

Budget Constraint

Introduction. These workouts are designed to build your skills in describing economic situations with graphs and algebra. Budget sets are a good place to start, because both the algebra and the graphing are very easy. Where there are just two goods, a consumer who consumes x_1 units of good 1 and x_2 units of good 2 is said to consume the *consumption bundle*, (x_1, x_2) . Any consumption bundle can be represented by a point on a two-dimensional graph with quantities of good 1 on the horizontal axis and quantities of good 2 on the vertical axis. If the prices are p_1 for good 1 and p_2 for good 2, and if the consumer has income m , then she can afford any consumption bundle, (x_1, x_2) , such that $p_1x_1 + p_2x_2 \leq m$. On a graph, the *budget line* is just the line segment with equation $p_1x_1 + p_2x_2 = m$ and with x_1 and x_2 both nonnegative. The budget line is the boundary of the *budget set*. All of the points that the consumer can afford lie on one side of the line and all of the points that the consumer cannot afford lie on the other.

If you know prices and income, you can construct a consumer's budget line by finding two commodity bundles that she can "just afford" and drawing the straight line that runs through both points.

Example: Myrtle has 50 dollars to spend. She consumes only apples and bananas. Apples cost 2 dollars each and bananas cost 1 dollar each. You are to graph her budget line, where apples are measured on the horizontal axis and bananas on the vertical axis. Notice that if she spends all of her income on apples, she can afford 25 apples and no bananas. Therefore her budget line goes through the point $(25, 0)$ on the horizontal axis. If she spends all of her income on bananas, she can afford 50 bananas and no apples. Therefore her budget line also passes through the point $(0, 50)$ on the vertical axis. Mark these two points on your graph. Then draw a straight line between them. This is Myrtle's budget line.

What if you are not told prices or income, but you know two commodity bundles that the consumer can just afford? Then, if there are just two commodities, you know that a unique line can be drawn through two points, so you have enough information to draw the budget line.

Example: Laurel consumes only ale and bread. If she spends all of her income, she can just afford 20 bottles of ale and 5 loaves of bread. Another commodity bundle that she can afford if she spends her entire income is 10 bottles of ale and 10 loaves of bread. If the price of ale is 1 dollar per bottle, how much money does she have to spend? You could solve this problem graphically. Measure ale on the horizontal axis and bread on the vertical axis. Plot the two points, $(20, 5)$ and $(10, 10)$, that you know to be on the budget line. Draw the straight line between these points and extend the line to the horizontal axis. This point denotes the amount of

ale Laurel can afford if she spends all of her money on ale. Since ale costs 1 dollar a bottle, her income in dollars is equal to the largest number of bottles she can afford. Alternatively, you can reason as follows. Since the bundles (20, 5) and (10, 10) cost the same, it must be that giving up 10 bottles of ale makes her able to afford an extra 5 loaves of bread. So bread costs twice as much as ale. The price of ale is 1 dollar, so the price of bread is 2 dollars. The bundle (20, 5) costs as much as her income. Therefore her income must be $20 \times 1 + 5 \times 2 = 30$.

When you have completed this workout, we hope that you will be able to do the following:

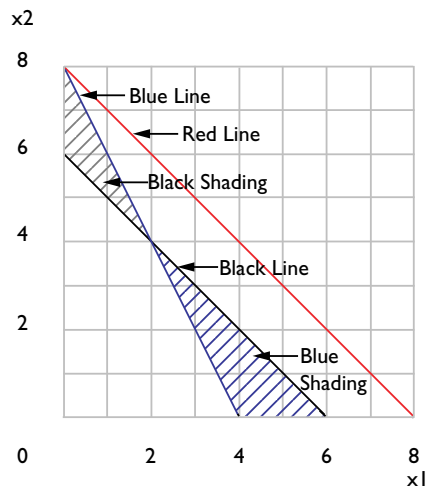
- Write an equation for the budget line and draw the budget set on a graph when you are given prices and income or when you are given two points on the budget line.
- Graph the effects of changes in prices and income on budget sets.
- Understand the concept of *numeraire* and know what happens to the budget set when income and all prices are multiplied by the same positive amount.
- Know what the budget set looks like if one or more of the prices is negative.
- See that the idea of a “budget set” can be applied to constrained choices where there are other constraints on what you can have, in addition to a constraint on money expenditure.

2.1 (0) You have an income of \$40 to spend on two commodities. Commodity 1 costs \$10 per unit, and commodity 2 costs \$5 per unit.

(a) Write down your budget equation. $10x_1 + 5x_2 = 40$.

(b) If you spent all your income on commodity 1, how much could you buy? **4**.

(c) If you spent all of your income on commodity 2, how much could you buy? **8**. Use blue ink to draw your budget line in the graph below.



(d) Suppose that the price of commodity 1 falls to \$5 while everything else stays the same. Write down your new budget equation. $5x_1 + 5x_2 = 40$. On the graph above, use red ink to draw your new budget line.

(e) Suppose that the amount you are allowed to spend falls to \$30, while the prices of both commodities remain at \$5. Write down your budget equation. $5x_1 + 5x_2 = 30$. Use black ink to draw this budget line.

(f) On your diagram, use blue ink to shade in the area representing commodity bundles that you can afford with the budget in Part (e) but could not afford to buy with the budget in Part (a). Use black ink or pencil to shade in the area representing commodity bundles that you could afford with the budget in Part (a) but cannot afford with the budget in Part (e).

2.2 (0) On the graph below, draw a budget line for each case.

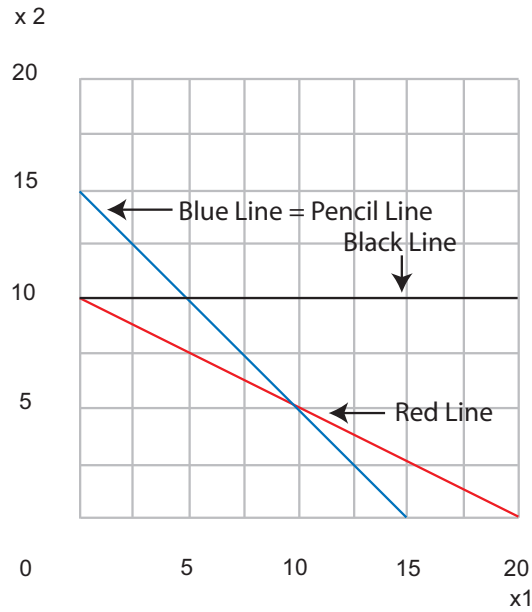
8 BUDGET CONSTRAINT (Ch. 2)

(a) $p_1 = 1, p_2 = 1, m = 15$. (Use blue ink.)

(b) $p_1 = 1, p_2 = 2, m = 20$. (Use red ink.)

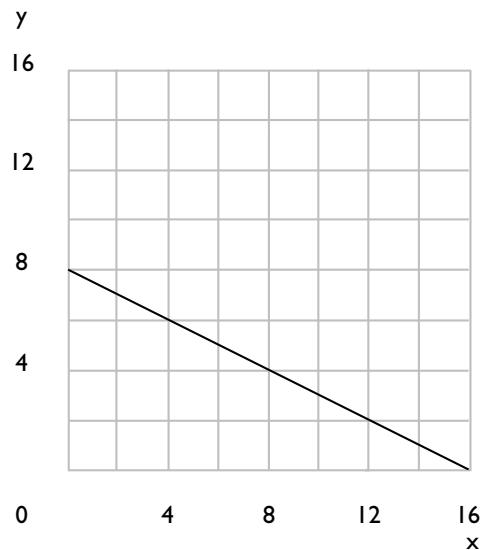
(c) $p_1 = 0, p_2 = 1, m = 10$. (Use black ink.)

(d) $p_1 = p_2, m = 15p_1$. (Use pencil or black ink. Hint: How much of good 1 could you afford if you spend your entire budget on good 1?)



2.3 (0) Your budget is such that if you spend your entire income, you can afford either 4 units of good x and 6 units of good y or 12 units of x and 2 units of y .

(a) Mark these two consumption bundles and draw the budget line in the graph below.



(b) What is the ratio of the price of x to the price of y ? $1/2$.

(c) If you spent all of your income on x , how much x could you buy?

16.

(d) If you spent all of your income on y , how much y could you buy?

8.

(e) Write a budget equation that gives you this budget line, where the price of x is 1. $x + 2y = 16$.

(f) Write another budget equation that gives you the same budget line, but where the price of x is 3. $3x + 6y = 48$.

2.4 (1) Murphy was consuming 100 units of X and 50 units of Y . The price of X rose from 2 to 3. The price of Y remained at 4.

(a) How much would Murphy's income have to rise so that he can still exactly afford 100 units of X and 50 units of Y ? **\$100.**

2.5 (1) If Amy spent her entire allowance, she could afford 8 candy bars and 8 comic books a week. She could also just afford 10 candy bars and 4 comic books a week. The price of a candy bar is 50 cents. Draw her budget line in the box below. What is Amy's weekly allowance? **\$6.**

Comic books



2.6 (0) In a small country near the Baltic Sea, there are only three commodities: potatoes, meatballs, and jam. Prices have been remarkably stable for the last 50 years or so. Potatoes cost 2 crowns per sack, meatballs cost 4 crowns per crock, and jam costs 6 crowns per jar.

(a) Write down a budget equation for a citizen named Gunnar who has an income of 360 crowns per year. Let P stand for the number of sacks of potatoes, M for the number of crocks of meatballs, and J for the number of jars of jam consumed by Gunnar in a year. $2P + 4M + 6J =$
360.

(b) The citizens of this country are in general very clever people, but they are not good at multiplying by 2. This made shopping for potatoes excruciatingly difficult for many citizens. Therefore it was decided to introduce a new unit of currency, such that potatoes would be the *numeraire*. A sack of potatoes costs one unit of the new currency while the same relative prices apply as in the past. In terms of the new currency, what is the price of meatballs? **2 crowns.**

(c) In terms of the new currency, what is the price of jam? **3 crowns.**

(d) What would Gunnar's income in the new currency have to be for him to be exactly able to afford the same commodity bundles that he could afford before the change? **180 crowns.**

(e) Write down Gunnar's new budget equation. $P + 2M + 3J =$
180. Is Gunnar's budget set any different than it was before the change?
No.

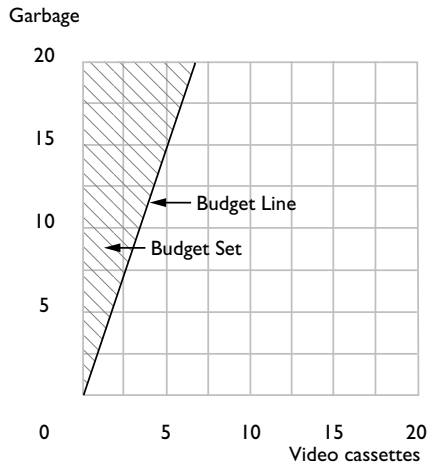
2.7 (0) Edmund Stench consumes two commodities, namely garbage and punk rock video cassettes. He doesn't actually eat the former but keeps it in his backyard where it is eaten by billy goats and assorted vermin. The reason that he accepts the garbage is that people pay him \$2 per sack for taking it. Edmund can accept as much garbage as he wishes at that price. He has no other source of income. Video cassettes cost him \$6 each.

(a) If Edmund accepts zero sacks of garbage, how many video cassettes can he buy? **0.**

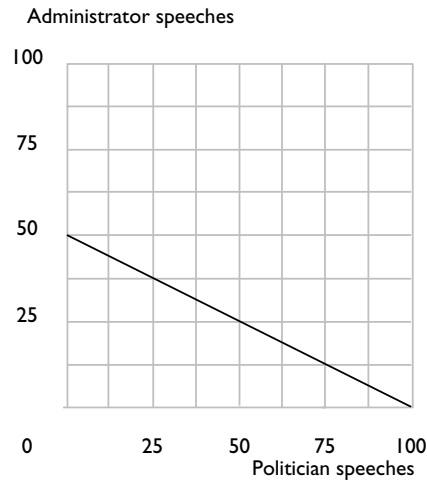
(b) If he accepts 15 sacks of garbage, how many video cassettes can he buy? **5.**

(c) Write down an equation for his budget line. $6C - 2G = 0.$

(d) Draw Edmund's budget line and shade in his budget set.



2.8 (0) If you think Edmund is odd, consider his brother Emmett. Emmett consumes speeches by politicians and university administrators. He is paid \$1 per hour for listening to politicians and \$2 per hour for listening to university administrators. (Emmett is in great demand to help fill empty chairs at public lectures because of his distinguished appearance and his ability to refrain from making rude noises.) Emmett consumes one good for which he must pay. We have agreed not to disclose what that good is, but we can tell you that it costs \$15 per unit and we shall call it Good X. In addition to what he is paid for consuming speeches, Emmett receives a pension of \$50 per week.



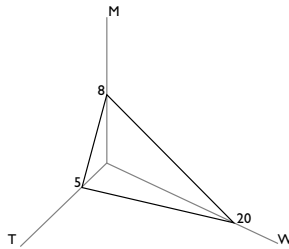
(a) Write down a budget equation stating those combinations of the three commodities, Good X , hours of speeches by politicians (P), and hours of speeches by university administrators (A) that Emmett could afford to consume per week. $15X - 1P - 2A = 50$.

(b) On the graph above, draw a two-dimensional diagram showing the locus of consumptions of the two kinds of speeches that would be possible for Emmett if he consumed 10 units of Good X per week.

2.9 (0) Jonathan Livingstone Yuppie is a prosperous lawyer. He has, in his own words, “outgrown those confining two-commodity limits.” Jonathan consumes three goods, unblended Scotch whiskey, designer tennis shoes, and meals in French gourmet restaurants. The price of Jonathan’s brand of whiskey is \$20 per bottle, the price of designer tennis shoes is \$80 per pair, and the price of gourmet restaurant meals is \$50 per meal. After he has paid his taxes and alimony, Jonathan has \$400 a week to spend.

(a) Write down a budget equation for Jonathan, where W stands for the number of bottles of whiskey, T stands for the number of pairs of tennis shoes, and M for the number of gourmet restaurant meals that he consumes. $20W + 80T + 50M = 400$.

(b) Draw a three-dimensional diagram to show his budget set. Label the intersections of the budget set with each axis.



(c) Suppose that he determines that he will buy one pair of designer tennis shoes per week. What equation must be satisfied by the combinations of restaurant meals and whiskey that he could afford? $20W + 50M = 320$.

2.10 (0) Martha is preparing for exams in economics and sociology. She has time to read 40 pages of economics and 30 pages of sociology. In the same amount of time she could also read 30 pages of economics and 60 pages of sociology.

(a) Assuming that the number of pages per hour that she can read of either subject does not depend on how she allocates her time, how many pages of sociology could she read if she decided to spend all of her time on sociology and none on economics? **150 pages.** (Hint: You have two points on her budget line, so you should be able to determine the entire line.)

(b) How many pages of economics could she read if she decided to spend all of her time reading economics? **50 pages.**

2.11 (1) Harry Hype has \$5,000 to spend on advertising a new kind of dehydrated sushi. Market research shows that the people most likely to buy this new product are recent recipients of M.B.A. degrees and lawyers who own hot tubs. Harry is considering advertising in two publications, a boring business magazine and a trendy consumer publication for people who wish they lived in California.

Fact 1: Ads in the boring business magazine cost \$500 each and ads in the consumer magazine cost \$250 each.

Fact 2: Each ad in the business magazine will be read by 1,000 recent M.B.A.'s and 300 lawyers with hot tubs.

Fact 3: Each ad in the consumer publication will be read by 300 recent M.B.A.'s and 250 lawyers who own hot tubs.

Fact 4: Nobody reads more than one ad, and nobody who reads one magazine reads the other.

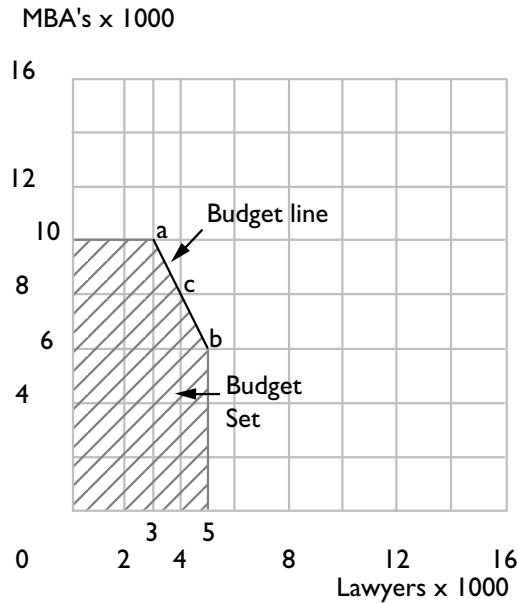
(a) If Harry spends his entire advertising budget on the business publication, his ad will be read by **10,000** recent M.B.A.'s and by **3,000** lawyers with hot tubs.

(b) If he spends his entire advertising budget on the consumer publication, his ad will be read by **6,000** recent M.B.A.'s and by **5,000** lawyers with hot tubs.

(c) Suppose he spent half of his advertising budget on each publication. His ad would be read by **8,000** recent M.B.A.'s and by **4,000** lawyers with hot tubs.

(d) Draw a "budget line" showing the combinations of number of readings by recent M.B.A.'s and by lawyers with hot tubs that he can obtain if he spends his entire advertising budget. Does this line extend all the way to the axes? **No.** Sketch, shade in, and label the budget set, which includes all the combinations of MBA's and lawyers he can reach if he spends *no more than* his budget.

(e) Let M stand for the number of instances of an ad being read by an M.B.A. and L stand for the number of instances of an ad being read by a lawyer. This budget line is a line segment that lies on the line with equation $M + 2L = 16$. With a fixed advertising budget, how many readings by M.B.A.'s must he sacrifice to get an additional reading by a lawyer with a hot tub? **2.**



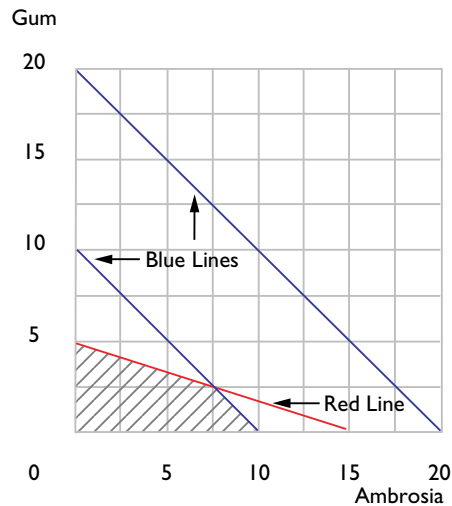
2.12 (0) On the planet Mungo, they have two kinds of money, blue money and red money. Every commodity has two prices—a red-money price and a blue-money price. Every Mungoan has two incomes—a red income and a blue income.

In order to buy an object, a Mungoan has to pay that object's red-money price in red money and its blue-money price in blue money. (The shops simply have two cash registers, and you have to pay at both registers to buy an object.) It is forbidden to trade one kind of money for the other, and this prohibition is strictly enforced by Mungo's ruthless and efficient monetary police.

- There are just two consumer goods on Mungo, ambrosia and bubble gum. All Mungoans prefer more of each good to less.
- The blue prices are 1 bcu (bcu stands for blue currency unit) per unit of ambrosia and 1 bcu per unit of bubble gum.
- The red prices are 2 rcus (red currency units) per unit of ambrosia and 6 rcus per unit of bubble gum.

(a) On the graph below, draw the red budget (with red ink) and the blue budget (with blue ink) for a Mungoan named Harold whose blue income is 10 and whose red income is 30. Shade in the "budget set" containing all of the commodity bundles that Harold can afford, given

its* two budget constraints. Remember, Harold has to have enough blue money *and* enough red money to pay both the blue-money cost and the red-money cost of a bundle of goods.



(b) Another Mungoan, Gladys, faces the same prices that Harold faces and has the same red income as Harold, but Gladys has a blue income of 20. Explain how it is that Gladys will not spend its entire blue income no matter what its tastes may be. (Hint: Draw Gladys’s budget lines.)

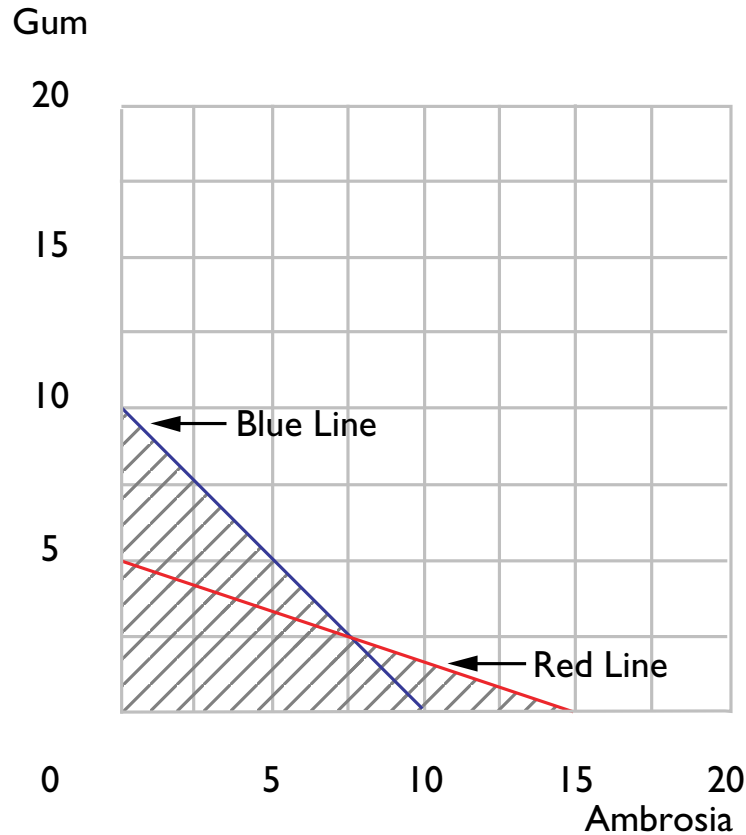
The blue budget line lies strictly outside the red budget line, so to satisfy both budgets, one must be strictly inside the red budget line.

(c) A group of radical economic reformers on Mungo believe that the currency rules are unfair. “Why should everyone have to pay two prices for everything?” they ask. They propose the following scheme. Mungo will continue to have two currencies, every good will have a blue price and a red price, and every Mungoan will have a blue income and a red income. But nobody has to pay both prices. Instead, everyone on Mungo must declare itself to be either a Blue-Money Purchaser (a “Blue”) or a Red-Money Purchaser (a “Red”) before it buys anything at all. Blues must make all of their purchases in blue money at the blue prices, spending only their blue incomes. Reds must make all of their purchases in red money, spending only their red incomes.

Suppose that Harold has the same income after this reform, and that prices do not change. Before declaring which kind of purchaser it will be,

* We refer to all Mungoans by the gender-neutral pronoun, “it.” Although Mungo has two sexes, neither of them is remotely like either of ours.

Harold contemplates the set of commodity bundles that it could afford by making one declaration or the other. Let us call a commodity bundle “attainable” if Harold can afford it by declaring itself to be a “Blue” and buying the bundle with blue money or if Harold can afford the bundle by declaring itself to be a “Red” and buying it with red money. On the diagram below, shade in all of the attainable bundles.



2.13 (0) Are Mungoan budgets really so fanciful? Can you think of situations on earth where people must simultaneously satisfy more than one budget constraint? Is money the only scarce resource that people use up when consuming? Consumption of many commodities takes time as well as money. People have to simultaneously satisfy a time budget and a money budget. Other examples--people may have a calorie budget or a cholesterol budget or an alcohol-intake budget.

