$

6. $-\frac{1}{8}x = 7$

$\boxed{x = -56}$

7. $\frac{3}{5}x = -21$

$\frac{1}{5}x = -7$

$\boxed{x = -35}$

8. $-\frac{4}{3}x = -32$

$\frac{1}{3}x = 8$

$\boxed{x = 24}$

9. $x - 3 = -7$

$\boxed{x = -4}$

10. $-2x = 7 - 3x$

$\boxed{x = 7}$

11. $x+6=-2$

$\boxed{x=-8}$

12. $3x=9+2x$

$\boxed{x=9}$

13. $4-x=9-2x$

$\boxed{x=5}$

14. $2x+7=x-5$

$\boxed{x=-12}$

15. $x+0.6x=32$

$1.6x=32$

$\boxed{x=20}$

16. $x-0.3x=210$

$0.7x=210$

$\boxed{x=300}$

17. $x-0.04x=192$

$0.96x=192$

$\boxed{x=200}$

18. $x+0.07x=64.20$

$1.07x=64.20$

$\boxed{x=60}$

B. 1. $3x+5=7x-11$

$-4x=-16$

$\boxed{x=4}$

LS: $3x+5 = 3(4)+5$

$= 12+5$

$= 17$

RS: $7x-11 = 7(4)-11$

$= 28-11$

$= 17$

2. $5-4x=-4-x$

$-3x=-9$

$\boxed{x=3}$

LS: $5-4x=5-(4)(3)$

$=5-12$

$=-7$

RS: $=-4-x$

$=-4-3$

$=-7$

3. $2-3x-9=2x-7+3x$

$-3x-7=5x-7$

$-8x=0$

$\boxed{x=0}$

LS: $=2-3x-9$

$=2-3(0)-9$

$=-7$

RS: $=2x-7+3x$

$=2(0)-7+3(0)$

$=-7$

4. $4x-8-9x=10+2x-4$

$-5x-8=6+2x$

$-7x=14$

$\boxed{x=-2}$

LS: $=4x-8-9x$

$=4(-2)-8-9(-2)$

$=-8-8+18=2$

RS: $=10+2x-4$

$=10+2(-2)-4$

$=10-4-4=2$

5. $3x+14=4x+9$

$-x=-5$

$\boxed{x=5}$

LS: $=3x+14$

$=3(5)+14$

$=15+14=29$

RS: $=4x+9$

$=4(5)+9$

$=20+9=29$

6. $16x-12=6x-32$

$10x=-20$

$\boxed{x=-2}$

LS: $=16x-12$

$=16(-2)-12$

$=-32-12=-44$

RS: $=6x-32$

$=6(-2)-32$

$=-12-32=-44$

7. $5+3+4x=5x+12-25$

$+4x-5x=+12-25-5-3$

$-x=-21$

$\boxed{x=21}$

LS: $=5+3+4x$

$=8+4(21)$

$=8+84=92$

RS: $=5x+12-25$

$=5(21)-13$

$=105-13=92$

8. $-3+2x+5=5x-36+14$

$+2x-5x=-36+14+3-5$

$-3x=-24$

$\boxed{x=8}$

LS: $=-3+2x+5$

$=-3+2(8)+5$

$=-3+16+5$

$=18$

RS: $=5x-36+14$

$=5(8)-36+14$

$=40-36+14=18$

***Exercise* 2.6**

A. 1. $12x-4(9x-20)=320$

$12x-36x+80=320$

$-24x=240$

$\boxed{x=-10}$

LS $=12(-10)-4[9(-10)-20]$

$=-120-4[-90-20]$

$=-120+440$

$=320$

RS $=320$

2. $5(x-4)-3(2-3x)=-54$

$5x-20-6+9x=-54$

$14x-26=-54$

$14x=-28$

$\boxed{x=-2}$

LS $=5[-2-4]-3[2-3(-2)]$

$=5(-6)-3(2+6)$

$=-30-24$

$=-54$

RS $=-54$

3. $3(2x-5)-2(2x-3)=-15$

$$6x-15-4x+6=-15$$
$$2x-9=-15$$
$$2x=-6$$
$$\boxed{x=-3}$$

$$\begin{aligned}\text{LS} &= 3[2(-3)-5]-2[2(-3)-3]\\ &= 3[-65]-2[-6-3]\\ &= 3(-11)-2(-9)\\ &= -33+18\\ &= -15\end{aligned}$$

$$\text{RS} = -15$$

4. $17-3(2x-7)=7x-3(2x-1)$

$$17-6x+21=7x-6x+3$$
$$-6x+38=x+3$$
$$-7x=-35$$
$$\boxed{x=5}$$

$$\begin{aligned}\text{LS} &= 17-3[2(5)-7]\\ &= 17-3[10-7]\\ &= 17-9=8\end{aligned}$$

$$\begin{aligned}\text{RS} &= 7(5)-3[2(5)-1]\\ &= 35-3[10-1]\\ &= 35-27=8\end{aligned}$$

5. $4x+2(2x-3)=18$

$$4x+4x-6=18$$
$$8x=24$$
$$\boxed{x=3}$$

$$\begin{aligned}\text{LS} &= 4(3)+2[2(3)-3]\\ &= 12+2[6-3]\\ &= 12+6\\ &= 18\end{aligned}$$

$$\text{RS} = 18$$

6. $-3(1-11x)+(8x-15)=187$

$$-3+33x+8x-15=187$$
$$33x+8x=187+3+15$$
$$41x=205$$
$$\boxed{x=5}$$

$$\text{LS}=-3[(1-11(5)]+[8(5)-15]$$
$$=-3[-54]+25$$
$$=162+25$$
$$=187$$
$$\text{RS}=187$$

7. $10x-4(2x-1)=32$

$$10x-8x+4=32$$
$$2x=28$$
$$\boxed{x=14}$$

$$\text{LS}=10(14)-4[2(14)-1]$$
$$=140-4[27]$$
$$=140-108$$
$$=32$$
$$\text{RS}=32$$

8. $-2(x-4)+12(3-2x)=-8$

$$-2x+8+36-24x=-8$$
$$-2x-24x=-8-8-36$$
$$-26x=-52$$
$$\boxed{x=2}$$

$$\text{LS}=-2(2-4)+12[3-2(2)]$$
$$=-4+8+36-48$$
$$=-8$$
$$\text{RS}=-8$$

B. 1. $x-\frac{1}{4}x=15$

$$4x-x=60$$
$$3x=60$$
$$\boxed{x=20}$$

2. $x+\frac{5}{8}x=26$

$8x+5x=208$

$13x=208$

$\boxed{x=16}$

3. $\frac{2}{3}x-\frac{1}{4}=-\frac{7}{4}-\frac{5}{6}x$

$8x-3=-21-10x$

$18x=-18$

$\boxed{x=-1}$

4. $\frac{5}{3}-\frac{2}{5}x=\frac{1}{6}x-\frac{1}{30}$

$50-12x=5x-1$

$-17x=-51$

$\boxed{x=3}$

5. $\frac{3}{4}x+4=\frac{113}{24}-\frac{2}{3}x$

$18x+96=113-16x$

$34x=17$

$\boxed{x=\frac{1}{2}}$

6. $2-\frac{3}{2}x=\frac{2}{3}x+\frac{31}{9}$

$36-27x=12x+62$

$-39x=26$

$\boxed{x=-\frac{2}{3}}$

C. 1. $\frac{3}{4}(2x-1)-\frac{1}{3}(5-2x)=-\frac{55}{12}$

$$9(2x-1)-4(5-2x)=-55$$
$$18x-9-20+8x=-55$$
$$26x-29=-55$$
$$26x=-26$$
$$\boxed{x=-1}$$

2. $\frac{4}{5}(4-3x)+\frac{53}{40}=\frac{3}{10}x-\frac{7}{8}(2x-3)$

$$32(4-3x)+53=12x-35(2x-3)$$
$$128-96x+53=12x-70x+105$$
$$-96x+181=-58x+105$$
$$-38x=-76$$
$$\boxed{x=2}$$

3. $\frac{2}{3}(2x-1)-\frac{3}{4}(3-2x)=2x-\frac{20}{9}$

$$24(2x-1)-27(3-2x)=72x-80$$
$$48x-24-81+54x=72x-80$$
$$102x-105=72x-80$$
$$30x=25$$
$$\boxed{x=\frac{5}{6}}$$

4. $\frac{4}{3}(3x-2)-\frac{3}{5}(4x-3)=\frac{11}{60}+3x$

$$80(3x-2)-36(4x-3)=11+180x$$
$$240x-160-144x+108=11+180x$$
$$96x-52=11+180x$$
$$-84x=63$$
$$\boxed{x=-\frac{3}{4}}$$

D. 1. $y=mx+b$

$$y-b=mx$$
$$\boxed{x=\frac{y-b}{m}}$$

2. $r = \dfrac{M}{S}$

$Sr = M$

$\boxed{S = \dfrac{M}{r}}$

3. $\text{PV} = \dfrac{\text{PMT}}{i}$

$\boxed{\text{PMT} = \text{PV}i}$

4. $\text{I} = \text{P}\,rt$

$\boxed{t = \dfrac{\text{I}}{\text{P}\,r}}$

5. $\text{S} = \text{P}(1 + rt)$

$\dfrac{\text{S}}{\text{P}} = 1 + rt$

$\dfrac{\text{S}}{\text{P}} - 1 = rt$

$r = \dfrac{\dfrac{\text{S}}{\text{P}} - 1}{t}$

$r = \dfrac{\dfrac{\text{S} - \text{P}}{\text{P}}}{t}$

$\boxed{r = \dfrac{\text{S} - \text{P}}{\text{P}t}}$

6. $\text{PV} = \text{FV}(1 + i)^{-n}$

$\dfrac{\text{PV}}{\text{FV}} = (1 + i)^{-n}$

$\left[\dfrac{\text{PV}}{\text{FV}}\right]^{-\frac{1}{n}} = 1 + i$

$\left[\dfrac{\text{FV}}{\text{PV}}\right]^{\frac{1}{n}} = 1 + i$

$\boxed{i = \left[\dfrac{\text{FV}}{\text{PV}}\right]^{\frac{1}{n}} - 1}$

***Exercise* 2.7**

A. 1. Let the cost be $\$x$.

$$\text{Selling price} = \$\left(x + \frac{3}{4}x\right)$$

$$\therefore x + \frac{3}{4}x = 49.49$$
$$4x + 3x = 197.96$$
$$7x = 197.96$$
$$x = 28.28$$

The cost was $\boxed{\$28.28.}$

2. Let the regular selling price be $\$x$.

$$\text{Sale price} = \$\left(x - \frac{1}{3}x\right)$$

$$\therefore x - \frac{1}{3}x = 576$$
$$3x - x = 1728$$
$$2x = 1728$$
$$x = 864$$

The regular selling price was $\boxed{\$864.}$

3. Let the price be $\$x$.

$$\text{Total} = \$x + 0.05x$$
$$\therefore x + 0.05x = \$36.75$$
$$1.05x = \$36.75$$
$$x = 35.00$$

The price was $\boxed{\$35.00.}$

4. Let the regular price be $\$x$.

$$\text{Sale price} = \$(x - 0.40x)$$
$$\therefore x - 0.40x = 11.34$$
$$0.60x = 11.34$$
$$x = 18.90$$

The regular selling price was $\boxed{\$18.90.}$

5. Let the last month's index be x.

This month's index $= x - \frac{1}{12}x$

$$\therefore x - \frac{1}{12}x = 176$$
$$12x - x = 2112$$
$$11x = 2112$$
$$x = 192$$

Last month the index was $\boxed{192.}$

6. Let the original hourly wage be \$$x$.

New hourly wage $= \$\left(x + \frac{1}{8}x\right)$

$$\therefore x + \frac{1}{8}x = 12.78$$
$$8x + x = 102.24$$
$$9x = 102.24$$
$$x = 11.36$$

The hourly wage before the increase was $\boxed{\$11.36.}$

7. Let Vera's sales be \$$x$.

Tai's sales $= \$(3x - 140)$

Total sales $= \$(x + 3x - 140)$

$$\therefore x + 3x - 140 = 940$$
$$4x = 1080$$
$$x = 270$$

Tai's sales $= 3(270) - 140 = \boxed{\$670.}$

8. Let the shorter piece be x cm.

Length of longer piece $= (2x + 15)$ cm.

Total length $= (x + 2x + 15)$ cm

$$\therefore x + 2x + 15 = 90$$
$$3x = 75$$
$$x = 25$$

The longer piece is 2(25) cm + 15 cm $= \boxed{65 \text{ cm.}}$

9. Let the cost of a ticket be $\$x$.

Total $= \$(x+5.00)\times 1.05\times 2$

$$\therefore (x+5.00)\times 1.05\times 2 = 197.40$$
$$(x+5.00)\times 2.10 = 197.40$$
$$(x+5.00) = 94.00$$
$$x = 89.00$$

The cost per ticket is $\boxed{\$89.00.}$

10. Let Ken's investment be $\$x$.

Martina's investment $= \$\left(\frac{2}{3}x+2500\right)$

Total investment $= \$\left(x+\frac{2}{3}x+2500\right)$

$$\therefore x+\frac{2}{3}x+2500 = 55\,000$$
$$\frac{5x}{3} = 52\,500$$
$$x = 31\,500$$

Martina's investment is $\frac{2}{3}\times 31\,500+2500 = \boxed{\$23\,500.}$

11. Let the number of chairs produced by the first shift be x.

Number of chairs produced by the second shift $= \frac{4}{3}x-60.$

Total production $= x+\frac{4}{3}x-60 = 2320.$

$$\therefore x+\frac{4}{3}x-60 = 2320$$
$$\frac{7}{3}x = 2380$$
$$x = 1020$$

Production by the second shift is $\frac{4}{3}\times 1020-60 = \boxed{1300.}$

12. Let the number of type A lights be x.

Number of type B lights $= 60-x.$

Value of type A lights $= \$40x.$

Value of type B lights $= \$(60-x)50.$

$$\therefore 40x + 50(60-x) = 2580$$
$$40x + 3000 - 50x = 2580$$
$$-10x = -420$$
$$x = 42$$

The number of type B lights is $\boxed{18.}$

13. Let the number of units of Product A be x;

then the number of units of Product B is $60-x$.

The number of hours for Product A is $4x$;

The number of hours for Product B is $3(60-x)$.

$$\therefore 4x + 3(60-x) = 200$$
$$4x + 180 - 3x = 200$$
$$x = 20$$

Production of Product A is $\boxed{20 \text{ units.}}$

14. Let the number of dimes be x.

Number of nickels $= 3x - 4$

Number of quarters $= \frac{3}{4}x + 1$

Value of the dimes $= 10x$ cents

Value of nickels $= 5(3x-4)$ cents

Value of quarters $= 25\left(\frac{3}{4}x + 1\right)$ cents

$$\therefore 10x + 5(3x-4) + 25\left(\frac{3}{4}x + 1\right) = 880$$
$$10x + 15x - 20 + \frac{75}{4}x + 25 = 880$$
$$25x + \frac{75}{4}x = 875$$
$$175x = 3500$$
$$x = 20$$

Alick has $\boxed{20 \text{ dimes, } 56 \text{ nickels, and } 16 \text{ quarters.}}$

15. Let the number of \$12 tickets be x.

Number of \$8 tickets $= 3x + 10$

Number of \$15 tickets $= \frac{4}{5}x - 3$

Value of the \$12 tickets $= \$12x$

Value of the \$8 tickets $= \$8(3x + 10)$

Value of the \$15 tickets $= \$15\left(\frac{4}{5}x - 3\right)$

$$\therefore 12x + 8(3x + 10) + 15\left(\frac{4}{5}x - 3\right) = 1475$$

$$12x + 24x + 80 + 12x - 45 = 1475$$

$$48x = 1440$$

$$x = 30$$

Sales were 30 \$12 tickets, 100 \$ 8 tickets, and 21 \$15 tickets.

16. Let the number of medium pizzas be x.

Number of large pizzas $= 3x - 1$

Number of small pizzas $= 2x + 1$

Value of medium pizzas $= \$15x$

Value of large pizzas $= \$18(3x - 1)$

Value of small pizzas $= \$11(2x + 1)$

$$\therefore 15x + 18(3x - 1) + 11(2x + 1) = 539$$

$$15x + 54x - 18 + 22x + 11 = 539$$

$$91x = 546$$

$$x = 6$$

Sales were 6 medium pizzas, 17 large pizzas, and 13 small pizzas.

Review Exercise

1. (a) $3x-4y-3y-5x=\boxed{-2x-7y}$

   (b) $2x-0.03x=\boxed{1.97x}$

   (c) $(5a-4)-(3-a)=5a-4-3+a=\boxed{6a-7}$

   (d) $-(2x-3y)-(-4x+y)+(y-x)=-2x+3y+4x-y+y-x=\boxed{x+3y}$

   (e) $(5a^2-2b-c)-(3c+2b-4a^2)$

   $=5a^2-2b-c-3c-2b+4a^2=\boxed{9a^2-4b-4c}$

   (f) $-(2x-3)-(x^2-5x+2)=-2x+3-x^2+5x-2=\boxed{-x^2+3x+1}$

2. (a) $3(-5a)=\boxed{-15a}$

   (b) $-7m(-4x)=\boxed{28mx}$

   (c) $14m\div(-2m)=\boxed{-7}$

   (d) $(-15a^2b)\div(5a)=\boxed{-3ab}$

   (e) $-6(-3x)(2y)=\boxed{36xy}$

   (f) $4(-3a)(b)(-2c)=\boxed{24abc}$

   (g) $-4(3x-5y-1)=\boxed{-12x+20y+4}$

   (h) $x(1-2x-x^2)=\boxed{x-2x^2-x^3}$

   (i) $(24x-16)\div(-4)=\boxed{-6x+4}$

   (j) $(21a^2-12a)\div 3a=\boxed{7a-4}$

   (k) $4(2a-5)-3(3-6a)$

   $=8a-20-9+18a$

   $=\boxed{26a-29}$

   (l) $2a(x-a)-a(3x+2)-3a(-5x-4)$

   $=2ax-2a^2-3ax-2a+15ax+12a$

   $=\boxed{14ax-2a^2+10a}$

(m) $(m-1)(2m-5)$

$= 2m^2 - 2m - 5m + 5$

$= \boxed{2m^2 - 7m + 5}$

(n) $(3a-2)(a^2-2a-3)$

$= 3a^3 - 2a^2 - 6a^2 + 4a - 9a + 6$

$= \boxed{3a^3 - 8a^2 - 5a + 6}$

(o) $3(2x-4)(x-1) - 4(x-3)(5x+2)$

$= 3(2x^2 - 4x - 2x + 4) - 4(5x^2 - 15x + 2x - 6)$

$= 6x^2 - 18x + 12 - 20x^2 + 52x + 24$

$= \boxed{-14x^2 + 34x + 36}$

(p) $-2a(3m-1)(m-4) - 5a(2m+3)(2m-3)$

$= -2a(3m^2 - m - 12m + 4) - 5a(4m^2 + 6m - 6m - 9)$

$= -6am^2 + 26am - 8a - 20am^2 + 45a$

$= \boxed{-26am^2 + 26am + 37a}$

3. (a) for $x = -2,\ y = 5,$

$3xy - 4x - 5y = 3(-2)(5) - 4(-2) - 5(5) = -30 + 8 - 25 = \boxed{-47}$

(b) for $a = -\frac{1}{4}, b = \frac{2}{3},$

$-5(2a - 3b) - 2(a + 5b)$

$= -10a + 15b - 2a - 10b$

$= -12a + 5b$

$= -12\left(-\frac{1}{4}\right) + 5\left(\frac{2}{3}\right) = 3 + 3\frac{1}{3} = \boxed{6\frac{1}{3}}$

(c) for $\mathrm{N} = 12, \mathrm{C} = 432, \mathrm{P} = 1800, n = 35,$

$$\frac{2\mathrm{NC}}{\mathrm{P}(n+1)} = \frac{\overset{1}{2} \times 12 \times \overset{48}{432}}{\underset{100}{\overset{900}{1800}} \times (35+1)} = \frac{\overset{1}{12} \times \overset{16}{48}}{100 \times \underset{1}{\overset{3}{36}}} = \frac{16}{100} = \boxed{0.16}$$

(d) for $\mathrm{I} = 600, r = 0.15, \mathrm{P} = 7300,$

$$\frac{365\,\mathrm{I}}{r\mathrm{P}} = \frac{\overset{1}{365}\times\overset{2}{\overset{30}{600}}}{\underset{1}{\overset{15}{0.15}}\times\underset{0.01}{7300}} = \frac{2}{0.01} = \boxed{200}$$

(e) for $\mathrm{A} = \$720, \mathrm{d} = 0.135, \mathrm{t} = \frac{280}{365},$

$$\mathrm{A}(1-dt) = \$720\left(1-0.135\times\frac{280}{365}\right) = \$720(1-0.103562) = 645.435616 = \boxed{\$645.44}$$

(f) for $\mathrm{S} = 2755, r = 0.17, t = \frac{219}{365},$

$$\frac{\mathrm{S}}{1+rt} = \frac{2755}{1+0.17\times\frac{219}{365}} = \frac{2755}{1+0.034\times 3} = \frac{2755}{1+0.102} = \boxed{2500}$$

4. (a) $(-3)^5 = \boxed{-243}$

(b) $\left(\frac{2}{3}\right)^4 = \boxed{\frac{16}{81}}$

(c) $(-5)^0 = \boxed{1}$

(d) $(-3)^{-1} = \boxed{-\frac{1}{3}}$

(e) $\left(\frac{2}{5}\right)^{-4} = \left(\frac{5}{2}\right)^4 = \boxed{\frac{625}{16}}$

(f) $(1.01)^0 = \boxed{1}$

(g) $(-3)^5(-3)^4 = (-3)^9 = \boxed{-19\,683}$

(h) $4^7 \div 4^2 = 4^5 = \boxed{1024}$

(i) $\left[(-3)^2\right]^5 = (-3)^{10} = \boxed{59\,049}$

(j) $(m^3)^4 = \boxed{m^{12}}$

(k) $\left(\frac{2}{3}\right)^3\left(\frac{2}{3}\right)^7\left(\frac{2}{3}\right)^{-6} = \left(\frac{2}{3}\right)^4 = \boxed{\frac{16}{81}}$

(l) $\left(-\frac{5}{4}\right)^5 \div \left(-\frac{5}{4}\right)^3 = \left(-\frac{5}{4}\right)^2 = \boxed{\frac{25}{16}}$

(m) $(1.03^{50})(1.03^{100}) = \boxed{1.03^{150}}$

(n) $(1+i)^{180} \div (1+i)^{100} = \boxed{(1+i)^{80}}$

(o) $\left[(1.05)^{30}\right]^5 = \boxed{1.05^{150}}$

(p) $(-2xy)^4 = \boxed{16x^4y^4}$

(q) $\left(\frac{a^2b}{3}\right)^{-4} = \left(\frac{3}{a^2b}\right)^4 = \boxed{\frac{81}{a^8b^4}}$

(r) $(1+i)^{-n} = \boxed{\frac{1}{(1+i)^n}}$

5. (a) $\sqrt{0.9216} = \boxed{0.96}$

(b) $\sqrt[6]{1.075} = \boxed{1.012126}$

(c) $14.974458^{1/40} = \boxed{1.07}$

(d) $1.08^{-5/12} = \frac{1}{1.08^{5/12}} = \boxed{0.968442}$

(e) $\ln 3 = \boxed{1.098612}$

(f) $\ln 0.05 = \boxed{-2.995732}$

(g) $\ln\left(\frac{5500}{1.10^{16}}\right) = \ln 5500 - \ln 1.10^{16}$

$= \ln 5500 - 16 \ln 1.10$

$= 8.612503 - 16(0.095310)$

$= 8.612503 - 1.524963$

$= \boxed{7.087540}$

(h) $\ln\left[375(1.01)\left(\dfrac{1-1.01^{-72}}{0.01}\right)\right] = \ln 375 + \ln 1.01 + \ln\,(1-1.01^{-72}) - \ln 0.01$

$= \ln 375 + \ln 1.01 + \ln\,(1-0.488496) - \ln 0.01$

$= \ln 375 + \ln 1.01 + \ln 0.511504 - \ln 0.01$

$= 5.926926 + 0.009950 - 0.670400 - (-4.605170)$

$= \boxed{9.871647}$

6. (a) $9x = -63$

$\boxed{x = -7}$

(b) $0.05x = 44$

$5x = 4400$

$\boxed{x = 880}$

(c) $-\dfrac{1}{7}x = 3$

$-x = 21$

$\boxed{x = -21}$

(d) $\dfrac{5}{6}x = -15$

$\dfrac{1}{6}x = -3$

$\boxed{x = -18}$

(e) $x - 8 = -5$

$x - 8 + 8 = -5 + 8$

$\boxed{x = 3}$

(f) $x + 9 = -2$

$x + 9 - 9 = -2 - 9$

$\boxed{x = -11}$

(g) $x + 0.02x = 255$

$1.02x = 255$

$\boxed{x = 250}$

(h) $x - 0.1x = 36$

$0.9x = 36$

$9x = 360$

$\boxed{x = 40}$

(i) $4x - 3 = 9x + 2$

$-5x = 5$

$\boxed{x = -1}$

(j) $9x - 6 - 3x = 15 + 4x - 7$

$6x - 6 = 8 + 4x$

$2x = 14$

$\boxed{x = 7}$

(k) $x - \frac{1}{3}x = 26$

$\frac{2}{3}x = 26$

$\frac{1}{3}x = 13$

$\boxed{x = 39}$

(l) $x + \frac{3}{8}x = 77$

$\frac{11}{8}x = 77$

$\frac{1}{8}x = 7$

$\boxed{x = 56}$

7. (a) $-9(3x - 8) - 8(9 - 7x) = 5 + 4(9x + 11)$

$-27x + 72 - 72 + 56x = 5 + 36x + 44$

$29x = 49 + 36x$

$-7x = 49$

$\boxed{x = -7}$

Check LS $= -9[3(-7) - 8] - 8[9 - 7(-7)] = -9(-29) - 8(58) = -203$

RS $= 5 + 4[9(-7) + 11] = 5 + 4(-52) = 5 - 208 = -203$

(b) $21x-4-7(5x-6)=8x-4(5x-7)$

$$21x-4-35x+42=8x-20x+28$$
$$-14x+38=-12x+28$$
$$-2x=-10$$
$$\boxed{x=5}$$

Check LS $=21(5)-4-7[5(5)-6]=105-4-7(19)=101-133=-32$

RS $=8(5)-4[5(5)-7]=40-4(18)=40-72=-32$

(c) $\frac{5}{7}x+\frac{1}{2}=\frac{5}{14}+\frac{2}{3}x$

$$42\left(\frac{5}{7}x\right)+42\left(\frac{1}{2}\right)=42\left(\frac{5}{14}\right)+42\left(\frac{2}{3}x\right)$$
$$6(5x)+21(1)=3(5)+14(2x)$$
$$30x+21=15+28x$$
$$2x=-6$$
$$\boxed{x=-3}$$

Check LS $=\frac{5}{7}(-3)+\frac{1}{2}=\frac{-30+7}{14}=-\frac{23}{14}$

RS $=\frac{5}{14}+\frac{2}{3}(-3)=\frac{5}{14}-2=-\frac{23}{14}$

(d) $\frac{4x}{3}+2=\frac{9}{8}-\frac{x}{6}$

$$8(4x)+24(2)=3(9)-4(x)$$
$$32x+48=27-4x$$
$$36x=-21$$
$$\boxed{x=-\frac{7}{12}}$$

Check LS $=\frac{4}{3}\left(-\frac{7}{12}\right)+2=-\frac{28}{36}+2=-\frac{7}{9}+\frac{18}{9}=\frac{11}{9}$

RS $=\frac{9}{8}-\frac{1}{6}\left(-\frac{7}{12}\right)=\frac{9}{8}+\frac{7}{72}=\frac{81+7}{72}=\frac{88}{72}=\frac{11}{9}$

(e) $\frac{7}{6}(6x-7)-\frac{3}{8}(7x+15)=25$

$$56(6x-7)-15(7x+15)=40(25)$$
$$336x-392-105x-225=1000$$
$$231x-617=1617$$
$$\boxed{x=7}$$

$$\text{Check LS}=\frac{7}{5}[6(7)-7]-\frac{3}{8}[7(7)+15]$$
$$=\frac{7}{5}(35)-\frac{3}{8}(64)=7(7)-24=49-24=25$$
$$\text{RS}=25$$

(f) $\frac{5}{9}(7-6x)-\frac{3}{4}(3-15x)=\frac{1}{12}(3x-5)-\frac{1}{2}$

$$20(7-6x)-27(3-15x)=3(3x-5)-18$$
$$140-120x-81+405x=9x-15-18$$
$$285x+59=9x-33$$
$$276x=-92$$
$$\boxed{x=-\frac{1}{3}}$$

$$\text{Check LS}=\frac{5}{9}\left[7-6\left(-\frac{1}{3}\right)\right]-\frac{3}{4}\left[3-15\left(-\frac{1}{3}\right)\right]$$
$$=\frac{5}{9}(7+2)-\frac{3}{4}(3+5)$$
$$=5-6=-1$$
$$\text{RS}=\frac{1}{12}\left[3\left(-\frac{1}{3}\right)-5\right]-\frac{1}{2}$$
$$=\frac{1}{12}(-6)-\frac{1}{2}$$
$$=-\frac{1}{2}-\frac{1}{2}=-1$$

(g) $\frac{5}{6}(4x-3)-\frac{2}{5}(3x+4)=5x-\frac{16}{15}(1-3x)$

$$25(4x-3)-12(3x+4)=150x-32(1-3x)$$
$$100x-75-36x-48=150x-32+96x$$
$$64x-123=246x-32$$
$$-182x=91$$
$$\boxed{x=-\frac{1}{2}}$$

$$\text{Check LS}=\frac{5}{6}\left[4\left(-\frac{1}{2}\right)-3\right]-\frac{2}{5}\left[3\left(-\frac{1}{2}\right)+4\right]$$
$$=\frac{5}{6}(-2-3)-\frac{2}{5}\left[-\frac{3}{2}+4\right]$$
$$=\frac{5}{6}(-5)-\frac{2}{5}\left(\frac{5}{2}\right)=-\frac{25}{6}-1=-\frac{31}{6}$$
$$\text{RS}=5\left(-\frac{1}{2}\right)-\frac{16}{15}\left[1-3\left(-\frac{1}{2}\right)\right]$$
$$=-\frac{5}{2}-\frac{16}{15}\left(1+\frac{3}{2}\right)=-\frac{5}{2}-\frac{16}{15}\left(\frac{5}{2}\right)$$
$$=-\frac{5}{2}-\frac{8}{3}=\frac{-15-16}{6}=-\frac{31}{6}$$

8. (a) $\text{I}=\text{P}rt$

$$\boxed{r=\frac{\text{I}}{\text{P}t}}$$

(b) $\text{S}=\text{P}(1+rt)$

$$\frac{S}{P} = 1 + rt$$

$$\frac{S}{P} - 1 = rt$$

$$t = \frac{\frac{S}{P} - 1}{r}$$

$$t = \frac{\frac{S - P}{P}}{r}$$

$$\boxed{t = \frac{S - P}{Pr}}$$

(c) $D = rL$

$$\boxed{r = \frac{D}{L}}$$

(d) $\text{FV} = \text{PMT}\left[\frac{(1+p)^n - 1}{p}\right]$

$$\boxed{\text{PMT} = \left[\frac{FVp}{(1+p)^n - 1}\right]}$$

9. (a) Let the size of the workforce be x.

$$\text{Number laid off} = \frac{1}{6}x$$

$$\text{Number after the layoff} = x - \frac{1}{6}x$$

$$\therefore x - \frac{1}{6}x = 690$$

$$\frac{5}{6}x = 690$$

$$5x = 4140$$

$$x = 828$$

$\therefore$ the number laid off is $\frac{1}{6} \times 828 = \boxed{138.}$

(b) Let last year's average property value be \$$x$.

$$\text{Current average value} = \$\left(x + \frac{2}{7}x\right)$$

$$\therefore x + \frac{2}{7}x = 81\ 450$$

$$\frac{9}{7}x = 81\ 450$$

$$\frac{1}{7}x = 9050$$

$$x = 63\ 350$$

$\therefore$ Last year's average value was $\boxed{\$63\ 350.}$

(c) Let the quoted price be $\$x$.

$$\therefore x + \frac{1}{20}x = 2457$$

$$\frac{21}{20}x = 2457$$

$$\frac{1}{20}x = 117$$

$$x = 2340$$

$\therefore$ The gratuities $= \frac{1}{20}$ of $2340 = \boxed{\$117.}$

(d) Let the value of the building be $\$x$.

Value of the land $= \$\frac{1}{3}x - 2000$

Total value of the property $= \$x + \frac{1}{3}x - 2000$

$$\therefore x + \frac{1}{3}x - 2000 = 184\ 000$$

$$\frac{4}{3}x = 186\ 000$$

$$\frac{1}{3}x = 46\ 500$$

$$x = 139\ 500$$

The value assigned to land is $\$(184\ 000 - 139\ 500) = \boxed{\$44\ 500.}$

(e) Let the cost of power be $\$x$.

Cost of heat $= \$\left(\frac{3}{4}x + 22\right)$

Cost of water $= \$\left(\frac{1}{3}x - 11\right)$

Total cost $= x + \frac{3}{4}x + 22 + \frac{1}{3}x - 11 = 2010 + 10\%$ of 2010.

$12x + 9x + 4x = 12(2010 + 201 - 11)$

$25x = 26\,400$

$x = 1056$

Cost of heat $= \frac{3}{4} \times 1056 + 22 =$ \$814

Cost of power $=$ \$1056

Cost of water $= \frac{1}{3} \times 1056 - 11 =$ \$341

(f) Let the amount allocated to newspaper advertising be $\$x$.

Amount allocated to TV advertising $= \$(3x + 1000)$

Amount allocated to direct selling $= \frac{3}{4}[x + 3x + 1000]$

$\therefore x + 3x + 1000 + \frac{3}{4}[4x + 1000] = 87\,500$

$4x + \frac{3}{4}[4x + 1000] = 86\,500$

$16x + 12x + 3000 = 346\,000$

$28x = 343\,000$

$x = 12\,250$

The amount allocated to newspaper advertising is \$12 250; the amount allocated to TV advertising is \$37 750; the amount allocated to direct selling is \$37 500

(g) Let the number of minutes on Machine B be x.

Time on Machine A $= \frac{4}{5}x - 3$ minutes

Time on Machine C $= \frac{5}{6}\left(x + \frac{4}{5}x - 3\right)$ minutes

Total time $= x + \frac{4}{5}x - 3 + \frac{5}{6}\left(x + \frac{4}{5}x - 3\right)$ minutes

$$\therefore x+\frac{4}{5}x-3+\frac{5}{6}\left(x+\frac{4}{5}x-3\right)=77$$

$$30x+24x-90+25\left(x+\frac{4}{5}x-3\right)=30(77)$$

$$54x-90+25x+20x-75=2310$$

$$99x-165=2310$$

$$99x=2475$$

$$x=25$$

Time on Machine B is 25 minutes; time on Machine A is $\frac{4}{5}(25)-3=17$ minutes; time on Machine C is $\frac{5}{6}(25+17)=$ 35 minutes.

(h) Let the number of pairs of superlight poles be x.

Number of pairs of ordinary poles $=72-x$

Value of superlight poles $=\$30x$

Value of ordinary poles $=\$16(72-x)$

Total value of all poles $=\$30x+16(72-x)$

$$30x+16(72-x)=1530$$

$$30x+1152-16x=1530$$

$$14x=378$$

$$x=27$$

The number of pairs of superlight poles is 27; the number of pairs of ordinary poles is 45.

(i) Let the number of \$2 coins be x.

Number of \$1 coins $=\frac{3}{5}x+1$

Number of quarters $=4\left(x+\frac{3}{5}x+1\right)$

Value of the \$2 coins $=\$2x$

Value of the \$1 coins $=\$\left[\frac{3}{5}x+1\right]$

Value of the quarters $=\$\frac{1}{4}(4)\left(x+\frac{3}{5}x+1\right)=x+\frac{3}{5}x+1$

$$\begin{aligned}\text{Total value} = 2x + \frac{3}{5}x + 1 + x + \frac{3}{5}x + 1 &= 107\\ 10x + 3x + 5 + 5x + 3x + 5 &= 535\\ 21x + 10 &= 535\\ 21x &= 525\\ x &= 25\end{aligned}$$

The number of \$2 coins is 25; the number of \$1 coins is $\left(\frac{3}{5}\times 25 + 1\right) = 16;$ the number of quarters is $4(25+16) = \boxed{164.}$

Self-Test

1. (a) $4-3x-6-5x=\boxed{-2-8x}$

   (b) $(5x-4)-(7x+5)=5x-4-7x-5=\boxed{-2x-9}$

   (c) $-2(3a-4)-5(2a+3)$

   $=-6a+8-10a-15$

   $=\boxed{-16a-7}$

   (d) $-6(x-2)(x+1)$

   $=-6(x^2-2x+x-2)$

   $=-6(x^2-x-2)$

   $=\boxed{-6x^2+6x+12}$

2. (a) For $x=-3,\ y=5$

   $2x^2-5xy-4y^2$

   $=2(-3)^2-5(-3)(5)-4(5)^2$

   $=18+75-100$

   $=\boxed{-7}$

   (b) For $a=\frac{2}{3}, b=-\frac{3}{4}$

   $3(7a-4b)-4(5a+3b)$

   $=21a-12b-20a-12b$

   $=a-24b$

   $=\frac{2}{3}-24\left(-\frac{3}{4}\right)$

   $=\frac{2}{3}+18$

   $=\boxed{18\frac{2}{3}}$

   (c) For N = 12, C = 400, P = 2000, n = 24

   $\frac{2\text{NC}}{\text{P}(n+1)}=\frac{(2)(12)(400)}{2000(24+1)}=\frac{2(12)(400)}{2000(25)}=\boxed{0.192}$

(d) For I = 324, P = 5400, $r = 0.15$

$$\frac{\mathrm{I}}{\mathrm{P}r} = \frac{324}{5400 \times 0.15} = \boxed{0.4}$$

(e) For S = 1606, $d = 0.125$, $t = \dfrac{240}{365}$

$$\begin{aligned} \mathrm{S}(1-dt) &= 1606\left(1 - 0.125 \times \frac{240}{365}\right) \\ &= 1606(1 - 0.082192) \\ &= 1606(0.917808) \\ &= \boxed{1474} \end{aligned}$$

(f) For S = 1566, $r = 0.10$, $t = \dfrac{292}{365}$

$$\begin{aligned} \frac{\mathrm{S}}{1+rt} &= \frac{1566}{1 + 0.10 \times \frac{292}{365}} \\ &= \frac{1566}{1 + 0.08} \\ &= \boxed{1450} \end{aligned}$$

3. (a) $(-2)^3 = \boxed{-8}$

(b) $\left(-\dfrac{2}{3}\right)^2 = \boxed{\dfrac{4}{9}}$

(c) $(4)^0 = \boxed{1}$

(d) $(3)^2(3)^5 = (3)^7 = \boxed{2187}$

(e) $\left(\dfrac{4}{3}\right)^{-2} = \dfrac{1}{\left(\frac{4}{3}\right)^2} = \dfrac{1}{\frac{16}{9}} = \boxed{\dfrac{9}{16}}$

(f) $(-x^3)^5 = \boxed{-x^{15}}$

4. (a) $\sqrt[10]{1.35} = 1.35^{\frac{1}{10}} = 1.35^{0.10} = \boxed{1.030465}$

(b) $\dfrac{1 - 1.03^{-40}}{0.03} = \dfrac{1 - 0.306557}{0.03} = \dfrac{0.693443}{0.03} = \boxed{23.114772}$

(c) $\ln 1.025 = \boxed{0.024693}$

(d) $\ln (3.00e^{-0.2})$

$= \ln 3.00 + \ln e^{-0.2}$

$= \ln 3.00 - 0.2 \ln e$

$= 1.098612 - 0.2$

$= \boxed{0.898612}$

(e) $\ln \left(\dfrac{600}{1.06^{11}} \right)$

$= \ln 600 - \ln 1.06^{11}$

$= \ln 600 - 11 \ln 1.06$

$= 6.396930 - 11(0.058269)$

$= 6.396930 - 0.640958$

$= \boxed{5.755972}$

(f) $\ln \left[250 \left(\dfrac{1.07^5 - 1}{0.07} \right) \right]$

$= \ln 250 + \ln (1.07^5 - 1) - \ln 0.07$

$= \ln 250 + \ln 0.402552 - \ln 0.07$

$= 5.521461 - 0.909932 - (-2.659260)$

$= 5.521461 - 0.909932 + 2.659260$

$= \boxed{7.270789}$

5. (a) $\dfrac{1}{81} = \left(\dfrac{1}{3} \right)^{n-2}$

$\dfrac{1}{3^4} = \left(\dfrac{1}{3} \right)^{n-2}$

$\left(\dfrac{1}{3} \right)^4 = \left(\dfrac{1}{3} \right)^{n-2}$

Since the bases are common

$4 = n - 2$

$\boxed{n = 6}$

(b) $\frac{5}{2} = 40\left(\frac{1}{2}\right)^{n-1}$

$\frac{1}{2} = 8\left(\frac{1}{2}\right)^{n-1}$

$\frac{1}{16} = \left(\frac{1}{2}\right)^{n-1}$

$\left(\frac{1}{2}\right)^{4} = \left(\frac{1}{2}\right)^{n-1}$

$4 = n-1$

$\boxed{n = 5}$

6. (a) $-\frac{2}{3}x = 24$

$x = 24\left(-\frac{3}{2}\right)$

$\boxed{x = -36}$

(b) $x - 0.06x = 8.46$

$0.94x = 8.46$

$\boxed{x = 9}$

(c) $0.2x - 4 = 6 - 0.3x$

$0.5x = 10$

$\boxed{x = 20}$

(d) $(3-5x) - (8x-1) = 43$

$3 - 5x - 8x + 1 = 43$

$-13x = 39$

$\boxed{x = -3}$

(e) $4(8x-2) - 5(3x+5) = 18$

$32x - 8 - 15x - 25 = 18$

$17x - 33 = 18$

$17x = 51$

$\boxed{x = 3}$

(f) $x+\frac{3}{10}x+\frac{1}{2}+x+\frac{3}{5}x+1=103$

$$2x+\frac{3}{10}x+\frac{3}{5}x+\frac{3}{2}=103$$
$$20x+3x+6x+15=1030$$
$$29x=1015$$
$$\boxed{x=35}$$

(g) $x+\frac{4}{5}x-3+\frac{5}{6}\left(x+\frac{4}{5}x-3\right)=77$

$$30x+24x-90+25\left(x+\frac{4}{5}x-3\right)=30(77)$$
$$54x-90+25x+20x-75=2310$$
$$99x-165=2310$$
$$99x=2475$$
$$\boxed{x=25}$$

(h) $\frac{2}{3}(3x-1)-\frac{3}{4}(5x-3)=\frac{9}{8}x-\frac{5}{6}(7x-9)$

$$16(3x-1)-18(5x-3)=27x-20(7x-9)$$
$$48x-16-90x+54=27x-140x+180$$
$$-42x+38=-113x+180$$
$$71x=142$$
$$\boxed{x=2}$$

7. (a) $\mathrm{I}=\mathrm{P}rt$

$$\boxed{\mathrm{P}=\frac{\mathrm{I}}{rt}}$$

(b) $\mathrm{S}=\frac{P}{1-dt}$

$$\frac{\mathrm{S}}{P}=\frac{1}{1-dt}$$
$$\frac{P}{\mathrm{S}}=1-dt$$
$$dt=1-\frac{P}{\mathrm{S}}$$

$$d = \frac{1 - \frac{P}{\mathrm{S}}}{t}$$

$$d = \frac{\frac{\mathrm{S} - P}{\mathrm{S}}}{t}$$

$$\boxed{d = \frac{\mathrm{S} - P}{\mathrm{S}t}}$$

8. (a) Let the regular selling price be $\$x$.

Reduction in price $= \$\frac{1}{5}x$

$$\therefore x - \frac{1}{5}x = 192$$
$$\frac{4}{5}x = 192$$
$$x = 240$$

The regular selling price is $\boxed{\$240.}$

(b) Let the floor space occupied by shipping be x.

Floor space occupied by weaving $= 2x + 400$

Total floor space $= x + 2x + 400$

$$\therefore x + 2x + 400 = 6700$$
$$3x = 6300$$
$$x = 2100$$

The floor space occupied by weaving is $2(2100) + 400 = \boxed{4600 \text{ square metres.}}$

(c) Let the number of units of Product A be x.

Number of units of Product B $= 95 - x$

Number of hours for Product A $= 3x$

Number of hours for Product B $= 5(95 - x)$

$$\therefore 3x + 5(95 - x) = 395$$
$$3x + 475 - 5x = 395$$
$$-2x = -80$$
$$x = 40$$

The number of units of Product B is $95 - 40 = \boxed{55.}$

(d) Let the sum of money invested in the bank be \$$x$.

Sum of money invested in the credit union $= \$\frac{2}{3}x + 500$

Yield on the bank investment $= \$\frac{1}{12}x$

Yield on the credit union investment $= \$\frac{1}{9}\left(\frac{2}{3}x + 500\right)$

$$\therefore \frac{1}{12}x + \frac{1}{9}\left(\frac{2}{3}x + 500\right) = 1000$$

$$3x + 4\left(\frac{2}{3}x + 500\right) = 36\,000$$

$$3x + \frac{8}{3}x + 2000 = 36\,000$$

$$\frac{17x}{3} = 34\,000$$

$$17x = 102\,000$$

$$x = 6000$$

The sum of money invested in the credit union certificate is

$$\$\left(\frac{2}{3} \times 6000 + 500\right) = \boxed{\$4500.}$$

Challenge Problems

1. Counting a nickel as a quarter overstates the total by \$0.20; for x nickels, the total must be reduced by \$0.20x.

   Counting a dime as a penny understates the total by \$0.09; for x dimes, the total must be increased by \$0.09x.

   The total adjustment $= -0.20x + 0.09x = -\$0.11x$

   The clerk must reduce the total by \$0.11x.

2. The number of rotations in 5;

The distance per rotation $= \frac{4000}{5} = 800$ km;

each tire will be used for four rotations for a total distance of 3200 km. (See table below.)

Rotation	*Tire A*	*Tire B*	*Tire C*	*Tire D*	*Tire E*	*Distance travelled*
1	800	800	800	800	—	800
2	800	800	800	—	800	800
3	800	800	—	800	800	800
4	800	—	800	800	800	800
5	—	800	800	800	800	800
Total	3200	3200	3200	3200	3200	4000

3. The lowest possible two-digit number is 10;

the highest possible two-digit number is 99.

For a difference in value of \$17.82, the two-digit numbers must differ by 18, such as 10 and 28, 11 and 29, etc.

The lowest possible correct value of the cheque is \$10.28;

the largest possible correct value of the cheque is \$81.99.

In either case the difference between is \$17.82.

(a) FALSE In the possible correct cheque value \$81.99, the x-value 81 is greater than 70.

(b) TRUE In the possible correct cheque value \$18.36, the y-value 36 equals $2x$.

(c) TRUE A cheque cannot have zero cents.

(d) FALSE Let the correct amount be \$A;

then the incorrect amount is \$2A;

the difference is \$A;

$$A = 17.82$$

For the correct value \$17.82, the incorrect cheque value \$82.17 is unequal to 2(\$17.82).

(e) FALSE In the possible correct amount \$10.28, the sum of the digits is $1+0+2+8=11$, which is not divisible by 9.

Case Study

1. $73 566 – $49 355 = $24 211
2. The contributions continue until the 65th year. Therefore, total contributions (65 – 45) × 12 months per year × $100 = $24 000.
3. The contributions continue until the 65th year.
   a. Total contributions (65 – 45) × 12 months per year × $100 = $24 000. Total value of TFSA = $29 529. Therefore, interest earned is $29 529 – 24 000 = $5529.
   b. Total contributions (65 – 45) × 12 months per year × $250 = $60 000. Total value of TFSA = $73 566. Therefore, interest earned is $73 566 – 60 000 = $13 566.
4. Annual salary of $48 000/12 months = $4000.00 per month.
   a. $150/$4000 = 0.0375 or 3.75% of salary
   b. $250/$4000 = 0.0625 = 6.25% of salary