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Chapter 2

Basic Probability Theory

<u>Answers to Review Questions</u>

2.1

1. Strength of a beam, Wind velocity, Flood in a river, Earthquake magnitude, Traffic on a highway.

2. An experiment denotes the act of performing something the outcome of which is subject to uncertainty and not known exactly. An event represents the outcome of a single experiment.

3. If the outcome of an event precludes the occurrence of other events in a given experiment, the events are said to be mutually exclusive. Realizing a head and a tail simultaneously on tossing a coin, A student passing and failing an exam simultaneously.

4. The universal set denotes the set of all elements under consideration. A null set indicates a set with no elements.

5. The union of two seta A and B denotes the set of all elements that belong to A or B or both. The intersection of two sets A and B indicates the set of all elements that belong to both A and B.

6. The complement of a set A is the set of all elements of the universal set which do not belong to A.

7. The union and intersection of two sets are said to be commutative if $A \cup B = B \cup A$ and $A \cap B = B \cap A$. The union and intersection of two sets are said to be associative if $(A \cup B) \cup C = A \cup (B \cup C)$ and $(A \cap B) \cap C = A \cap (B \cap C)$. The union and intersection of three sets A, B and C are said to be distributive if $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$ and $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$.

8. de Morgan's rule states that the complement of union (or intersection) of sets A and B is equal to the intersection (union) of the complements of A and B.

9. A sample point denotes a possible outcome of an experiment.

10. The sample space denotes all possible outcomes of an experiment.

11. Venn diagram is a geometric figure that includes all the possible outcomes of an experiment (as sample points).

12. The probability of occurrence of an event is defined as the ratio of the number of occurrences of the event E to the total number of trials (N) as N tends to infinity.

13. The probability of occurrence of an event E is defined as a number P(E) such that P(E) obeys the following three axioms: (i) $P(E) \geq 0$, (ii) $P(E) = 1$ if E is a certain event, (iii) If E_1 and E_2 are mutually exclusive events, then $P(E_1 \cup E_2) = P(E_1) + P(E_2)$.

14. The occurrence of an event E_2 when it is known that E_1 has occurred is known as the conditional event and is denoted as $E_2|_{E1}$.

15. If the occurrence of an event A in no way affects the probability of occurrence of event B, then the events A and B are said to be statistically independent.

16. If $B_1, B_2, \ldots, B_n$ are a set of mutually exclusive and collectively exhaustive events, then the total probability theorem states that the probability of occurrence of another event A is given by $P(A) = \sum_{i=1}^{n} P(A \cap B_i)$.

17. If A and b are two events, Bayes' rule states that $P\left(A|_B\right)P(B) = P\left(B|_A\right)P(A)$.

18. (i) A building might collapse due to an earthquake (event B_1) , wind loading (event B_2) , or any other cause (event B_3). Let the probabilities of occurrence of these events be known at a given location. From the design of the building, let the probabilities, $P(A|_{B_i})$, i = 1, 2, 3, be known. If the building really collapses, Bayes' theorem helps in finding the cause of the collapse (among earthquake, wind loading or any other cause).

    (ii) Let a machine used to detect blowholes in castings be known to be 95% accurate both on castings that have blowholes and on those that do not have blowholes. Historically, let 1% of the castings be known to have blowholes. Then Bayes' rule can be used to find the probability that a specific casting has a blowhole, given the machine indicates that it has a blowhole.

2.2

1. F 2. T 3. T 4. F 5. T

2.3

1. independent 2. HH, HT, TH and TT 3. Sample 4. Zero 5. 0.25

6. 64   7. $26^3 10^2$ = 1,757,600   8. 26(25)(24)(10)(9) = 1,404,000   9. Exclusive

10. Both   11. Both   12. Excluding   13. 2/3   14. 4/52 = 1/13

15. 3/10   16. Zero   17. 1/3   18. 0.0081

2.4

1. (a)   2. (c)   3. (b)   4. (a)   5. (c)   6. (b)   7. (a)   8. (a)   9. (a)   10. (a)

2.5

1 – c   2 – e   3 – a   4 – b   5 – d

2.6

1 – b   2 – d   3 – a   4 – c

2.7

1 – c   2 – e   3 – b   4 – e   5 – a   6 – d

2.8

1 – e   2 - b   3 – c   4 – a   5 – d