2/1

$$\begin{cases} F_x = 600 \cos 40^\circ = \underline{460 \text{ N}} \\ F_y = -600 \sin 40^\circ = \underline{-386 \text{ N}} \end{cases}$$

$$\underline{\underline{F} = 460\underline{i} - 386\underline{j} \text{ N}}$$

2/2

$$\underline{F} = 400(-\cos 30^\circ \underline{i} + \sin 30^\circ \underline{j})$$

$$= \underline{-346\underline{i} + 200\underline{j} \text{ lb}}$$

Scalar components: $\begin{cases} F_x = -346 \text{ lb} \\ F_y = 200 \text{ lb} \end{cases}$

Vector components: $\begin{cases} \underline{F_x} = -346\underline{i} \text{ lb} \\ \underline{F_y} = 200\underline{j} \text{ lb} \end{cases}$

2/3

$$\mathbf{F} = 6.5\left(-\frac{12}{13}\mathbf{i} - \frac{5}{13}\mathbf{j}\right)$$

$$= \underline{-6\mathbf{i} - 2.5\mathbf{j} \text{ kN}}$$

(Note: Writing 6, rather than 6.00, indicates an exact result.)

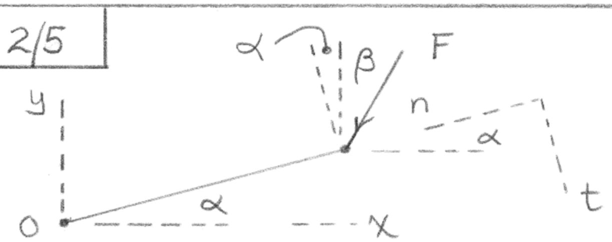
2/4

$$\underline{F} = F\underline{n}_{AB} = 3000\left[\frac{15\underline{i} + 8\underline{j}}{\sqrt{15^2 + 8^2}}\right]$$

$$= 2650\underline{i} + 1412\underline{j} \text{ lb}$$

Scalar components : $\begin{cases} F_x = 2650 \text{ lb} \\ F_y = 1412 \text{ lb} \end{cases}$

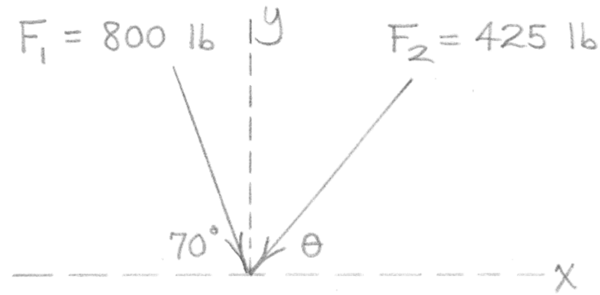
2/5



$$\begin{cases} F_x = -F\sin\beta \\ F_y = -F\cos\beta \end{cases}$$

$$\begin{cases} F_n = F\sin(\alpha+\beta) \\ F_t = F\cos(\alpha+\beta) \end{cases}$$

2/6



$R_x = \Sigma F_x = 800 \cos 70° - 425 \cos\theta = 0$

$\underline{\theta = 49.9°}$

$R_y = \Sigma F_y = -800 \sin 70° - 425 \sin 49.9°$

$= -1077$ lb

So $\underline{R = 1077 \text{ lb}}$

2/7

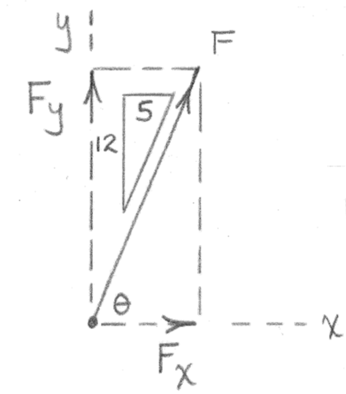
$$\underline{R} = (500 + 350\cos 60^\circ)\,\underline{i} + 350 \sin 60^\circ\,\underline{j}$$

$$\underline{\underline{R} = 675\,\underline{i} + 303\,\underline{j}\ \text{N}}$$

$$R = \sqrt{675^2 + 303^2} \longrightarrow \underline{R = 740\ \text{N}}$$

$$\theta_x = \cos^{-1}\left(\frac{R_x}{R}\right) = \cos^{-1}\left(\frac{675}{740}\right) \longrightarrow \underline{\theta_x = 24.2^\circ \text{ ABOVE } +x \text{ AXIS}}$$

2/8



$\cos\theta = \frac{5}{13}$, $\sin\theta = \frac{12}{13}$

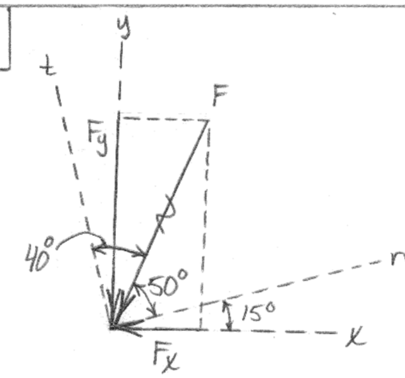
$F_y = F\sin\theta = F\frac{12}{13} = 70$

$\underline{F = 75.8 \text{ lb}}$

$F_x = F\cos\theta = 75.8\left(\frac{5}{13}\right)$

$= \underline{29.2 \text{ lb}}$

2/9

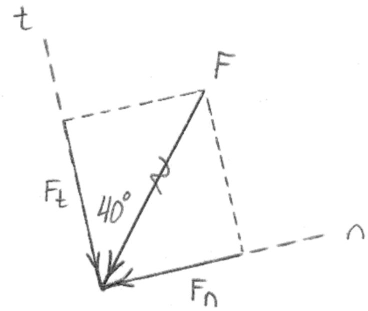


$$F_x = -F\cos 65^\circ = -13\cos 65^\circ$$

$$\underline{F_x = -5.49 \text{ kips}}$$

$$F_y = -F\sin 65^\circ = -13\sin 65^\circ$$

$$\underline{F_y = -11.78 \text{ kips}}$$



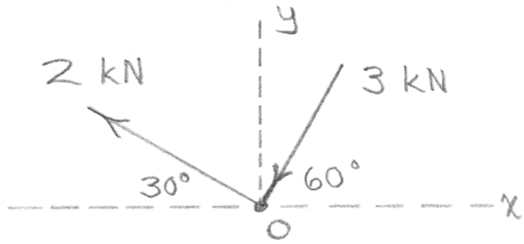
$$F_n = -F\sin 40^\circ = -13\sin 40^\circ$$

$$\underline{F_n = -8.36 \text{ kips}}$$

$$F_t = -F\cos 40^\circ = -13\cos 40^\circ$$

$$\underline{F_t = -9.96 \text{ kips}}$$

2/10



$$R_x = \Sigma F_x = -2\cos 30° - 3\cos 60° = -3.23\ \text{kN}$$

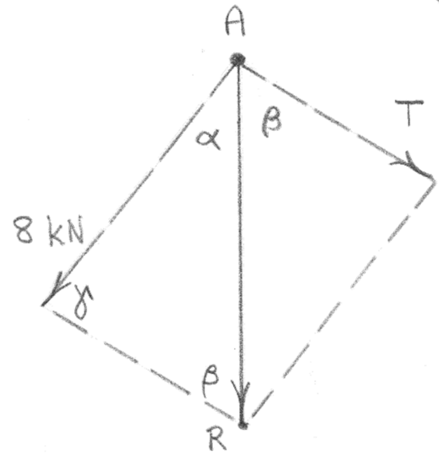
$$R_y = \Sigma F_y = 2\sin 30° - 3\sin 60° = -1.598\ \text{kN}$$

$$R = \sqrt{R_x^2 + R_y^2} = \underline{3.61\ \text{kN}}$$

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{-1.598}{-3.23}\right) = \underline{206°}$$

2/11

$$\begin{cases} \alpha = \tan^{-1} \frac{40}{50} = 38.7° \\ \beta = \tan^{-1} \frac{50}{30} = 59.0° \end{cases}$$



$$\gamma = 180° - \alpha - \beta$$

$$= 82.3°$$

$$\frac{\sin \beta}{8} = \frac{\sin \alpha}{T}$$

$$\underline{T = 5.83 \text{ kN}}$$

$$\frac{\sin \beta}{8} = \frac{\sin \gamma}{R}, \quad \underline{R = 9.25 \text{ kN}}$$

2/12

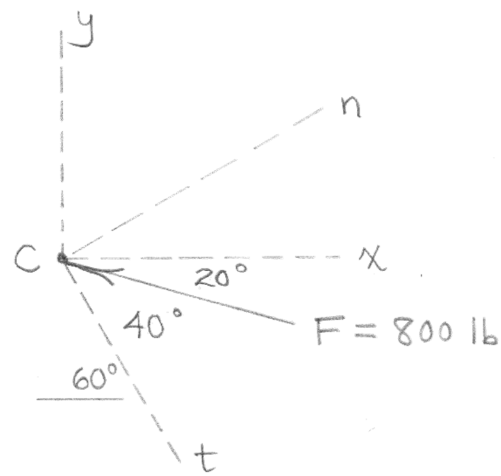
$$R_x = \Sigma F_x = 400 + 400 \cos 60° = 600 \text{ N}$$

$$R_y = \Sigma F_y = 400 \sin 60° = 346 \text{ N}$$

$$\Rightarrow \underline{R} = 600\underline{i} + 346\underline{j} \text{ N}$$

$$R = \sqrt{600^2 + 346^2} = 693 \text{ N}$$

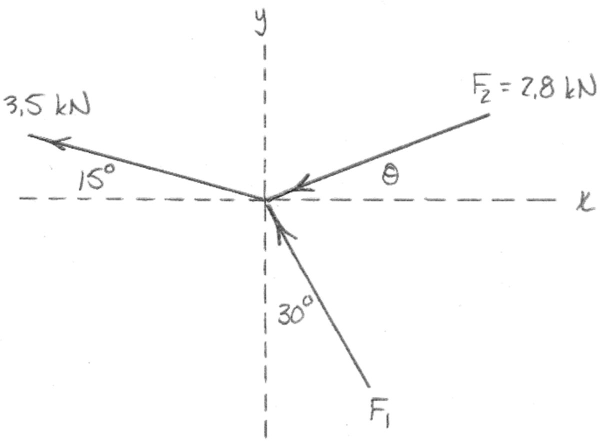
2/13



$$\begin{cases} F_x = -800 \cos 20° = \underline{-752 \text{ lb}} \\ F_y = 800 \sin 20° = \underline{274 \text{ lb}} \end{cases}$$

$$\begin{cases} F_n = -800 \sin 40° = \underline{-514 \text{ lb}} \\ F_t = -800 \cos 40° = \underline{-613 \text{ lb}} \end{cases}$$

2/14

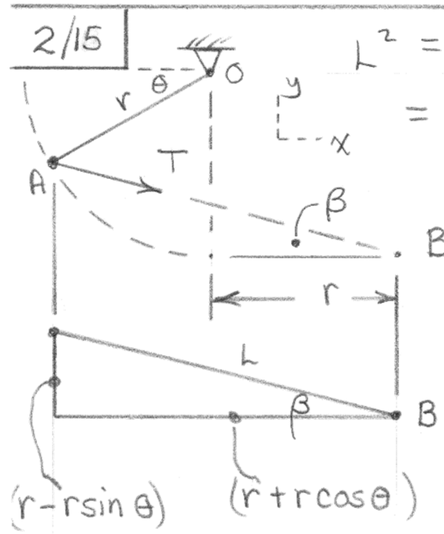


$$\begin{cases} R_x = \Sigma F_x : & -3.5\cos 15^\circ = -F_1 \sin 30^\circ - 2.8\cos\theta \quad (1) \\ R_y = \Sigma F_y : & 3.5 \sin 15^\circ = F_1 \cos 30^\circ - 2.8 \sin\theta \quad (2) \end{cases}$$

SOLVING (1) AND (2)...

$$\begin{cases} \underline{F_1 = 1.165 \text{ kN}} \\ \underline{\theta = 2.11^\circ} \end{cases} \quad \text{OR} \quad \begin{cases} \underline{F_1 = 3.78 \text{ kN}} \\ \underline{\theta = 57.9^\circ} \end{cases}$$

2/15



$$L^2 = (r - r\sin\theta)^2 + (r + r\cos\theta)^2$$

$$= r^2 - 2r^2\sin\theta + r^2\sin^2\theta + r^2 + 2r^2\cos\theta + r^2\cos^2\theta$$

$$= r^2(3 + 2\cos\theta - 2\sin\theta)$$

So $\cos\beta = \dfrac{r(1+\cos\theta)}{r\sqrt{3+2\cos\theta-2\sin\theta}}$

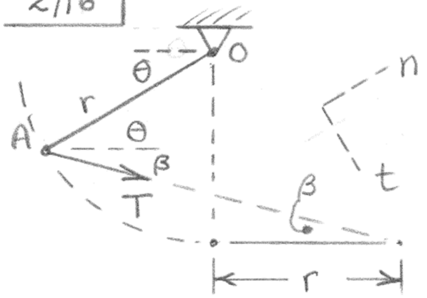
$$= \frac{1+\cos\theta}{\sqrt{3+2\cos\theta-2\sin\theta}}$$

$$\sin\beta = \frac{1-\sin\theta}{\sqrt{3+2\cos\theta-2\sin\theta}}$$

$$T_x = T\cos\beta = \underline{\frac{T(1+\cos\theta)}{\sqrt{3+2\cos\theta-2\sin\theta}}}$$

$$T_y = -T\sin\beta = \underline{\frac{T(\sin\theta-1)}{\sqrt{3+2\cos\theta-2\sin\theta}}}$$

2/16



From solution to previous problem:

$$\beta = \tan^{-1}\left[\frac{1-\sin\theta}{1+\cos\theta}\right]$$

$$\begin{cases} T_n = T\cos(\theta+\beta) \\ T_t = T\sin(\theta+\beta) \end{cases}$$

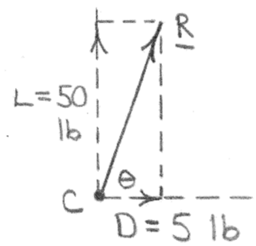
For $T = 100$ N and $\theta = 35°$:

$\beta = 13.19°$

$$\begin{cases} T_n = 66.7 \text{ N} \\ T_t = 74.5 \text{ N} \end{cases}$$

2/17

$$\frac{L}{D} = \frac{50}{D} = 10 \; ; \quad D = 5 \text{ lb}$$



$$R = \sqrt{L^2 + D^2} = \sqrt{50^2 + 5^2}$$

$$= \underline{50.2 \text{ lb}}$$

$$\theta = \tan^{-1}\left(\frac{L}{D}\right) = \tan^{-1}\left(\frac{50}{5}\right)$$

$$= \underline{84.3°}$$

2/18 Using the coordinates of the problem figure:

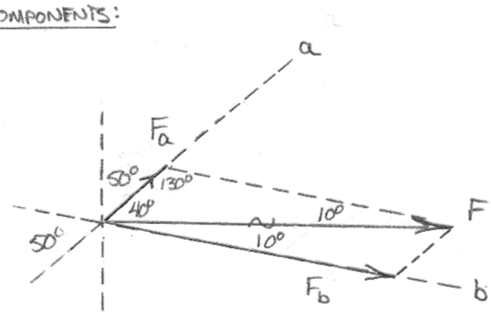
$$R_x = \Sigma F_x = 200 \cos 35° - 150 \sin 30°$$
$$= 88.8 \text{ N}$$

$$R_y = \Sigma F_y = 200 \sin 35° + 150 \cos 30°$$
$$= 245 \text{ N}$$

$$\therefore \underline{R} = 88.8\underline{i} + 245\underline{j} \text{ N}$$

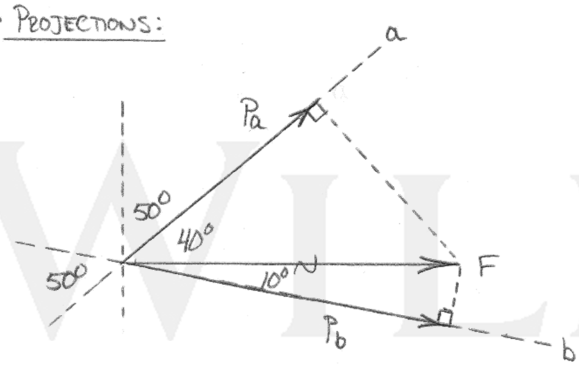
2/19 $F = 2.5$ kN

• COMPONENTS:



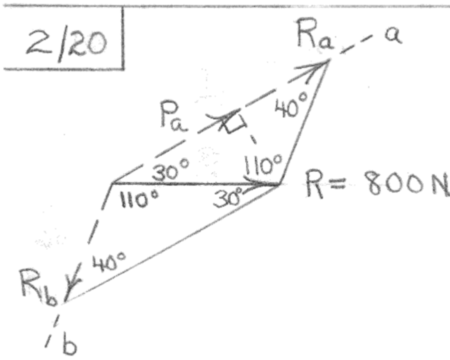
$$\frac{F}{\sin 130^\circ} = \frac{F_a}{\sin 10^\circ} = \frac{F_b}{\sin 40^\circ} \longrightarrow \begin{cases} \underline{F_a = 0.567 \text{ kN}} \\ \underline{F_b = 2.10 \text{ kN}} \end{cases}$$

• PROJECTIONS:



$$\begin{cases} P_a = F\cos 40^\circ = 2.5\cos 40^\circ \longrightarrow \underline{P_a = 1.915 \text{ kN}} \\ P_b = F\cos 10^\circ = 2.5\cos 10^\circ \longrightarrow \underline{P_b = 2.46 \text{ kN}} \end{cases}$$

2/20



Law of sines:

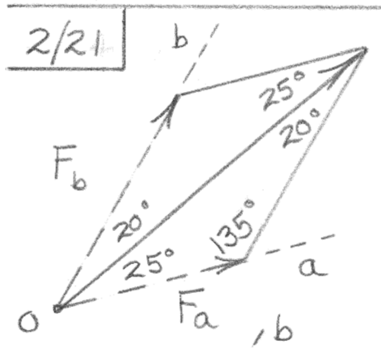
$$\frac{800}{\sin 40°} = \frac{R_a}{\sin 110°} = \frac{R_b}{\sin 30°}$$

$\underline{R_a = 1170 \text{ N}}$

$\underline{R_b = 622 \text{ N}}$

Projection $P_a = R\cos 30° = 800 \cos 30° = \underline{693 \text{ N}}$

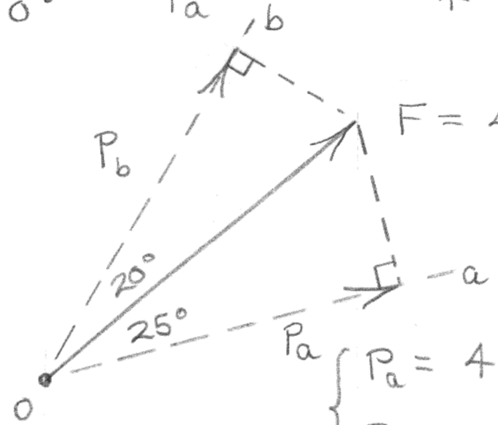
2/21



Components:

$$\frac{\sin 135^\circ}{4} = \frac{\sin 20^\circ}{F_a}, \quad \underline{F_a = 1.935 \text{ kN}}$$

$$\frac{\sin 135^\circ}{4} = \frac{\sin 25^\circ}{F_b}, \quad \underline{F_b = 2.39 \text{ kN}}$$



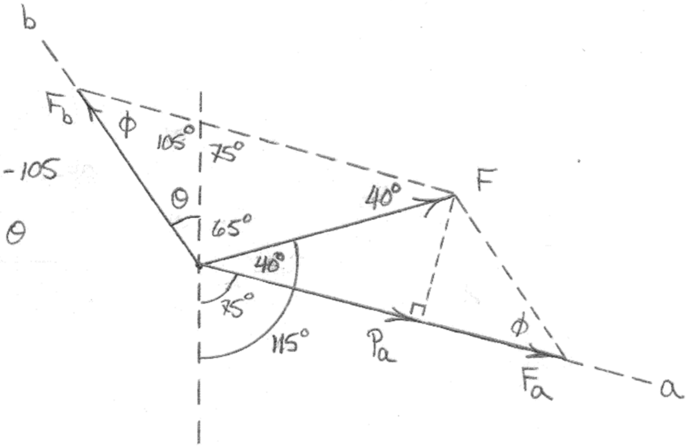
Projections:

$$\begin{cases} P_a = 4 \cos 25^\circ = \underline{3.63 \text{ kN}} \\ P_b = 4 \cos 20^\circ = \underline{3.76 \text{ kN}} \end{cases}$$

2/22

$\phi = 180 - \theta - 105$

$\phi = 75° - \theta$

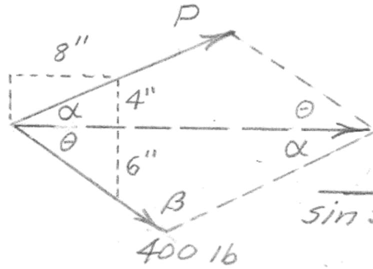


$P_a = F\cos 40° \longrightarrow 325 = F\cos 40° \longrightarrow \underline{F = 424 \text{ N}}$

$$\frac{F}{\sin\phi} = \frac{F_b}{\sin 40°} \longrightarrow \frac{424}{\sin(75°-\theta)} = \frac{325}{\sin 40°}$$

SOLVING THIS YIELDS... $\underline{\theta = 17.95° \text{ OR } -48.0°}$

2/23



$\alpha = \tan^{-1}\frac{4}{8} = 26.57°$

$\theta = \tan^{-1}\frac{6}{8} = 36.87°$

$\beta = 180 - (\alpha + \theta) = 116.57°$

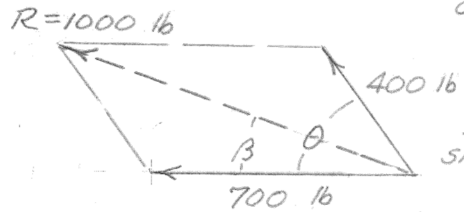
$$\frac{P}{\sin 36.87°} = \frac{400}{\sin 26.57°}$$

$$P = 400\,\frac{0.6}{0.4472} = \underline{537\ \text{lb}}$$

$$\frac{T}{\sin 116.56°} = \frac{400}{\sin 26.57°}, \quad T = 400\,\frac{0.8944}{0.4472} = \underline{800\ \text{lb}}$$

2/24 Law of cosines: $\overline{1000}^2 = \overline{400}^2 + \overline{700}^2 + 2(400)(700)\cos\theta$

$\cos\theta = 0.6250$, $\underline{\theta = 51.3^\circ}$



Law of sines:

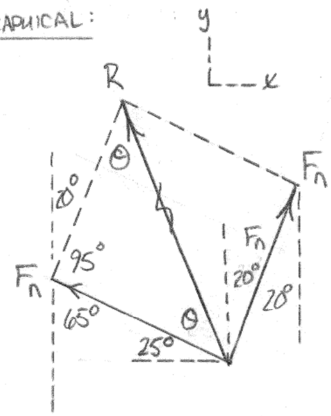
$$\frac{1000}{\sin(180-51.3)^\circ} = \frac{400}{\sin\beta}$$

$$\sin\beta = \frac{400}{1000}\,0.7806 = 0.312$$

$$\underline{\beta = 18.19^\circ}$$

2/25

• GRAPHICAL:



$$R = \sqrt{F_n^2 + F_n^2 - 2F_n^2 \cos 95°}$$

$$R = \sqrt{2(5500)^2[1 - \cos 95°]}$$

$$\underline{R = 8110 \text{ N}}$$

$$\theta = \frac{180° - 95°}{2} \longrightarrow \theta = 42.5°$$

$$\underline{R = 8110 \text{ N @ } 112.5° \text{ CCW FROM } +x \text{ AXIS}}$$

• VECTORS:

$$\underline{R} = (F_n \sin 20° - F_n \sin 65°)\underline{i} + (F_n \cos 20° + F_n \cos 65°)\underline{j}$$

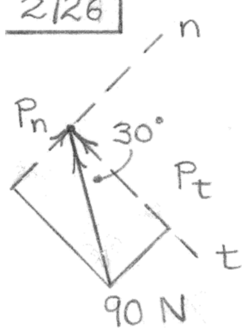
$$\underline{R} = 5500[(\sin 20° - \sin 65°)\underline{i} + (\cos 20° + \cos 65°)\underline{j}]$$

$$\underline{R} = -3100\underline{i} + 7490\underline{j} \text{ N}$$

$$R = \sqrt{3100^2 + 7490^2} \longrightarrow \underline{R = 8110 \text{ N}}$$

$$\theta_x = \cos^{-1}\left(\frac{R_x}{R}\right) = \cos^{-1}\left(\frac{-3100}{8110}\right) \longrightarrow \underline{\theta_x = 112.5° \text{ CCW FROM } +x \text{ AXIS}}$$

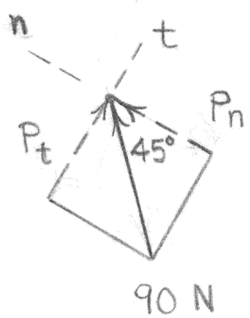
2/26



BC

$P_t = -90 \cos 30° = \underline{-77.9 \text{ N}}$

$P_n = 90 \sin 30° = \underline{45.0 \text{ N}}$

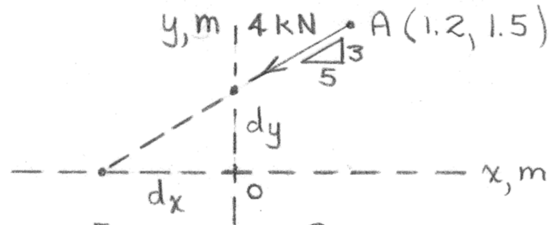


AB

$P_t = 90 \sin 45° = \underline{63.6 \text{ N}}$

$P_n = 90 \cos 45° = \underline{63.6 \text{ N}}$

2/27



$$\curvearrowleft + \; M_O = 4\left[\frac{5}{\sqrt{34}}(1.5) - \frac{3}{\sqrt{34}}(1.2)\right] = \underline{2.68 \text{ kN·m}}$$

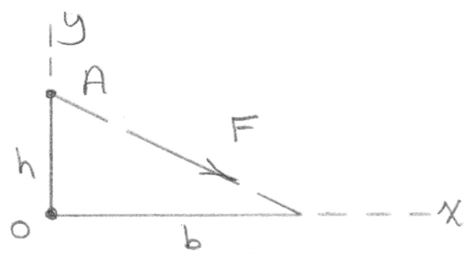
As a vector, $\underline{\mathbf{M}_O = 2.68\mathbf{k} \text{ kN·m}}$

$$\frac{1.5}{d_x + 1.2} = \frac{3}{5}, \quad d_x = 1.3 \text{ m}$$

$$\frac{d_y}{1.3} = \frac{3}{5}, \quad d_y = 0.78 \text{ m}$$

Coordinates of intercepts (in m): $\underline{(-1.3, 0), (0, 0.78)}$

2/28

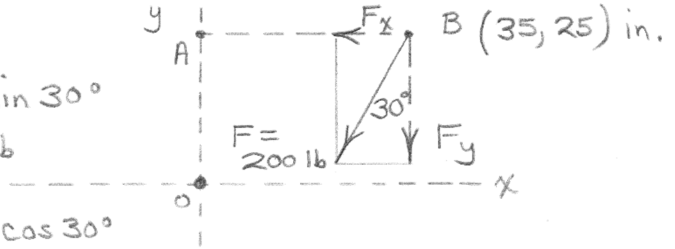


$$\underline{F} = F\left[\frac{b\underline{i} - h\underline{j}}{\sqrt{b^2+h^2}}\right]$$

Acting at A:

$$+\circlearrowright M_O = \frac{Fb}{\sqrt{h^2+b^2}}(h) = \underline{\frac{Fbh}{\sqrt{h^2+b^2}} \quad \text{CW}}$$

2/29



$|F_x| = 200 \sin 30° = 100$ lb

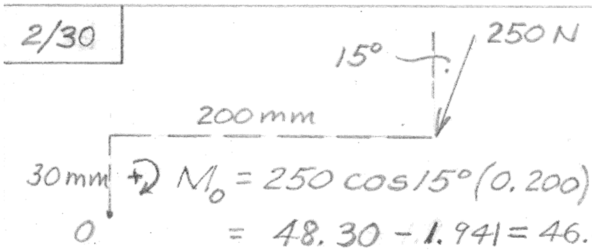
$|F_y| = 200 \cos 30° = 173.2$ lb

$+\circlearrowright M_A = 173.2(35) = 6060$ lb-in. (505 lb-ft) CW

$+\circlearrowright M_O = 173.2(35) - 100(25)$

$= 3560$ lb-in. (297 lb-ft) CW

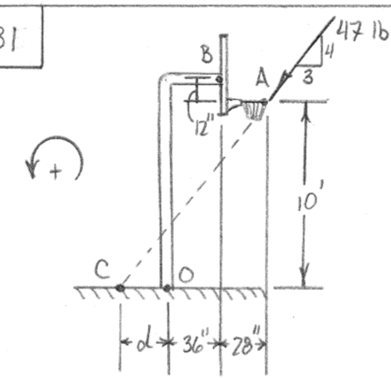
2/30



$$+\circlearrowleft\ M_O = 250\cos 15°(0.200) - 250\sin 15°(0.030)$$

$$= 48.30 - 1.941 = \underline{46.4\ \text{N}\cdot\text{m}}$$

2/31

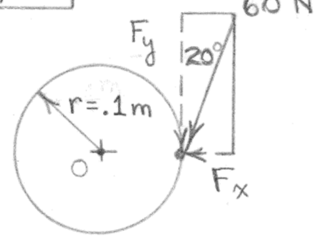


a) $M_O = 10\left(\frac{3}{5}47\right) - \left(\frac{36+28}{12}\right)\left(\frac{4}{5}47\right) = \underline{81.5} \longrightarrow \underline{M_O = 81.5 \text{ lb-ft CCW}}$

b) $M_B = -\frac{12}{12}\left(\frac{3}{5}47\right) - \frac{28}{12}\left(\frac{4}{5}47\right) = \underline{-115.9} \longrightarrow \underline{M_B = 115.9 \text{ lb-ft CW}}$

c) $M_C = 0 = 10\left(\frac{3}{5}47\right) - \left(\frac{36+28+d}{12}\right)\left(\frac{4}{5}47\right) \longrightarrow \underline{d = 26 \text{ in. LEFT OF O}}$

2/32



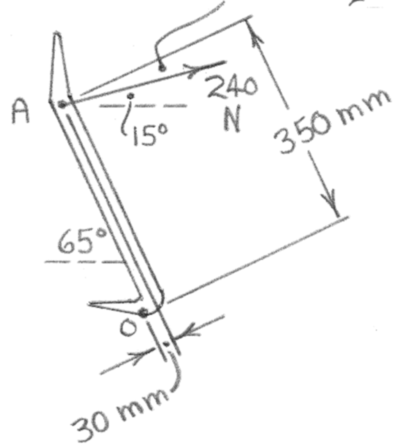
$$+\circlearrowleft M_O = r F_y$$

$$= (0.1)(60 \cos 20°)$$

$$= \underline{5.64 \text{ N}\cdot\text{m}}$$

2/33

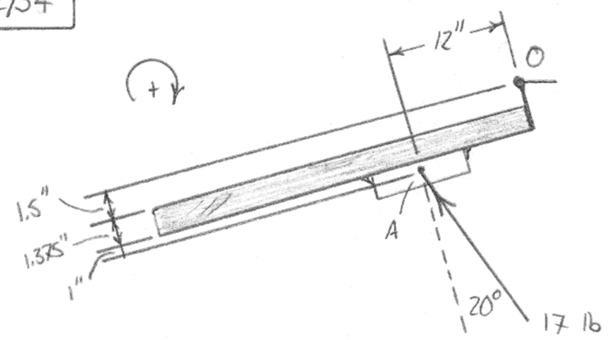
$25° - 15° = 10°$



$$\circlearrowright\!+\; M_O = (240 \cos 10°)(0.350) + (240 \sin 10°)(0.030)$$

$$\underline{M_O = 84.0 \text{ N·m CW}}$$

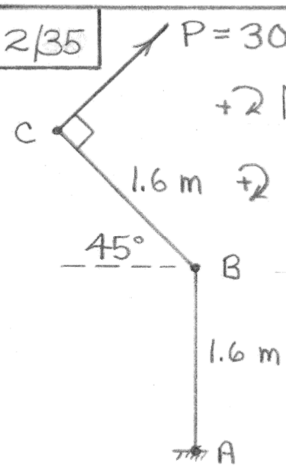
2/34



$$M_O = 12(17\cos 20^\circ) + (1 + 1.375 + 1.5)(17\sin 20^\circ)$$

$$\underline{M_O = 214 \text{ lb-in. CW}}$$

2/35

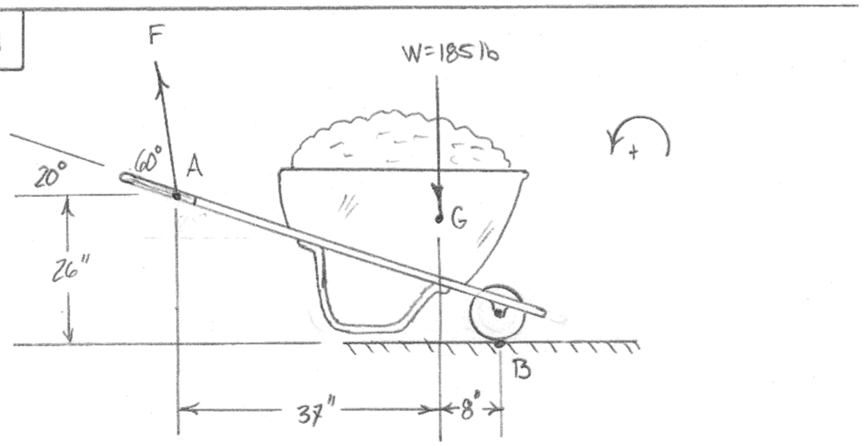


$\circlearrowleft + M_B = 30(1.6) = \underline{48 \text{ N}\cdot\text{m}}$

$\circlearrowleft + M_A = 30 \cos 45^\circ (1.6 + 1.6 \sin 45^\circ)$

$+ 30 \sin 45^\circ (1.6 \cos 45^\circ) = \underline{81.9 \text{ N}\cdot\text{m}}$

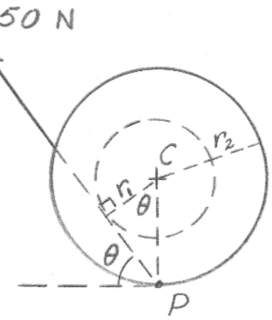
2/36



$\Sigma M_B = 0: \quad 185(8) - F\sin(20^\circ+60^\circ)(37+8) + F\cos(20^\circ+60^\circ)(26) = 0$

$\therefore \quad \underline{F = 37.2 \text{ lb}}$

2/37



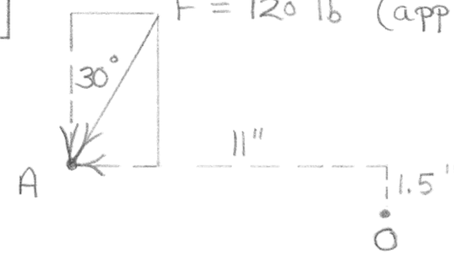
$$\circlearrowright + \; M_C = Tr_1 = 150(0.125)$$
$$= \underline{18.75 \text{ N·m CW}}$$

$$\cos\theta = \frac{r_1}{r_2} = \frac{125}{200}$$

$$\underline{\theta = 51.3°}$$

2/38

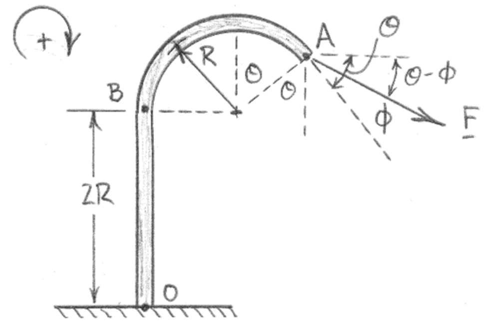
$F = 120$ lb (applied at A)



$$+\circlearrowleft M_O = 120\cos 30^\circ(11) + 120\sin 30^\circ(1.5)$$

$$\simeq \underline{1233 \text{ lb-in.}} \text{ or } \underline{102.8 \text{ lb-ft CCW}}$$

2/39



$$M_B = F\sin(\theta-\phi)(R+R\sin\theta) + F\cos(\theta-\phi)(R\cos\theta)$$

$$\underline{M_B = FR\left[\cos\phi + \sin(\theta-\phi)\right]}$$

$$M_O = F\sin(\theta-\phi)(R+R\sin\theta) + F\cos(\theta-\phi)(2R+R\cos\theta)$$

$$\underline{M_O = FR\left[2\cos(\theta-\phi) + \cos\phi + \sin(\theta-\phi)\right]}$$

IF $F = 750$ N, $R = 2.4$ m, $\theta = 30^\circ$, AND $\phi = 15^\circ$...

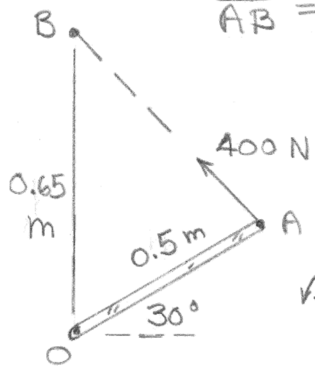
$\underline{M_B = 2200 \text{ N}\cdot\text{m CW}}$

$\underline{M_O = 5680 \text{ N}\cdot\text{m CW}}$

2/40

$$\overline{AB}^2 = 0.65^2 + 0.5^2 - 2(0.65)(0.5)\cos 60°$$

$$\overline{AB} = 0.589 \text{ m}$$



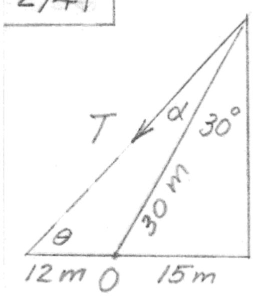
$$\frac{\sin 60°}{\overline{AB}} = \frac{\sin \measuredangle OAB}{0.65}$$

$$\measuredangle OAB = 72.7°$$

$$\curvearrowleft + \; M_O = 0.5(400 \sin 72.7°)$$

$$= \underline{191.0 \text{ N·m CCW}}$$

2/41



$$\theta = \tan^{-1}\frac{30(0.866)}{12+15} = 43.90^\circ$$

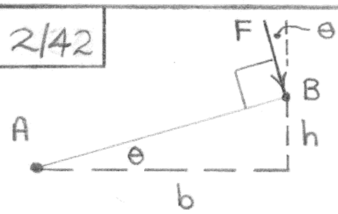
$$\alpha = 90^\circ - (30^\circ + 43.90^\circ) = 16.10^\circ$$

$$M_O = 72 \text{ kN}\cdot\text{m}$$

$$= T \sin 16.10^\circ (30) = 8.32T$$

$$T = \frac{72}{8.32} = \underline{8.65 \text{ kN}}$$

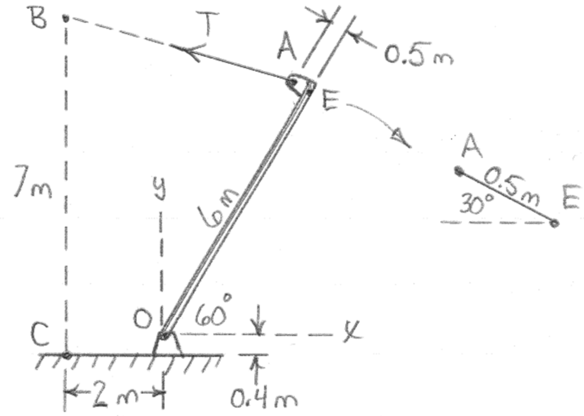
2/42



M_A is maximum when $\underline{F}$ is perpendicular to AB.

Thus $\underline{\theta = \tan^{-1}(h/b)}$

2/43 $T = 6.75$ kN



$$\underline{r}_{OA} = (6\cos 60° - 0.5\cos 30°)\underline{i} + (6\sin 60° + 0.5\sin 30°)\underline{j}$$

$$= 2.57\underline{i} + 5.45\underline{j} \text{ m}$$

$$\underline{r}_{OB} = -2\underline{i} + (7 - 0.4)\underline{j} = -2\underline{i} + 6.6\underline{j} \text{ m}$$

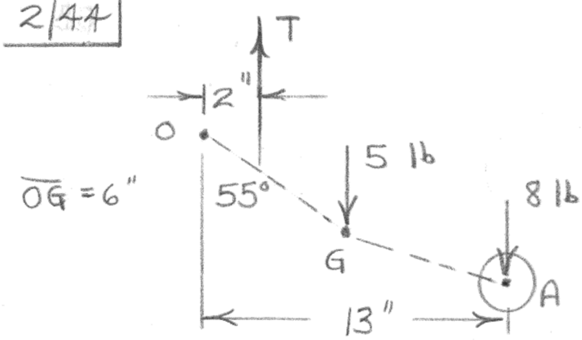
$$\underline{n}_{AB} = \frac{\underline{r}_{OB} - \underline{r}_{OA}}{|\underline{r}_{OB} - \underline{r}_{OA}|} \longrightarrow \underline{n}_{AB} = -0.970\underline{i} + 0.245\underline{j}$$

$$\underline{T} = T\underline{n}_{AB} = 6.75(-0.970\underline{i} + 0.245\underline{j}) = -6.54\underline{i} + 1.653\underline{j} \text{ kN}$$

$$\underline{M}_O = \underline{r}_{OA} \times \underline{T} = (2.57\underline{i} + 5.45\underline{j}) \times (-6.54\underline{i} + 1.653\underline{j})$$

$$\underline{\underline{M}_O = 39.9\,\underline{k} \text{ kN}\cdot\text{m}}$$

2/44



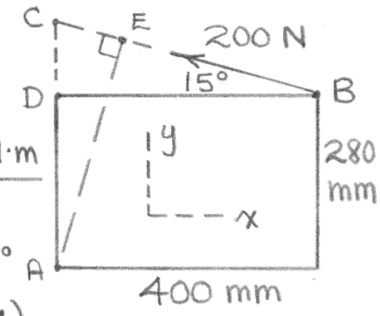
The combined moment about O of the 5-lb and 8-lb weights is

$$+\circlearrowright M_O = 5(6 \sin 55°) + 8(13) = 128.6 \text{ lb-in. (CW)}$$

$$+\circlearrowright \Sigma M_O = 0 : -T(2) + 128.6 = 0$$

$$\underline{T = 64.3 \text{ lb}}$$

2/45



(1) $M_A = 200 \cos 15° (0.280) + 200 \sin 15° (0.400) = \underline{74.8 \text{ N·m}}$

(2) $AE = 0.280 \cos 15° + 0.400 \sin 15° = 0.374$ m ; $M_A = 200(0.374) = \underline{74.8 \text{ N·m}}$

(3) Apply force at C

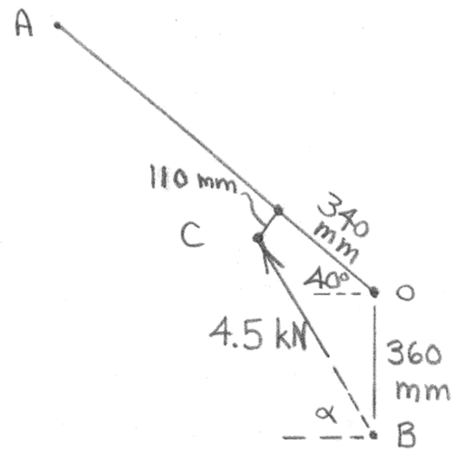
$CD = 0.400 \tan 15° = 0.1072$ m

$CA = 0.280 + 0.1072 = 0.387$ m

$M_A = (200 \cos 15°)(0.387) = \underline{74.8 \text{ N·m}}$

(4) $\underline{M}_A = \underline{r} \times \underline{F} = (0.400\underline{i} + 0.280\underline{j}) \times 200(-\cos 15°\underline{i} + \sin 15°\underline{j}) = \underline{74.8\underline{k} \text{ N·m}}$

2/46



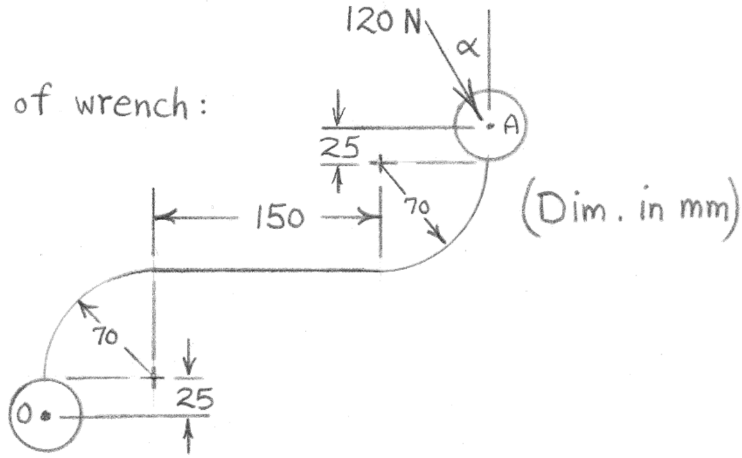
$$\alpha = \tan^{-1}\left[\frac{360 + 340 \sin 40° - 110 \sin 50°}{340 \cos 40° + 110 \cos 50°}\right]$$

$$= 56.2°$$

$$\stackrel{+}{\curvearrowright} M_O = 4.5\,(0.360 \cos 56.2°) = \underline{0.902 \text{ kN}\cdot\text{m CW}}$$

2/47

Elements of wrench:



$\alpha = 30°$:

$$+\circlearrowleft M_O = 120 \cos 30° [70 + 150 + 70]$$
$$+ 120 \sin 30° [25 + 70 + 70 + 25] = 41\,500 \text{ N·mm}$$

or $M_O = 41.5$ N·m CW

For maximum M_O:

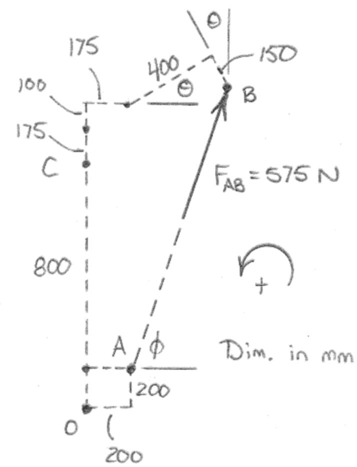
$$\alpha = \tan^{-1}\left[\frac{25+70+25+70}{70+150+70}\right] = 33.2°$$

$$(M_O)_{max} = 120\sqrt{(25+70+25+70)^2 + (70+150+70)^2}$$
$$= 41\,600 \text{ N·mm or } 41.6 \text{ N·m CW}$$

2/48

$$\phi = \tan^{-1}\left(\frac{800 + 175 + 100 + 400\sin\theta - 150\cos\theta}{175 + 400\cos\theta + 150\sin\theta - 200}\right) = 70.9^\circ$$

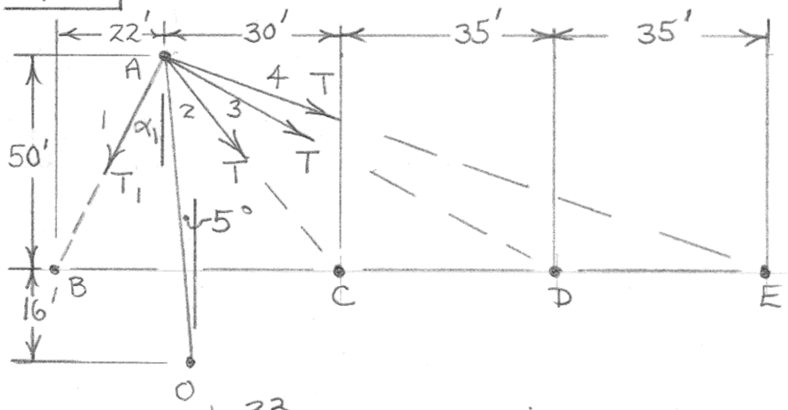
$\theta = 30^\circ$



$$M_O = 575\sin\phi(0.200) - 575\cos\phi(0.200) \longrightarrow \underline{M_O = 71.1 \text{ N·m CCW}}$$

$$M_C = 575\sin\phi(0.200) + 575\cos\phi(0.800) \longrightarrow \underline{M_C = 259 \text{ N·m CCW}}$$

2/49



$\alpha_1 = \tan^{-1}\frac{22}{50} = 23.7°$; Similarly, $\alpha_2 = 31.0°$,

$\alpha_3 = 52.4°$, $\alpha_4 = 63.4°$ (all relative to vertical)

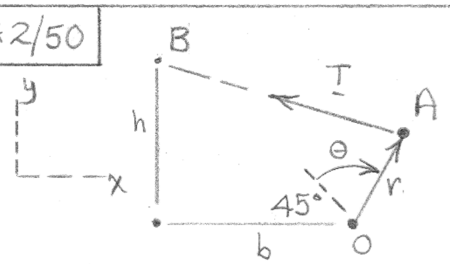
$$\circlearrowleft\Sigma M_O = 0: \left[T_1 \sin(\alpha_1 + 5°) - T\sin(\alpha_2 - 5°) - T\sin(\alpha_3 - 5°) - T\sin(\alpha_4 - 5°)\right]\frac{50+16}{\cos 5°} = 0$$

$\underline{T_1 = 4.21T}$

$$+\downarrow \Sigma F = P = T_1\cos(\alpha_1 + 5°) + T\left[\cos(\alpha_2 - 5°) \cos(\alpha_3 - 5°) + \cos(\alpha_4 - 5°)\right]$$

or $\underline{P = 5.79T}$ (with $T_1 = 4.21T$)

*2/50



$$\underline{M}_O = \underline{r}_{OA} \times \underline{T}$$

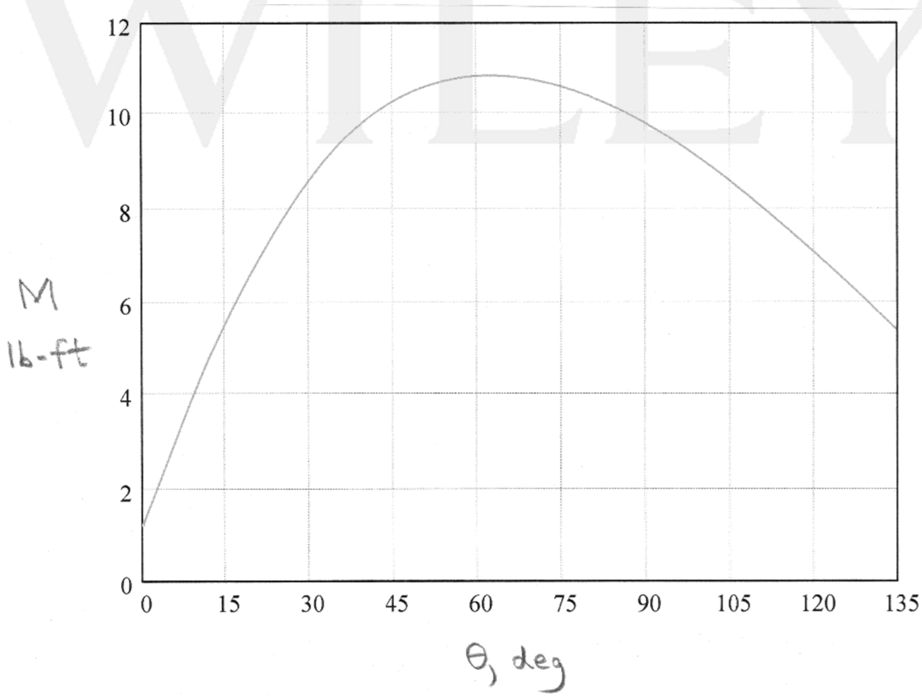
$$\underline{r}_{OA} = r\left[-\cos(\theta+45°)\underline{i} + \sin(\theta+45°)\underline{j}\right]$$

$\underline{T} = W\underline{n}_{AB}$, where

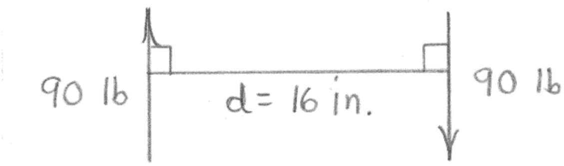
$$\underline{n}_{AB} = \frac{[-b + r\cos(\theta+45°)]\underline{i} + [h - r\sin(\theta+45°)]\underline{j}}{\left\{[-b + r\cos(\theta+45°)]^2 + [h - \sin(\theta+45°)]^2\right\}^{1/2}}$$

With $r = \frac{13}{12}$ ft, $h = \frac{21}{12}$ ft, $b = \frac{24}{12}$ ft, and $W = 10$ lb, vary θ and take the z-comp. of $\underline{M}_O$ to obtain the following plot.

Note that $M_{max} = 10.83$ lb-ft at $\theta = 62.1°$



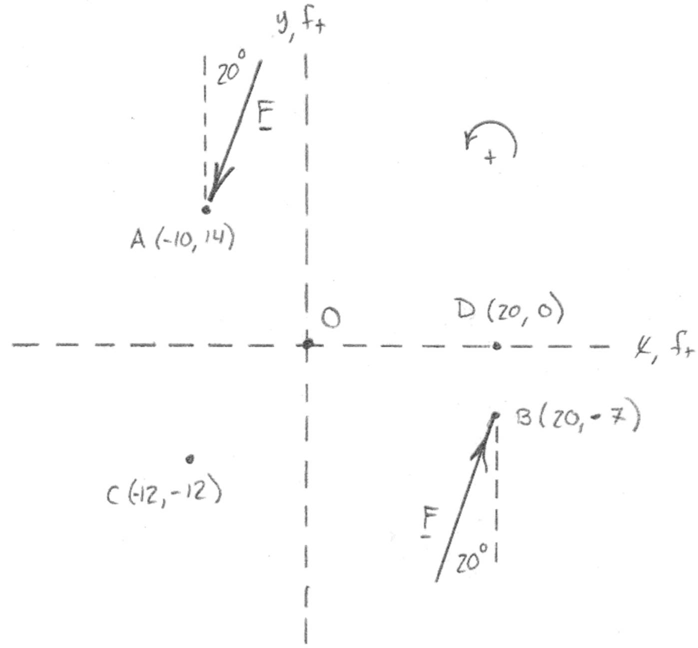
2/51 $\circlearrowright^{+}$ $M = M_O = M_A = Fd = 90(16) = 1440$ lb-in. CW



2/52 $\overset{+}{\circlearrowright}$ $M = Fd = 80(1.4) = \underline{112 \text{ lb-in. CW}}$

$\underline{(\text{or } 9.33 \text{ lb-ft CW})}$

2/53 $F = 65$ lb



a) $M_O = 65 \cos 20^\circ (20 + 10) + 65 \sin 20^\circ (14 + 7)$

$M_O = 2300$ lb-ft CCW

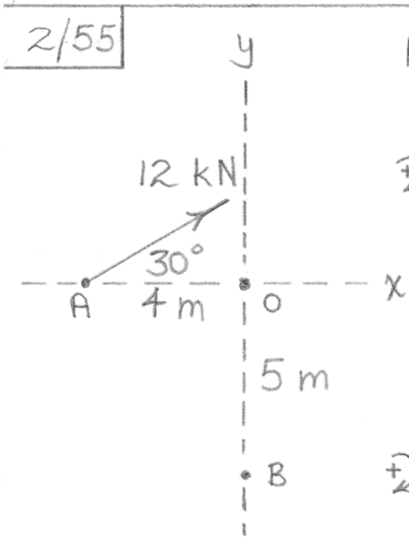
b) By inspection... $M_C = 2300$ lb-ft CCW

c) By inspection... $M_D = 2300$ lb-ft CCW

2/54 $\mathbf{R} = 6\mathbf{j}$ kN @ $x = \frac{400}{6000} = 0.0667$ m

or $x = 66.7$ mm

2/55



At O:

$F = \underline{12 \text{ kN} \angle 30°}$

$\circlearrowright + \; M_O = (12 \sin 30°)(4)$

$= \underline{24 \text{ kN}\cdot\text{m CW}}$

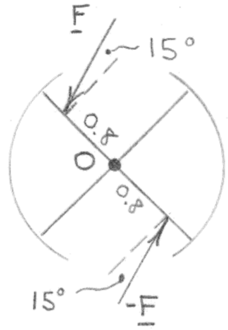
At B:

$F = \underline{12 \text{ kN} \angle 30°}$

$\circlearrowright + \; M_B = (12 \sin 30°)(4) + (12 \cos 30°)(5) = 76.0 \text{ kN}\cdot\text{m}$

$\underline{\text{CW}}$

2/56

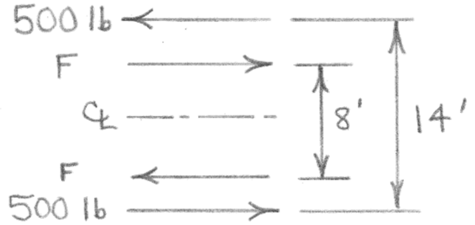


$\circlearrowleft + \; M_O = \Sigma Fd$

$25 = 2F(\cos 15°)(0.8)$

$F = \underline{16.18 \text{ N}}$

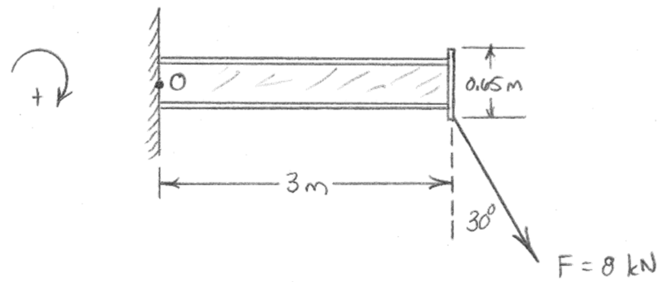
2/57



$$\circlearrowleft + \; \Sigma M = 500(14) - F(8) = 0$$

$$\underline{F = 875 \text{ lb}}$$

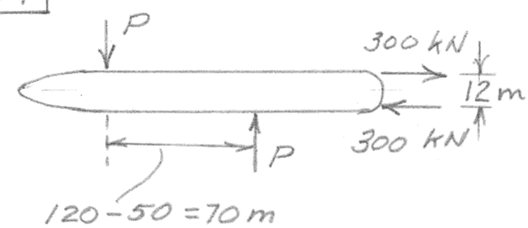
2/58



F = 8 kN @ 60° CW BELOW HORIZONTAL

$$M_O = 8\cos 30^\circ (3) - 8\sin 30^\circ \left(\frac{0.65}{2}\right) \longrightarrow M_O = 19.48 \text{ kN}\cdot\text{m CW}$$

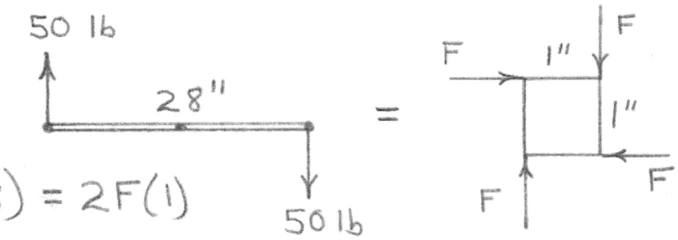
2/59



$70P = 300(12)$

$\underline{P = 51.4 \text{ kN}}$

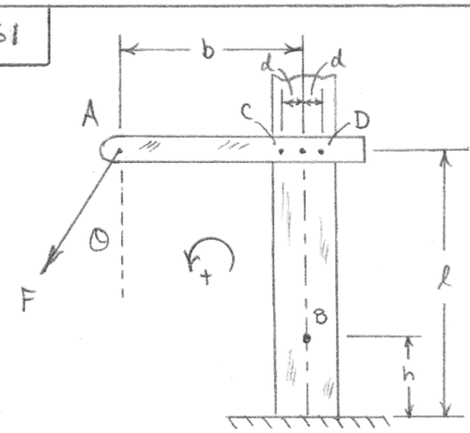
2/60



$M = 50(28) = 2F(1)$

$\underline{F = 700 \text{ lb}}$

2/61



$F = 425$ N
$\theta = 30°$
$b = 1.9$ m
$d = 0.2$ m
$h = 0.8$ m
$\ell = 2.75$ m

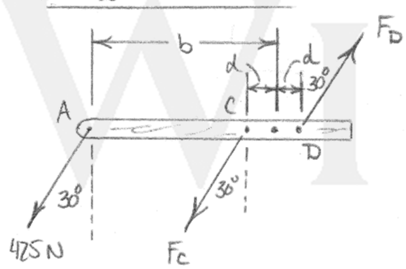
a) Force - Couple At B:

F = 425 N @ 120° CW Below Horizontal

$$M_B = F\cos\theta \, b + F\sin\theta \, (\ell - h) = 425\cos 30°(1.9) + 425\sin 30°(2.75 - 0.8)$$

$\therefore$ $M_B = 1114$ N·m CCW

b) Forces At C and D:



$\Sigma F_{\nearrow}$: $F_C - F_D = 425$

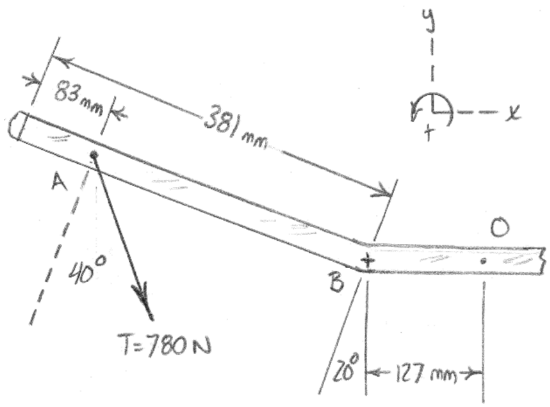
$\Sigma M_A = 0$: $F_d \cos 30°(b+d) - F_C \cos 30°(b-d) = 0$

Solving...

$F_C = 2230$ N @ 120° CW Below Horizontal

$F_D = 1806$ N @ 60° CCW Above Horizontal

2/62



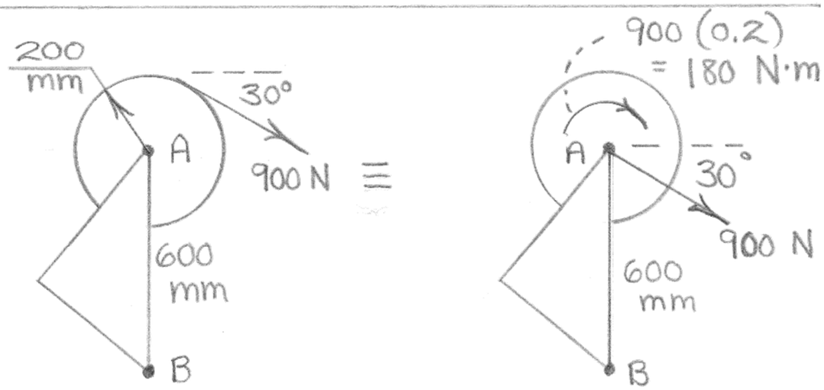
$\underline{T} = 780 \sin 20° \underline{i} - 780 \cos 20° \underline{j} \longrightarrow \underline{T} = 267 \underline{i} - 733 \underline{j}$ N

$\underline{M}_B = 780 \cos 40° \left(\frac{381 - 83}{1000}\right) \underline{k} \longrightarrow \underline{M}_B = 178.1 \underline{k}$ N·m

$\underline{M}_O = \underline{M}_B + T \cos 20° (\overline{OB}) = \left(178.1 + 780 \cos 20° \left(\frac{127}{1000}\right)\right) \underline{k}$

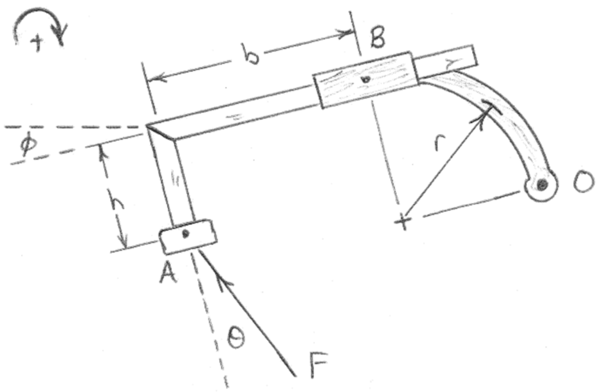
$\therefore \underline{M}_O = 271 \hat{k}$ N·m

2/63



$$+\circlearrowright M_B = 180 + 900\cos 30^\circ(0.6) = \underline{648\ \text{N·m CW}}$$

2/64 $b = 450\text{ mm}$, $h = 215\text{ mm}$, $r = 325\text{ mm}$, $F = 520\text{ N}$, $\theta = 15°$, $\phi = 10°$



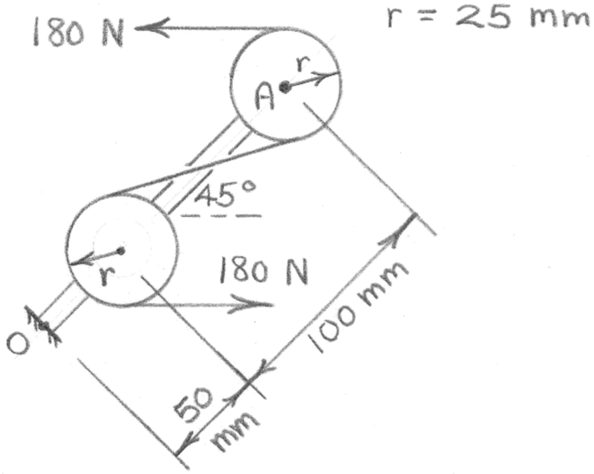
F = 520 N @ 115° CCW ABOVE HORIZONTAL

$$M_O = F\cos\theta(b+r) - F\sin\theta(r-h)$$

$$= 520\cos 15°\left(\frac{450+325}{1000}\right) - 520\sin 15°\left(\frac{325-215}{1000}\right)$$

$\therefore$ $M_O = 374\text{ N}\cdot\text{m}$ CW

2/65



The system at O is a couple.

$$\curvearrowleft + M = Fd = 180\,(100 \sin 45^\circ + 25 + 25)$$

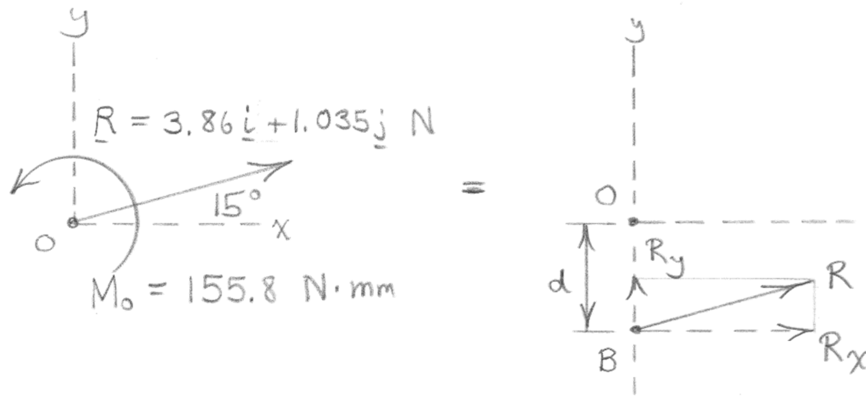
$$= 21\,700 \text{ N·mm or } \underline{21.7 \text{ N·m CCW}}$$

2/66 At O:

$\underline{R} = 4(\cos 15° \underline{i} + \sin 15° \underline{j}) = 3.86\underline{i} + 1.035\underline{j}$ N

$\circlearrowleft + M_O = 300 - 4\cos 15°(40) + 4\sin 15°(10)$

$= 155.8$ N·mm CCW

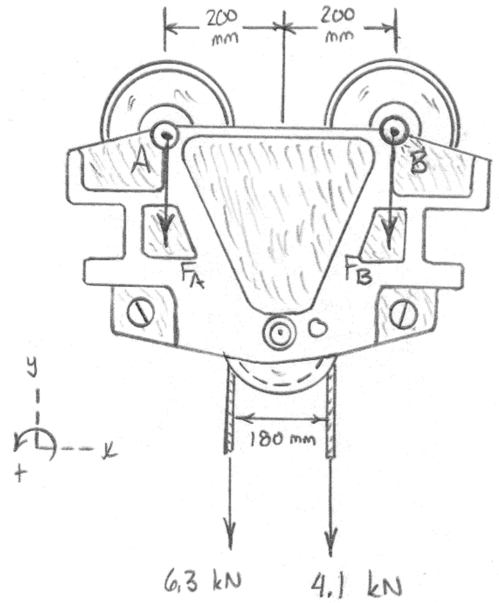


Condition: $R_x d = M_O$

$3.86 d = 155.8$, $d = 40.3$ mm

So $\underline{y = -40.3 \text{ mm}}$

2/67



F_A & F_B ARE DOWN AT A & B.

ΣF_y: $F_A + F_B = 6.3 + 4.1$

ΣM_O: $200 F_A - 200 F_B = 90(6.3) - 90(4.1)$

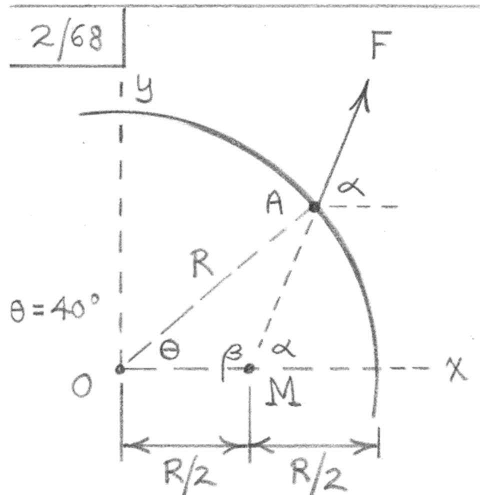
SOLVING...

$\underline{F_A = 5.70 \text{ kN}}$

$\underline{F_B = 4.70 \text{ kN}}$

(BOTH DOWN AS SHOWN)

2/68



$$\overline{AM}^2 = R^2 + (R/2)^2 - 2(R)\left(\frac{R}{2}\right)\cos 40°$$

$$= 0.484R^2, \quad \overline{AM} = 0.696R$$

$$\frac{\sin\beta}{R} = \frac{\sin 40°}{0.696R}, \quad \beta = 112.5°$$

$$\alpha = 180° - \beta = 180° - 112.5° = 67.5°$$

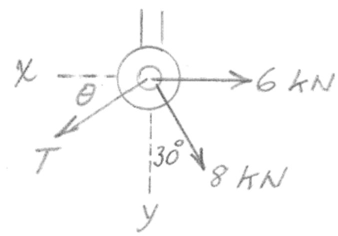
$$M_O = (F\sin\alpha)\frac{R}{2} = (F\sin 67.5°)\frac{R}{2}$$

$$= 0.462FR$$

So the force-couple system for $\theta = 40°$ is

$$\begin{cases} F \ \angle 67.5° \\ M_O = 0.462FR \quad \text{CCW} \end{cases}$$

2/69



$R = R_y = 15 = T\sin\theta + 8\cos 30°$

$R_x = 0 = T\cos\theta - 6 - 8\sin 30°$

So $T\sin\theta = 8.07$

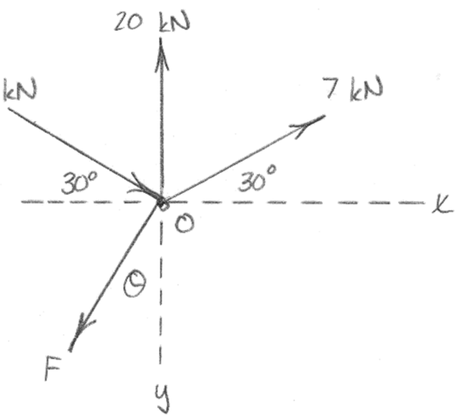
$T\cos\theta = 10$

Divide & set $\theta = \tan \frac{8.07}{10}$

$\underline{\theta = 38.9°}$

$T = \frac{10}{\cos 38.9°} = \underline{12.85 \text{ kN}}$

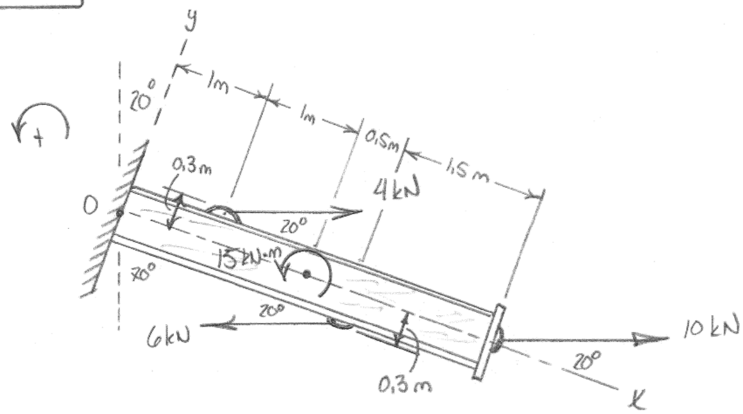
2/70 R = 9 kN, RIGHTWARD



$$\begin{cases} R_x = 9 = 11\cos 30^\circ + 7\cos 30^\circ - F\sin\theta \\ R_y = 0 = 11\sin 30^\circ - 7\sin 30^\circ + F\cos\theta - 20 \end{cases}$$

SOLVING... $\underline{F = 19.17 \text{ kN}}$ AND $\underline{\theta = 20.1^\circ}$

2/71



$$R = 10 + 4 - 6 \longrightarrow R = 8 \text{ kN}$$

$$\underline{R} = 8\cos 20^\circ \underline{i} + 8\sin 20^\circ \underline{j} \longrightarrow \underline{\underline{R} = 7.52\,\underline{i} + 2.74\,\underline{j} \text{ kN}}$$

$$M_O = 15 + 4\sin 20^\circ(1) - 6\sin 20^\circ(2) + 10\sin 20^\circ(4) - 4\cos 20^\circ(0.3) - 6\cos 20^\circ(0.3)$$

$$\therefore \underline{M_O = 22.1 \text{ kN}\cdot\text{m CCW}}$$

• Line-of-Action:

$$\underline{r} \times \underline{R} = \underline{M}_O \longrightarrow (x\underline{i} + y\underline{j}) \times (7.52\,\underline{i} + 2.74\,\underline{j}) = 22.1\,\underline{k}$$

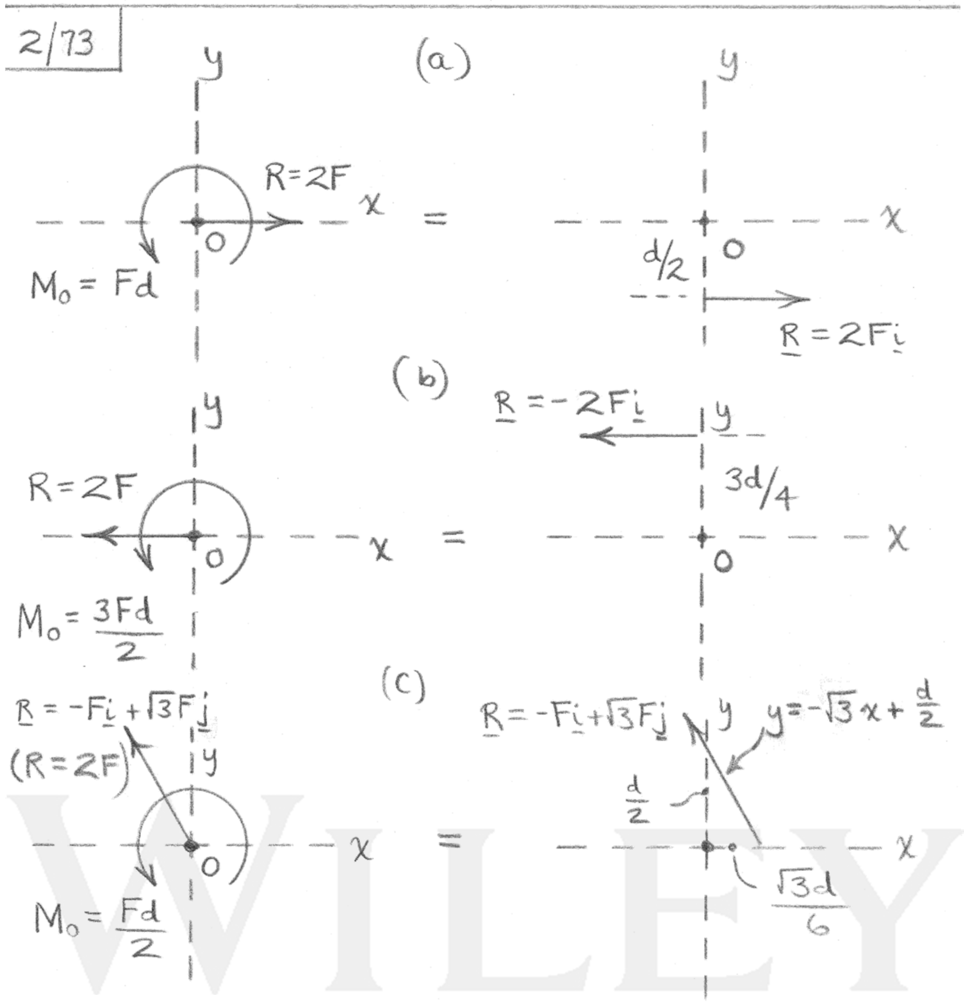
$$\underline{k}: \; 2.74x - 7.52y = 22.1$$

$$\therefore \quad \underline{y = 0.364x - 2.94 \text{ (m)}}$$

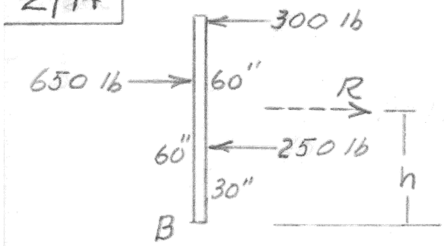
2/72 (a) $\underline{R} = -2F\underline{j}$, $\underline{M}_o = \underline{0}$

(b) $\underline{R} = \underline{0}$, $\underline{M}_o = Fd\underline{k}$ (+$\underline{k}$ is out)

(c) $\underline{R} = -F\underline{i} + F\underline{j}$, $\underline{M}_o = \underline{0}$


2/73
(a)
R=2F
M₀ = Fd
d/2
R = 2Fi
(b)
R = -2Fi
R=2F
3d/4
M₀ = 3Fd/2
(c)
R = -Fi + √3Fj
(R=2F)
M₀ = Fd/2
y = -√3x + d/2
d/2
√3d/6

2/74



$R = \Sigma F = 650 - 250 - 300$

$= 100 \text{ lb}$

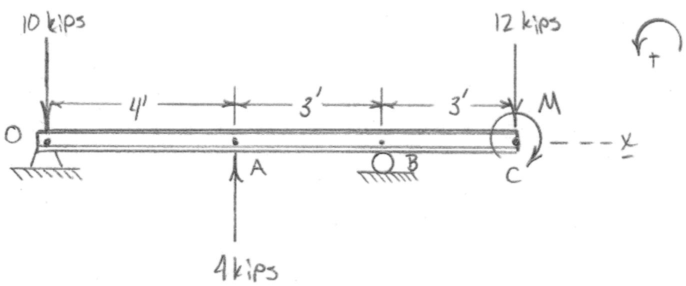
$Rh = \Sigma M_B$;

$100h = 650(60) - 300(90)$

$-250(30)$

$\underline{h = 45 \text{ in.}}$

2/75



$\underline{R = 18 \text{ kips DOWN}}$

$\Sigma M_B = 0: \quad 7(10) - 4(3) - 12(3) - M = 0 \longrightarrow M = 22 \text{ kip-ft CW}$

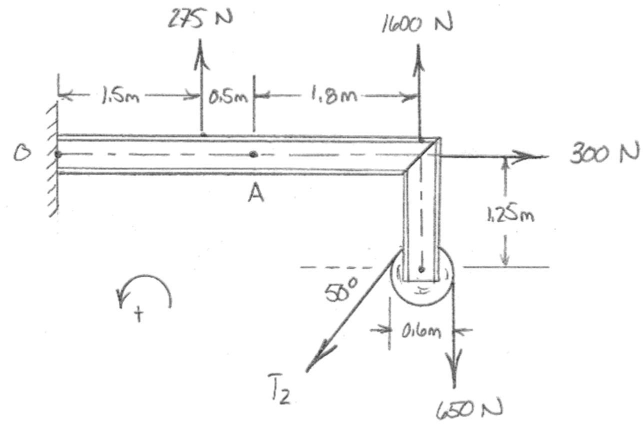
$M_O = 4(4) - 12(10) - 22 = -126$ so... $\underline{M_O = 126 \text{ kip-ft CW}}$

2/76 $M_O = 0$, so

$$\circlearrowleft^{+} M - 400(0.150\cos 30^\circ) - 320(0.300) = 0$$

$$\underline{M = 148.0\ \text{N}\cdot\text{m}}$$

2/77

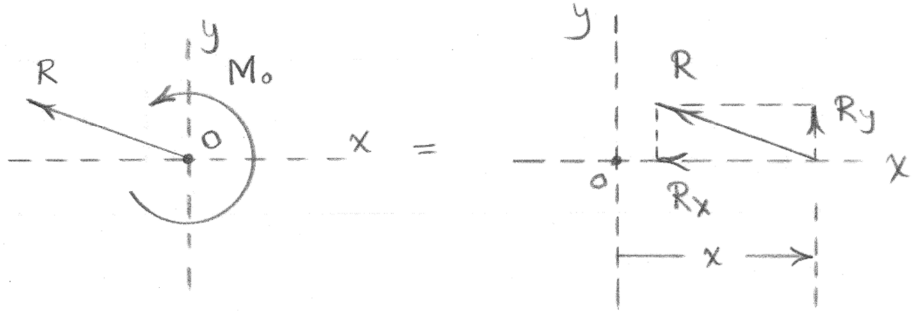


$$\Sigma M_A = 0: \quad -275(0.5) + 1.8(1600) - 650(1.8+0.3) + T_2(0.3) - T_2 \sin 50°(1.8) \ldots$$
$$- T_2 \cos 50°(1.25)$$

$$\underline{T_2 = 732 \text{ N}}$$

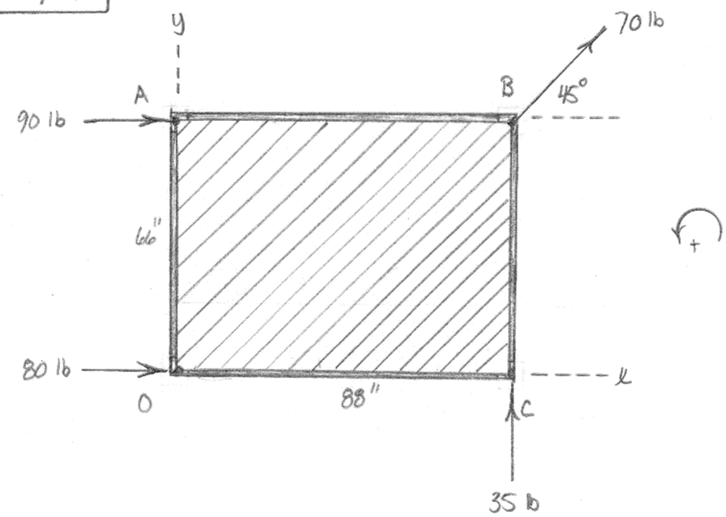
2/78 $\underline{\mathbf{R} = -50\mathbf{i} + 20\mathbf{j} \text{ lb}}$

$\curvearrowleft + \; M_O = -40(10) + 60(20) + 50(10) = 1300 \text{ lb-in.}$



$R_y x = M_O\,, \quad x = \dfrac{1300}{20} = \underline{65 \text{ in. (off pipe)}}$

2/79



$\underline{R} = (90 + 80 + 70\cos 45^\circ)\underline{i} + (35 + 70\sin 45^\circ)\underline{j}$

$\underline{R} = 219\,\underline{i} + 84.5\,\underline{j}$ lb

$M_O = -90(66) + 35(88) + 70\sin 45^\circ(88) - 70\cos 45^\circ(66) = -1771$

$M_O = 1771$ lb-in. CW

TO PRODUCE A CW MOMENT ABOUT O WITH R_x POSITIVE, R_x IS PLACED ABOVE O.

$R_x y = M_O \longrightarrow 219y = 1771 \longrightarrow y = 8.07$ in. ABOVE O

TO PRODUCE A CW MOMENT ABOUT O WITH R_y POSITIVE, R_y IS PLACED LEFT OF O.

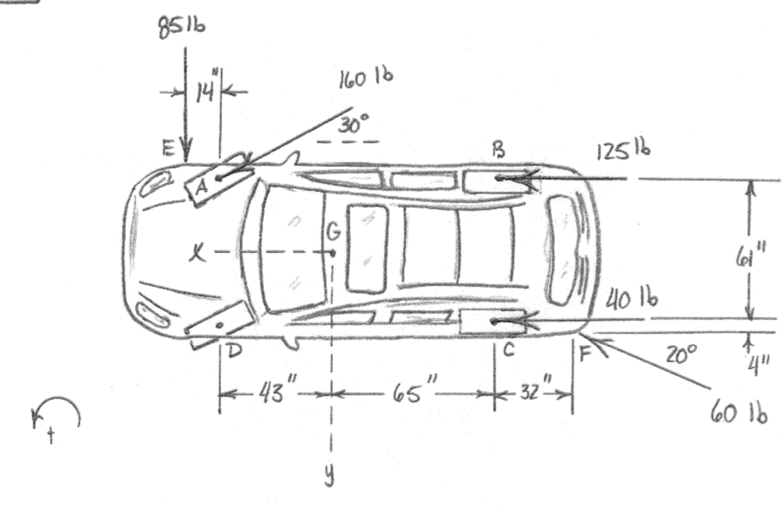
$R_y x = M_O \longrightarrow 84.5x = 1771 \longrightarrow x = 21.0$ in. LEFT OF O

2/80 $\Sigma M_O = 0$ since $\underline{R}$ passes through O.

$40(8) + 60(4) - 5P\cos 20° = 0$, $\underline{P = 119.2 \text{ lb}}$

Moment of 40-lb & 60-lb forces unaffected by θ so result for P is not dependent on θ.

2/81



$$\underline{R} = \Sigma \underline{F} = (160\cos 30° + 40 + 125 + 60\cos 20°)\underline{i} + (85 + 160\sin 30° - 60\sin 20°)\underline{j}$$

$$\therefore \quad \underline{\underline{R} = 360\,\underline{i} + 144.5\,\underline{j} \text{ lb}}$$

$$\Sigma M_G = 85(14+43) + 160\cos 30°\left(\tfrac{61}{2}\right) + 160\sin 30°(43) + 125\left(\tfrac{61}{2}\right) - 40\left(\tfrac{61}{2}\right) + 60\sin 20°(65+32) - 60\cos 20°\left(4+\tfrac{61}{2}\right)$$

$$\Sigma M_G = 15{,}150 \text{ lb-in. or } 1262 \text{ lb-ft CCW}$$

FOR CCW M_G WITH POSITIVE R_x, R IS PLACED ABOVE G, IN NEGATIVE y.

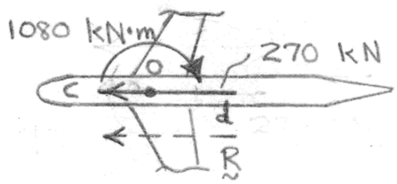
$M_G = R_x|y| \longrightarrow 15{,}150 = 360|y| \longrightarrow \underline{|y| = 42.1 \text{ in.} \quad \text{so} \quad (0, -42.1) \text{ in.}}$

FOR CCW M_G WITH POSITIVE R_y, R IS PLACED LEFT OF G, IN POSITIVE x.

$M_G = R_y x \longrightarrow 15{,}150 = 144.5\,x \longrightarrow \underline{x = 104.9 \text{ in.} \quad \text{so} \quad (104.9, 0) \text{ in.}}$

2/82 Force – Couple system at point O:

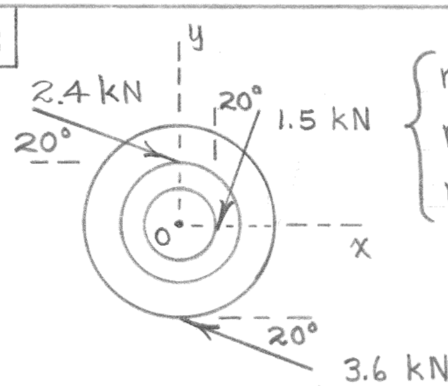
$$\begin{cases} R = 3(90) = \underline{270 \text{ kN } (\leftarrow)} \\ +\circlearrowright M_O = 12(90) = 1080 \text{ kN}\cdot\text{m} \end{cases}$$



$$d = \frac{M_O}{R} = \frac{1080}{270}$$

$$= \underline{4 \text{ m}}$$

2/83



$$\begin{cases} r_1 = 0.120 \text{ m} \\ r_2 = 0.200 \text{ m} \\ r_3 = 0.300 \text{ m} \end{cases}$$

$$\underline{R} = \Sigma \underline{F} = 2.4(c20^\circ \underline{i} - s20^\circ \underline{j}) + 1.5(-s20^\circ \underline{i} - c20^\circ \underline{j})$$

$$+ 3.6(-c20^\circ \underline{i} + s20^\circ \underline{j}) = -1.641\underline{i} - 0.999\underline{j} \text{ kN}$$

$$\Sigma M_O = (2.4(0.2) + 1.5(0.12) + 3.6(0.3)) \cos 20^\circ = 1.635 \text{ kN}\cdot\text{m}$$

$$\underline{r} \times \underline{R} = \underline{M}_O : (x\underline{i} + y\underline{j}) \times (-1.641\underline{i} - 0.999\underline{j}) = -1.635$$

$$\Rightarrow -0.999x + 1.641y = -1.635$$

Axis intercepts : $\underline{x = 1.637 \text{ m}, \; y = -0.997 \text{ m}}$

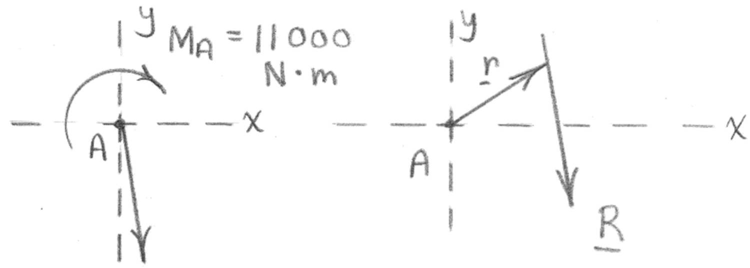
2/84 Equivalent force-couple system at A:

$\mathbf{R} = \Sigma\mathbf{F} = [-2(250) - 3(500)]\mathbf{j} + 400[\cos 30°\mathbf{i} - \sin 30°\mathbf{j}]$

$= \underline{346\mathbf{i} - 2200\mathbf{j}\ \text{N}}$

$\circlearrowright + \ M_A = 500[2.5 + 5 + 7.5] + 250[10] + 400(2.5)$

$= \underline{11\,000\ \text{N}\cdot\text{m}\ \ \text{CW}}$



$\mathbf{R} = 346\mathbf{i} - 2200\mathbf{j}$ N

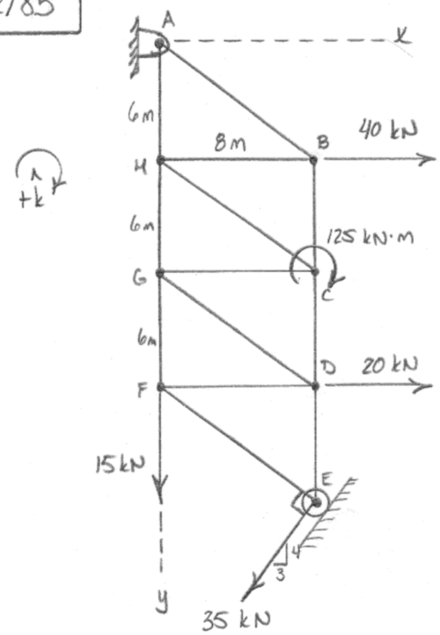
Condition: $\mathbf{M}_A = \mathbf{r} \times \mathbf{R}$

$-11\,000\mathbf{k} = (x\mathbf{i} + y\mathbf{j}) \times (346\mathbf{i} - 2200\mathbf{j})$

$= (-2200x - 346y)\mathbf{k}$

Set $y = 0$ & obtain $\underline{x = 5\ \text{m}}$ (!)

2/85



$$\underline{R} = \left(40 + 20 - \tfrac{3}{5}35\right)\underline{i} + \left(15 + \tfrac{4}{5}35\right)\underline{j}$$

$$\underline{R} = 39\underline{i} + 43\underline{j} \text{ kN}$$

$$\underline{M}_A = \left[-40(6) - 20(18) + 125 + 35(10) + \tfrac{3}{5}35(18)\right]\underline{k} \longrightarrow \underline{M}_A = 253\,\underline{k} \text{ kN}\cdot\text{m}$$

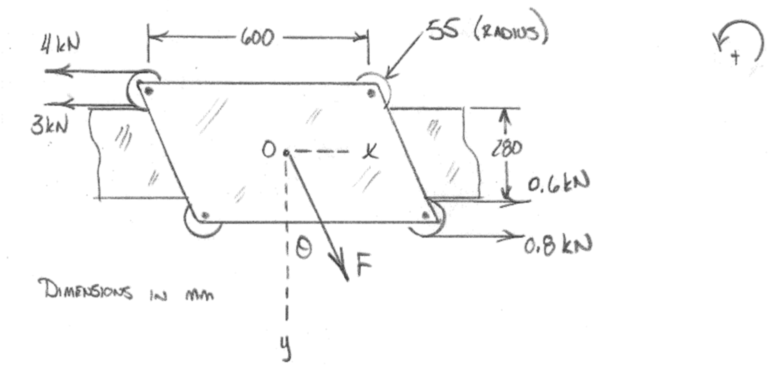
$$\underline{r} \times \underline{R} = \underline{M}_A \longrightarrow (x\underline{i} + y\underline{j}) \times (39\underline{i} - 43\underline{j}) = 253\underline{k}$$

$$\underline{k}: \; 43x - 39y = 253 \longrightarrow \underline{y = 1.103x - 6.49 \;\; (m)}$$

$\underline{x\text{-Axis}}$: $y = 0 = 1.103x - 6.49 \longrightarrow \underline{x = 5.88 \text{ m} \quad \text{so} \quad (5.88, 0)\text{ m}}$

$\underline{y\text{-Axis}}$: $x = 0 \longrightarrow \underline{y = -6.49 \text{ m} \quad \text{so} \quad (0, -6.49)\text{ m}}$

2/86 $F = 5\text{ kN}$, $\theta = 30^\circ$



$$\underline{R} = (0.8 + 0.6 + 5\sin 30^\circ - 4 - 3)\,\underline{i} + 5\cos 30^\circ\,\underline{j} \longrightarrow \underline{R} = -3.10\,\underline{i} + 4.33\,\underline{j}\ \text{kN}$$

$$\Sigma M_O = 0.6\left(\frac{140}{1000}\right) + 0.8\left(\frac{140+110}{1000}\right) + 3\left(\frac{140}{1000}\right) + 4\left(\frac{140+110}{1000}\right) =$$

$$\therefore\ \Sigma M_O = 1.704\ \text{kN}\cdot\text{m CCW}$$

FOR A CCW M_O WITH NEGATIVE R_x, R IS PLACED ABOVE O IN MINUS y.

$$R_x y = M_O \longrightarrow 3.10|y| = 1.704 \longrightarrow |y| = 0.550$$

$$\therefore\ \underline{|y| = 550\text{ mm ABOVE O OR } (0, -550)\ (\text{mm})}$$

2/87 $\underline{R} = \Sigma \underline{F} = 400(\cos 45° \underline{i} - \sin 45° \underline{j}) + 500(\sin 15° \underline{i} - \cos 15° \underline{j})$

$= \underline{412\underline{i} - 766\underline{j}\ \text{N}}$

$\circlearrowleft\!+\ M_O = (500 - 400)(0.060) = 6\ \text{N}\cdot\text{m}$

For the line of action of the standalone force:

$\underline{r} \times \underline{R} = \underline{M_O}$

$(x\underline{i} + y\underline{j}) \times (412\underline{i} - 766\underline{j}) = -6\underline{k}$

$-766x - 412y = -6$

$\begin{cases} \text{For } x = 0: & y = 0.01455\ \text{m} \quad \text{or} \quad \underline{y = 14.55\ \text{mm}} \\ \text{For } y = 0: & x = 0.00783\ \text{m} \quad \text{or} \quad \underline{x = 7.83\ \text{mm}} \end{cases}$

2/88 For a zero force-couple system at point O: $\overset{y}{\llcorner} x$

$$\underline{R} = \Sigma \underline{F} = (-F_C \sin 30^\circ + F_D \sin 30^\circ)\underline{i}$$
$$+ (50 - 10 - 100 - 50 + F_B$$
$$+ F_C \cos 30^\circ + F_D \cos 30^\circ)\underline{j} = \underline{0}$$

$$\Rightarrow F_C = F_D = F$$

$$\circlearrowleft M_O = -10(0.5) + 50(0.7) - 100(1.35) + F_B(2)$$
$$- 50(2.5) + 2F\cos 30^\circ(2.9) = 0$$

$$\underline{F = F_C = F_D = 6.42 \text{ N}}, \quad \underline{F_B = 98.9 \text{ N}}$$

2/89

$$\underline{F} = F\underline{n}$$

$$= 60\left[\frac{40\underline{i} - 50\underline{j} + 110\underline{k}}{\sqrt{40^2 + 50^2 + 110^2}}\right]$$

$$= \underline{18.86\underline{i} - 23.6\underline{j} + 51.9\underline{k} \text{ N}}$$

$$\cos\theta_y = \frac{F_y}{F} = \frac{-23.6}{60}, \quad \underline{\theta_y = 113.1^\circ}$$

2/90

$$\underline{T} = T\underline{n}_{BA}$$

$$\underline{T} = 12\left[\frac{-35\underline{i} + 25\underline{j} + 60\underline{k}}{\sqrt{35^2 + 25^2 + 60^2}}\right]$$

$$= \underline{-5.69\underline{i} + 4.06\underline{j} + 9.75\underline{k} \ \text{kN}}$$

2/91 $F_z = 5\cos 40° = 3.83$ kN

$F_h = 5\sin 40° = 3.21$ kN

$F_x = -3.21\sin 35° = -1.843$ kN

$F_y = 3.21\cos 35° = 2.63$ kN

So $\underline{F} = -1.843\underline{i} + 2.63\underline{j} + 3.83\underline{k}$ kN

$F_{OA} = \underline{F}\cdot\underline{n}_{OA}$

$= (-1.843\underline{i} + 2.63\underline{j} + 3.83\underline{k})\cdot(\cos 30°\underline{i} + \sin 30°\underline{j})$

$= -0.280$ kN (as a scalar)

$\underline{F}_{OA} = F_{OA}\,\underline{n}_{OA}$

$= -0.280(\cos 30°\underline{i} + \sin 30°\underline{j})$

$= -0.243\underline{i} - 0.1401\underline{j}$ kN

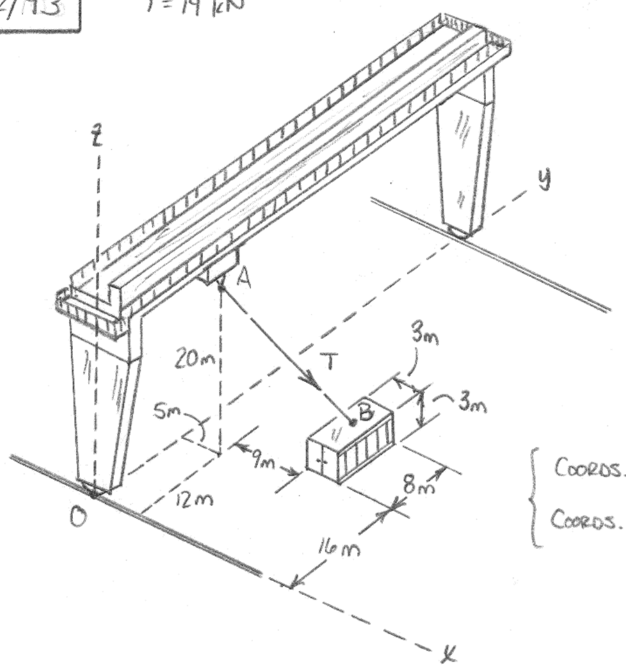
2/92

$$\underline{F} = F\underline{n} = 300\left[\frac{4\underline{i} - 8\underline{j} - 8\underline{k}}{\sqrt{4^2 + 8^2 + 8^2}}\right]$$

$$= 300\left[\frac{1}{3}\underline{i} - \frac{2}{3}\underline{j} - \frac{2}{3}\underline{k}\right] \text{ lb}$$

$F_x = 100$ lb, $F_y = -200$ lb, $F_z = -200$ lb

2/93 $T = 14$ kN



COORDS. OF A ARE (5, 12, 20) m
COORDS. OF B ARE (15.5, 20, 3) m

$$\underline{n}_{AB} = \frac{(15.5-5)\underline{i} + (20-12)\underline{j} + (3-20)\underline{k}}{\left[(15.5-5)^2 + (20-12)^2 + (3-20)^2\right]^{1/2}} \longrightarrow \underline{\underline{n}_{AB} = 0.488\underline{i} + 0.372\underline{j} - 0.790\underline{k}}$$

$$T_x = Tn_x = 14(0.488) \longrightarrow \underline{T_x = 6.83 \text{ kN}}$$

$$T_y = Tn_y = 14(0.372) \longrightarrow \underline{T_y = 5.20 \text{ kN}}$$

$$T_z = Tn_z = 14(-0.790) \longrightarrow \underline{T_z = -11.06 \text{ kN}}$$

2/94 $\underline{T} = T\underline{n}_{AB} = 2.4\left(\dfrac{2\underline{i} + \underline{j} - 5\underline{k}}{\sqrt{2^2 + 1^2 + 5^2}}\right)$

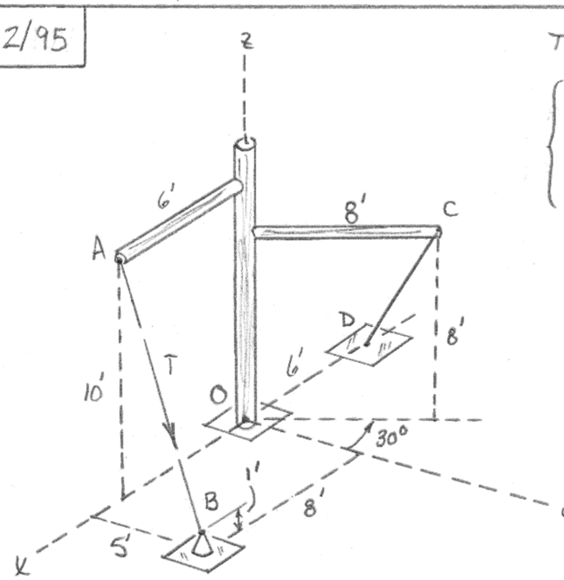
$= \underline{0.876\,\underline{i} + 0.438\,\underline{j} - 2.19\,\underline{k}\ \text{kN}}$

Projection $T_{AC} = \underline{T} \cdot \underline{n}_{AC}$

$= \left(0.876\underline{i} + 0.438\underline{j} - 2.19\underline{k}\right) \cdot \left(\dfrac{2\underline{i} - 2\underline{j} - 5k}{\sqrt{2^2 + 2^2 + 5^2}}\right)$

$= \underline{2.06\ \text{kN}}$

2/95



$T = 1750$ lb

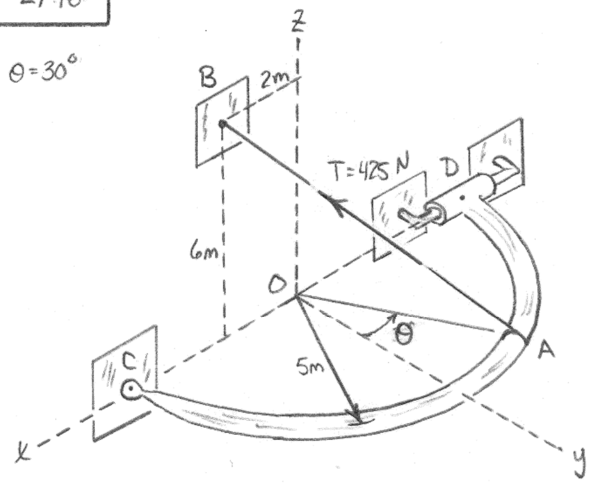
$$\begin{cases} A(6, 0, 10) \text{ ft} \\ B(8, 5, 1) \text{ ft} \end{cases}$$

$$\underline{n}_{AB} = \frac{(8-6)\underline{i} + (5-0)\underline{j} + (1-10)\underline{k}}{\left[(8-6)^2 + 5^2 + (1-10)^2\right]^{1/2}} \longrightarrow \underline{n}_{AB} = 0.1907\underline{i} + 0.477\underline{j} - 0.858\underline{k}$$

$$\begin{cases} \theta_x = \cos^{-1}(n_x) = \cos^{-1}(0.1907) \longrightarrow \underline{\theta_x = 79.0^\circ} \\ \theta_y = \cos^{-1}(n_y) = \cos^{-1}(0.477) \longrightarrow \underline{\theta_y = 61.5^\circ} \\ \theta_z = \cos^{-1}(n_z) = \cos^{-1}(-0.858) \longrightarrow \underline{\theta_z = 149.1^\circ} \end{cases}$$

2/96

$\theta = 30°$



$$\begin{cases} A(-5\sin 30,\ 5\cos 30,\ 0)\ \text{m} \\ B(2, 0, 6)\ \text{m} \end{cases}$$

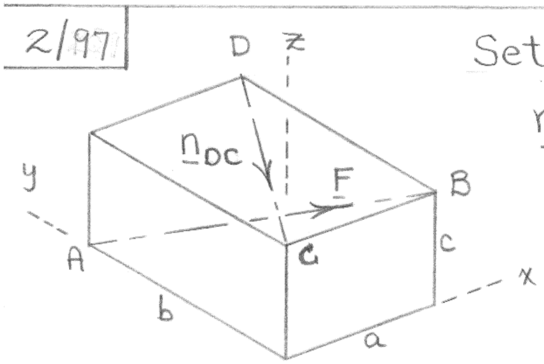
$$\underline{n}_{AB} = \frac{(2+5\sin 30)\underline{i} + (0-5\cos 30)\underline{j} + (6-0)\underline{k}}{[(2+5\sin 30)^2 + (5\cos 30)^2 + 6^2]^{1/2}}$$

$$\therefore \quad \underline{n}_{AB} = 0.520\underline{i} - 0.5\underline{j} + 0.693\underline{k}$$

$$\underline{T}_A = T\underline{n}_{AB} = 425(0.520\hat{i} - 0.5\hat{j} + 0.693\hat{k}) \longrightarrow \underline{\underline{T}_A = 221\underline{i} - 212\underline{j} + 294\underline{k}\ \text{N}}$$

$$\underline{T}_B = -\underline{T}_A \longrightarrow \underline{\underline{T}_B = -221\underline{i} + 212\underline{j} - 294\underline{k}\ \text{N}}$$

2/97



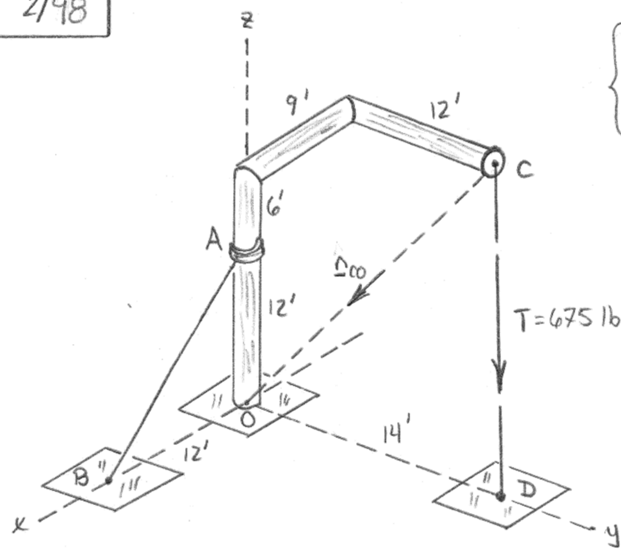
Set up x-y-z axes shown

$$\underline{n}_{DC} = \frac{\overline{DC}}{\overline{DC}} = \frac{-a\underline{i} - b\underline{j}}{\sqrt{a^2+b^2}}$$

$$\underline{F} = F\underline{n}_{AB} = F\frac{\overline{AB}}{\overline{AB}} = F\frac{a\underline{i} - b\underline{j} + c\underline{k}}{\sqrt{a^2+b^2+c^2}}$$

$$F_{DC} = \underline{F}\cdot\underline{n}_{DC} = \frac{(b^2-a^2)F}{\sqrt{(a^2+b^2)}\sqrt{(a^2+b^2+c^2)}}$$

2/98



$C(-9, 12, 18)$

$D(0, 14, 0)$

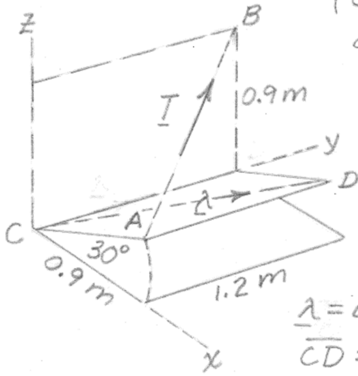
$$n_{CD} = \frac{(0+9)\underline{i} + (14-12)\underline{j} + (0-18)\underline{k}}{\left[9^2 + (14-12)^2 + (-18)^2\right]^{1/2}} \longrightarrow \hat{n}_{CD} = 0.445\underline{i} + 0.0989\underline{j} - 0.890\underline{k}$$

$$\underline{T} = T\underline{n}_{CD} = 675(0.445\underline{i} + 0.0989\underline{j} - 0.890\underline{k}) \longrightarrow \underline{T} = 300\underline{i} + 66.8\underline{j} - 601\underline{k} \text{ lb}$$

$$\underline{n}_{CO} = \frac{9\underline{i} - 12\underline{j} - 18\underline{k}}{\sqrt{9^2 + 12^2 + 18^2}} \longrightarrow \underline{n}_{CO} = \frac{3}{\sqrt{61}}\underline{i} - \frac{4}{\sqrt{61}}\underline{j} - \frac{6}{\sqrt{61}}\underline{k}$$

$$T_{CO} = \underline{T}\cdot\underline{n}_{CO} = (300\underline{i} + 66.8\underline{j} - 601\underline{k})\cdot\left(\frac{3}{\sqrt{61}}\underline{i} - \frac{4}{\sqrt{61}}\underline{j} - \frac{6}{\sqrt{61}}\underline{k}\right) \longrightarrow \underline{T_{CO} = 543 \text{ lb}}$$

2/99 $T = 150$ N



Coordinates of A are $(0.9\cos 30°, 0, 0.9\sin 30°)$

or $(0.779, 0, 0.45)$ m

$\overline{AB} = \sqrt{0.779^2 + 1.2^2 + (0.9 - 0.45)^2}$

$= 1.50$ m

$l = -0.779/1.5$, $m = 1.2/1.5$, $n = \frac{0.45}{1.5}$

$\underline{T} = \frac{100}{1.5}(-0.779\underline{i} + 1.2\underline{j} + 0.45\underline{k})$ N

$\underline{\lambda}$ = unit vector along CD

$\overline{CD} = \sqrt{0.9^2 + 1.2^2} = 1.5$ m

$l = 0.779/1.5$, $m = 1.2/1.5$, $n = 0.45/1.5$

$\lambda = \frac{1}{1.5}(0.779\underline{i} + 1.2\underline{j} + 0.45\underline{k})$

$T_{CD} = \underline{T}\cdot\underline{\lambda} = \frac{100}{1.5}(-0.779\underline{i} + 1.2\underline{j} + 0.45\underline{k}) \cdot \frac{1}{1.5}(0.779\underline{i} + 1.2\underline{j} + 0.45\underline{k})$

$= \frac{100}{2.25}(-0.779^2 + 1.2^2 + 0.45^2) = \frac{100}{2.25}(-0.6075 + 1.44 + 0.2025)$

$= \frac{400}{9}(1.035) = \underline{46.0 \text{ N}}$

2/100

$$\underline{F} = F\underline{n}_{AB} = 200\left[\frac{-12\underline{i} + 24\underline{j} + 8\underline{k}}{\sqrt{12^2 + 24^2 + 8^2}}\right]$$

$$= -85.7\underline{i} + 171.4\underline{j} + 57.1\underline{k} \quad \text{lb}$$

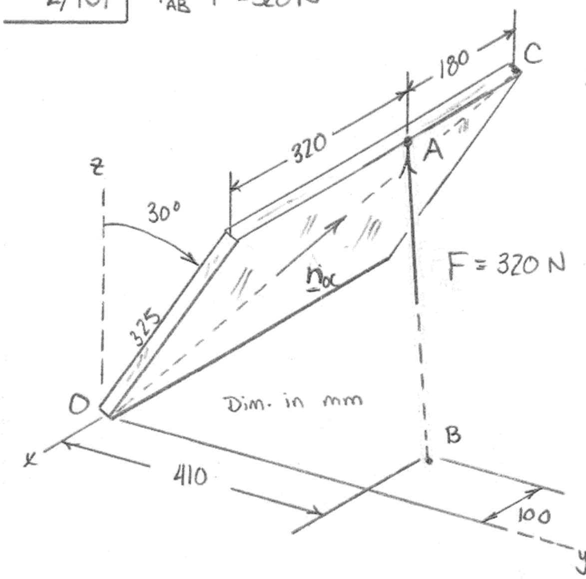
$\underline{OC} = 12\underline{i} + 24\underline{j}$ in.

The angle θ between $\underline{F}$ and $\underline{OC}$ is

$$\theta = \cos^{-1}\frac{\underline{F}\cdot\underline{OC}}{F(\overline{OC})} = \cos^{-1}\left[\frac{-85.7(12) + 171.4(24)}{200\left(\sqrt{12^2+24^2}\right)}\right]$$

$$= \underline{54.9^\circ}$$

2/101 $F_{AB} = F = 320\text{ N}$



$$\begin{cases} A\,(-320,\ 325\sin 30^\circ,\ 325\cos 30^\circ) \\ B\,(-100,\ 410,\ 0) \\ C\,(-500,\ 325\sin 30^\circ,\ 325\cos 30^\circ) \end{cases}$$

$$\underline{n}_{BA} = \frac{(-320+100)\underline{i} + (325\sin 30^\circ - 410)\underline{j} + 325\cos 30^\circ\,\underline{k}}{\left[(-320+100)^2 + (325\sin 30^\circ - 410)^2 + (325\cos 30^\circ)^2\right]^{1/2}}$$

$$\therefore\quad \underline{n}_{BA} = -0.506\underline{i} - 0.569\underline{j} + 0.648\underline{k}$$

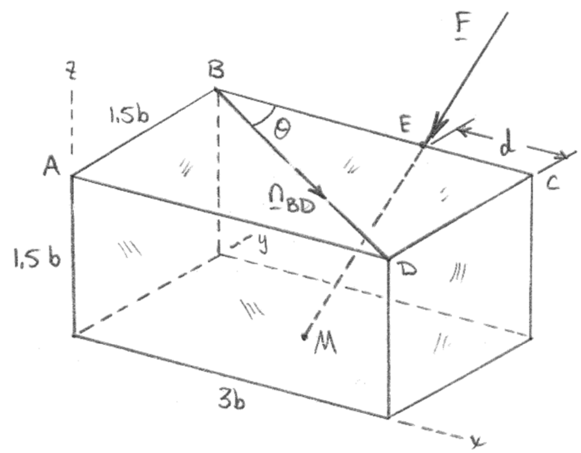
$$\underline{F} = F\underline{n}_{BA} = 320(-0.506\underline{i} - 0.569\underline{j} + 0.648\underline{k}) \longrightarrow \underline{F} = -162.0\underline{i} - 182.2\underline{j} + 207\underline{k}\text{ N}$$

$$\underline{n}_{OC} = \frac{-500\underline{i} + 325\sin 30^\circ\underline{j} + 325\cos 30^\circ\underline{k}}{\sqrt{500^2 + 325^2}} \longrightarrow \underline{n}_{OC} = -0.838\underline{i} + 0.272\underline{j} + 0.472\underline{k}$$

$$F_{OC} = \underline{F}\cdot\underline{n}_{OC} = (-162.0\underline{i} - 182.2\underline{j} + 207\underline{k})\cdot(-0.838\underline{i} + 0.272\underline{j} + 0.472\underline{k})$$

$$\therefore\quad \underline{F_{OC} = 184.0\text{ N}}$$

2/102



B $(0, 1.5b, 1.5b)$

D $(3b, 0, 1.5b)$

E $(3b-d, 1.5b, 1.5b)$

M $\left(\frac{3b}{2}, \frac{3b}{4}, 0\right)$

$$\theta = \tan^{-1}\left(\frac{1.5b}{3b}\right) = 26.6^\circ$$

$$\underline{n}_{BD} = \cos\theta\,\underline{i} - \sin\theta\,\underline{j} \longrightarrow \underline{n}_{BD} = 0.894\,\underline{i} - 0.447\,\underline{j}$$

$$\underline{n}_{EM} = \frac{\left(\frac{3b}{2} - 3b + d\right)\underline{i} + \left(\frac{3b}{4} - 1.5b\right)\underline{j} + (0 - 1.5b)\underline{k}}{\left[\left(\frac{3b}{2} - 3b + d\right)^2 + \left(\frac{3b}{4} - 1.5b\right)^2 + (-1.5b)^2\right]^{1/2}}$$

$$\therefore \underline{n}_{EM} = \frac{(4d - 6b)\underline{i} - 3b\,\underline{j} - 6b\,\underline{k}}{\sqrt{81b^2 - 48bd + 16d^2}} \quad \text{AND} \quad \underline{F} = F\underline{n}_{EM}$$

$$F_{BD} = \underline{F} \cdot \underline{n}_{BD} = F\left(\frac{(4d-6b)\underline{i} - 3b\underline{j} - 6b\underline{k}}{\sqrt{81b^2 - 48bd + 16d^2}}\right) \cdot \left(0.894\underline{i} - 0.447\underline{j}\right)$$

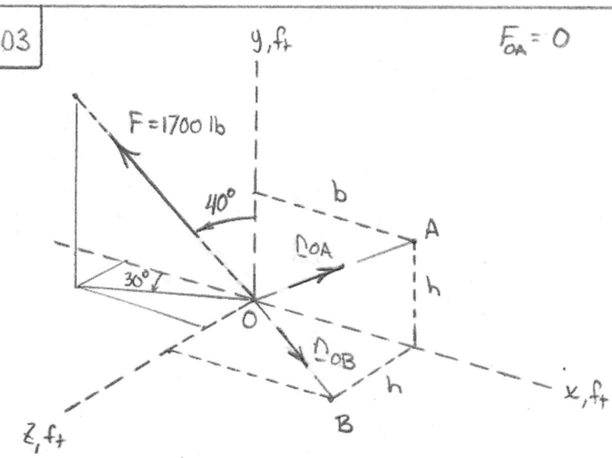
$$\therefore F_{BD} = \frac{(8d - 9b)F}{\sqrt{5}\sqrt{81b^2 - 48bd + 16d^2}}$$

IF $d = \frac{b}{2}$ ... $F_{BD} = -0.286\,F$

IF $d = \frac{5b}{2}$ ... $F_{BD} = 0.630\,F$

2/103

$F_{OA} = 0$ AND $b = 6$ ft.



$$F_x = -1700 \sin 40° \cos 30° = -946 \text{ lb}$$

$$F_y = 1700 \cos 40° = 1302 \text{ lb}$$

$$F_z = 1700 \sin 40° \sin 30° = 546 \text{ lb}$$

$$\underline{F} = -946\underline{i} + 1302\underline{j} + 546\underline{k} \text{ lb}$$

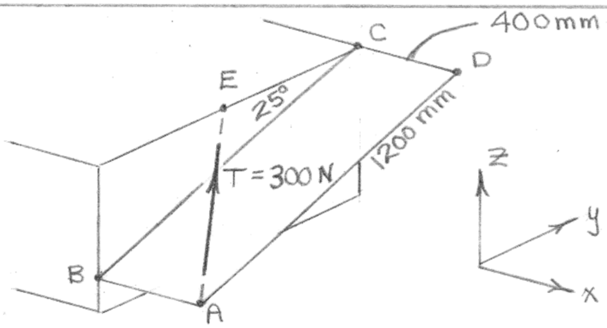
$$\underline{n}_{OA} = \frac{b\underline{i} + h\underline{j}}{\sqrt{b^2+h^2}} \quad \text{so...} \quad \underline{F} \cdot \underline{n}_{OA} = (-946\underline{i} + 1302\underline{j} + 546\underline{k}) \cdot \frac{6\underline{i} + h\underline{j}}{\sqrt{6^2+h^2}}$$

$$F_{OA} = \underline{F} \cdot \underline{n}_{OA} = 0 = \frac{1302\,(h - 4.36)}{\sqrt{36 + h^2}} \quad \text{so...} \quad h = 4.36 \text{ ft}$$

$$\underline{n}_{OB} = \frac{b\underline{i} + h\underline{k}}{\sqrt{b^2+h^2}} = 0.809\underline{i} + 0.588\underline{k}$$

$$F_{OB} = \underline{F} \cdot \underline{n}_{OB} = (-946\underline{i} + 1302\underline{j} + 546\underline{k}) \cdot (0.809\underline{i} + 0.588\underline{k}) \longrightarrow \underline{F_{OB} = -444 \text{ lb}}$$

2/104



$$\underline{T} = T\underline{n}_{AE} = 300\left[\frac{-400\underline{i} + 544\underline{j} + 507\underline{k}}{\sqrt{400^2 + 544^2 + 507^2}}\right]$$

$$= 300\left[-0.474\underline{i} + 0.644\underline{j} + 0.601\underline{k}\right] \text{ N}$$

$$\underline{n}_{BC} = \cos 25°\,\underline{j} + \sin 25°\,\underline{k}$$

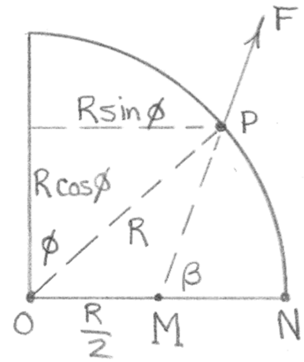
Carry out $T_{BC} = \underline{T} \cdot \underline{n}_{BC}$ to obtain

$$\underline{T_{BC} = 251 \text{ N}}$$

▶2/105

$F_x = F_{xy}\cos\theta$, $F_y = F_{xy}\sin\theta$

$F_z = F\sin\beta$, $F_{xy} = F\cos\beta$



$$\tan\beta = \frac{R\cos\phi}{R\sin\phi - \frac{R}{2}} = \frac{2\cos\phi}{2\sin\phi - 1}$$

$$\text{So } \sin\beta = \frac{2\cos\phi}{\sqrt{(2\sin\phi-1)^2+(2\cos\phi)^2}}$$

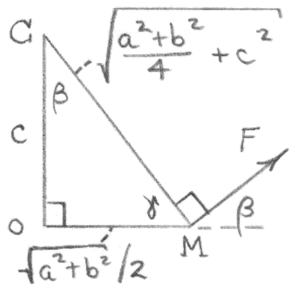
$$\cos\beta = \frac{2\sin\phi - 1}{\sqrt{(2\sin\phi-1)^2+(2\cos\phi)^2}}$$

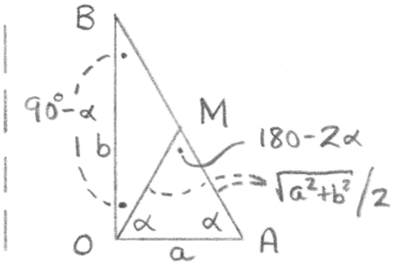
Note that $\sqrt{(2\sin\phi-1)^2+(2\cos\phi)^2} = \sqrt{5-4\sin\phi}$

So $\underline{F} = F\left[\cos\theta\cos\beta\,\underline{i} + \sin\theta\cos\beta\,\underline{j} + \sin\beta\,\underline{k}\right]$

$$= \frac{F}{\sqrt{5-4\sin\phi}}\left[(2\sin\phi-1)(\cos\theta\,\underline{i}+\sin\theta\,\underline{j}) + 2\cos\phi\,\underline{k}\right]$$

▸2/106





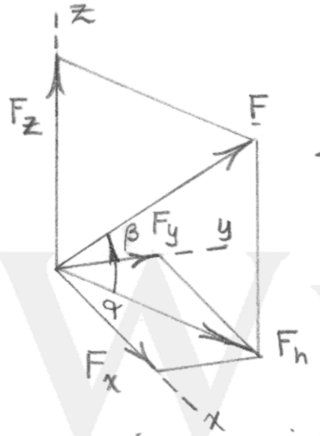
$$\tan\gamma = \frac{c}{\sqrt{a^2+b^2}/2} = \frac{2c}{\sqrt{a^2+b^2}}$$

$$\gamma + 90° + \beta = 180°$$

$$\beta = 90° - \gamma = 90° - \tan^{-1}\frac{2c}{\sqrt{a^2+b^2}}$$

$$\tan\alpha = \frac{b}{a}$$

$$\begin{cases} \cos\alpha = \dfrac{a}{\sqrt{a^2+b^2}} \\ \sin\alpha = \dfrac{b}{\sqrt{a^2+b^2}} \end{cases}$$



$$\begin{cases} F_z = F\sin\beta \\ F_h = F\cos\beta \\ F_x = F_h\cos\alpha = F\cos\beta\cos\alpha \\ F_y = F_h\sin\alpha = F\cos\beta\sin\alpha \end{cases}$$

Now simplify $\sin\beta$ & $\cos\beta$ expressions:

$$\sin\beta = \sin\left[90° - \tan^{-1}\frac{2c}{\sqrt{a^2+b^2}}\right]$$

$$= \sin 90° \cos\left[\tan^{-1}\frac{2c}{\sqrt{a^2+b^2}}\right] - \cos 90° \sin\left[\tan^{-1}\frac{2c}{\sqrt{a^2+b^2}}\right]$$

$$= \frac{\sqrt{a^2+b^2}}{\sqrt{a^2+b^2+4c^2}}$$

(Also see that $\sin\beta = \cos\gamma$!)

$$\cos\beta = \sin\gamma = \frac{c}{\sqrt{\frac{a^2+b^2}{4}+c^2}} = \frac{2c}{\sqrt{a^2+b^2+4c^2}}$$

Finally,

$$F_x = F\frac{2c}{\sqrt{a^2+b^2+4c^2}}\frac{a}{\sqrt{a^2+b^2}} = \frac{2acF}{\sqrt{a^2+b^2}\sqrt{a^2+b^2+4c^2}}$$

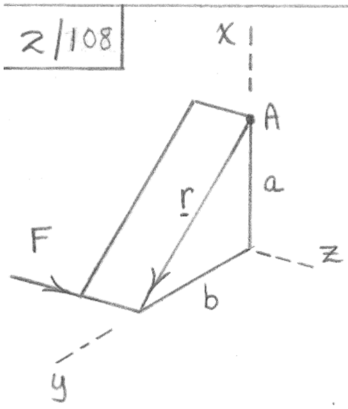
$$F_y = F\frac{2c}{\sqrt{a^2+b^2+4c^2}}\frac{b}{\sqrt{a^2+b^2}} = \frac{2bcF}{\sqrt{a^2+b^2}\sqrt{a^2+b^2+4c^2}}$$

$$F_z = F\sqrt{\frac{a^2+b^2}{a^2+b^2+4c^2}}$$

2/107

$$\begin{cases} \underline{M}_1 = \underline{-cF_1 \underline{j}} \\ \underline{M}_2 = cF_2 \underline{j} - bF_2 \underline{k} = \underline{F_2(c\underline{j} - b\underline{k})} \\ \underline{M}_3 = \underline{-bF_3 \underline{k}} \end{cases}$$

2/108



$$\mathbf{M}_A = \mathbf{r} \times \mathbf{F}$$

$$= (-a\mathbf{i} + b\mathbf{j}) \times F\mathbf{k}$$

$$= \underline{F(b\mathbf{i} + a\mathbf{j})}$$

2/109

$$\underline{M}_O = F(-c\underline{i} + a\underline{k})$$

$$\underline{M}_A = Fa\underline{k}$$

$$\underline{M}_{OB} = (\underline{M}_O \cdot \underline{n}_{OB})\underline{n}_{OB}$$

$$= \left[F(-c\underline{i} + a\underline{k}) \cdot \frac{a\underline{i} + b\underline{j}}{\sqrt{a^2+b^2}}\right] \frac{a\underline{i} + b\underline{j}}{\sqrt{a^2+b^2}}$$

$$= -\frac{Fac}{a^2+b^2}(a\underline{i} + b\underline{j})$$

2/110

$$\underline{M}_O = \underline{r}_{OA} \times \underline{F}$$

$$= (1.5\underline{j} + 0.75\underline{k}) \times 4(-\cos 30°\,\underline{i} + \sin 30°\,\underline{j})$$

$$= \underline{-1.5\underline{i} - 2.60\underline{j} + 5.20\underline{k} \text{ lb-in.}}$$

2/111 $R = \Sigma F = 250$ kips

$\underline{M} = -150(3)\underline{i} + (150-100)6\,\underline{j}$

$= -450\underline{i} + 300\underline{j}$ kip-in.

or $\underline{M} = (-0.450\underline{i} + 0.300\underline{j})10^6$ lb-in.

2/112 $\underline{M} = \underline{r} \times \underline{F}$

$= -0.5\underline{i} \times 400(\cos 15^\circ \underline{j} + \sin 15^\circ \underline{k})$

$= 51.8\underline{j} - 193.2\underline{k}$ N·m

2/113 $\overline{AB} = \sqrt{0.8^2 + 1.5^2 + 2^2} = 2.62 \text{ m}$

$\underline{T} = \frac{1.2}{2.62}(0.8\underline{i} + 1.5\underline{j} - 2\underline{k}) \text{ kN}$

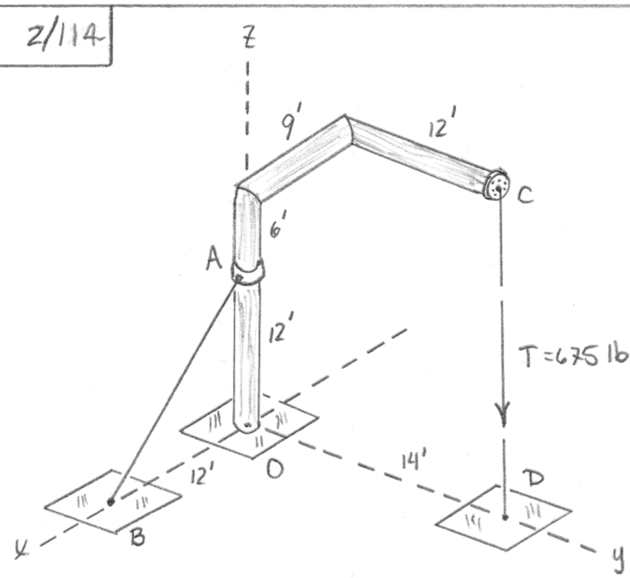
Take $\underline{r} = \overrightarrow{OA} = 1.6\underline{i} + 2\underline{k} \text{ m}$

$$\underline{M}_O = \underline{r} \times \underline{T} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1.6 & 0 & 2 \\ 0.8 & 1.5 & -2 \end{vmatrix} \frac{1.2}{2.62}$$

$$\underline{M}_O = 0.457(-3\underline{i} + 4.8\underline{j} + 2.40\underline{k}) \text{ kN}\cdot\text{m}$$

$$M_O = |\underline{M}_O| = 0.457\sqrt{3^2 + 4.8^2 + 2.40^2} = \underline{2.81 \text{ kN}\cdot\text{m}}$$

2/114



FROM 2/98 ... $\underline{T} = 300\underline{i} + 66.8\underline{j} - 601\underline{k}$ lb

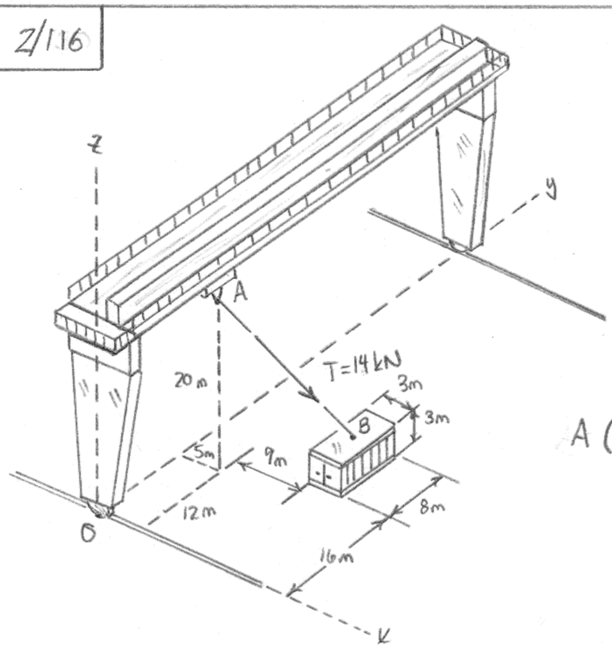
$$\underline{M}_O = \underline{r}_{OD} \times \underline{T} = 14\underline{j} \times (300\underline{i} + 66.8\underline{j} - 601\underline{k})$$

$$\underline{M}_O = -8410\underline{i} - 4210\underline{k} \text{ lb-ft}$$

2/115 $\underline{M} = -150(0.250+0.250)\underline{i} + 150(0.150)\underline{j}$

$= \underline{-75\underline{i} + 22.5\underline{j} \ \text{N}\cdot\text{m}}$

2/116



A (5, 12, 20) m

From 2/93 ... $\underline{T} = \underline{R} = 6.83\underline{i} + 5.20\underline{j} - 11.06\underline{k}$ kN

$$\underline{M}_O = \underline{r}_{OA} \times \underline{T} = (5\underline{i} + 12\underline{j} + 20\underline{k}) \times (6.83\underline{i} + 5.20\underline{j} - 11.06\underline{k})$$

$$\underline{M}_O = -237\underline{i} + 191.9\underline{j} - 55.9\underline{k} \text{ kN·m}$$

2/117

$$M_O = (250 \sin 60°)12 + (250 \cos 60°) \sin 40° (8 - 4.2)$$

$$= \underline{2900 \text{ lb-in.}}$$

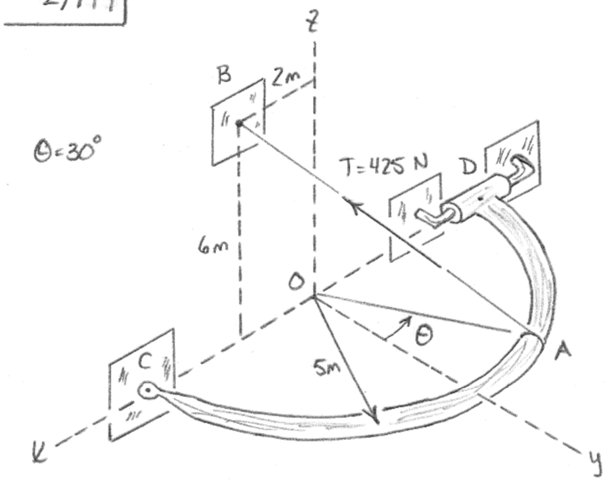
2/118

$$\underline{M}_O = \underline{r} \times \underline{F}$$

$$= (-6\underline{i} + 0.8\underline{j} + 1.2\underline{k}) \times (-400\underline{j})$$

$$= \underline{480\underline{i} + 2400\underline{k} \ \text{N}\cdot\text{m}}$$

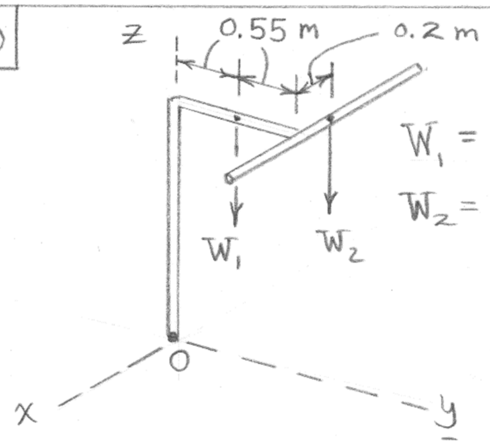
2/119



From 2/108 ... $\underline{T} = 221\underline{i} - 212\underline{j} + 294\underline{k}$ N

$M_{O_x} = r\cos\theta\, T_z = 5\cos 30°(294) \longrightarrow \underline{M_{O_x} = 1275 \text{ N·m}}$

2/120



$W_1 = 1.1(7)(9.81) = 75.5$ N

$W_2 = 2(7)(9.81) = 137.3$ N

$$\begin{cases} M_{O_x} = -75.5(0.55) - 137.3(1.1) = -192.6 \text{ N·m} \\ M_{O_y} = -137.3(0.2) = -27.5 \text{ N·m} \\ M_{O_z} = 0 \end{cases}$$

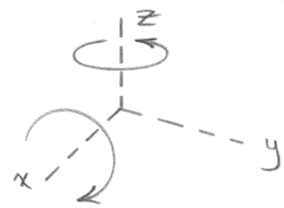
$\therefore$ $\underline{\mathbf{M}_O = -192.6\mathbf{i} - 27.5\mathbf{j} \text{ N·m}}$

$\underline{M_O = 194.6 \text{ N·m}}$

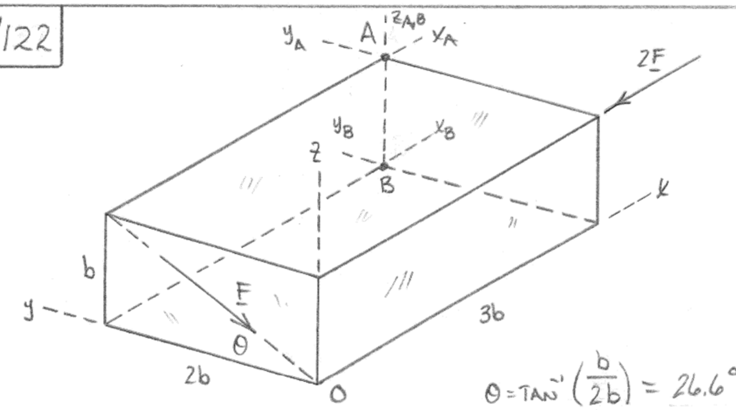
2/121 $\underline{M} = 1.2(40)\underline{k} - 1.2(50)\underline{i}$

$= -60\underline{i} + 48\underline{k}$ lb-in.

The spacecraft will begin to rotate about its x- and z axes.



2/122



$\theta = \tan^{-1}\left(\frac{b}{2b}\right) = 26.6°$

· FORCE $\underline{F}$

$$\underline{M}_A = F\cos\theta\,(3b)\underline{k} - F\sin\theta\,(3b)\underline{j} \longrightarrow \underline{M}_A = \frac{Fb}{\sqrt{5}}(-3\underline{j} + 6\underline{k})$$

$$\underline{M}_B = F\cos\theta\,(3b)\underline{k} - F\sin\theta(3b)\underline{j} + F\sin\theta\,(2b)\underline{i}$$

$$\therefore \quad \underline{M}_B = \frac{Fb}{\sqrt{5}}(2\underline{i} - 3\underline{j} + 6\underline{k})$$

· FORCE $2\underline{F}$

$$\underline{M}_A = -2F(2b)\underline{k} \longrightarrow \underline{M}_A = -4Fb\,\underline{k}$$

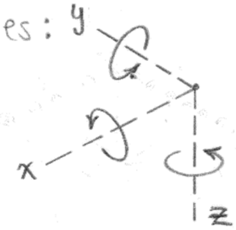
$$\underline{M}_B = -2F(2b)\underline{k} - 2F(b)\underline{j} \longrightarrow \underline{M}_B = -2Fb(\underline{j} + 2\underline{k})$$

2/123

$$\mathbf{M} = (1700)(2)\mathbf{i} - (1700)(30)\mathbf{j} - (1700)(30)\mathbf{k}$$

$$= 3400\mathbf{i} - 51\,000\mathbf{j} - 51\,000\mathbf{k} \text{ N·m}$$

The orbiter would acquire rotational motion about all three axes:



2/124

$$\underline{M}_o = 0\underline{i} - (200)(0.2 + 0.125 \sin 20^\circ)\underline{j}$$
$$- 200\,(0.125 \cos 20^\circ - 0.070)\,\underline{k}$$
$$= \underline{-48.6\underline{j} - 9.49\underline{k}\ \ \text{N}\cdot\text{m}}$$

There would be no z-component of $\underline{M}_o$ if

$d \cos 20^\circ - 70 = 0$, $\underline{d = 74.5 \text{ mm}}$

2/125

$$\underline{T} = T\left[\frac{-0.35\underline{i} - 0.45\cos 20°\,\underline{j} + (0.4 + 0.45\sin 20°)\,\underline{k}}{\sqrt{(0.35)^2 + (0.45\cos 20°)^2 + (0.4 + 0.45\sin 20°)^2}}\right]$$

$$= 143.4\left[-0.449\underline{i} - 0.542\underline{j} + 0.710\underline{k}\right] \text{ N}$$

Moment of this force about the x-axis is

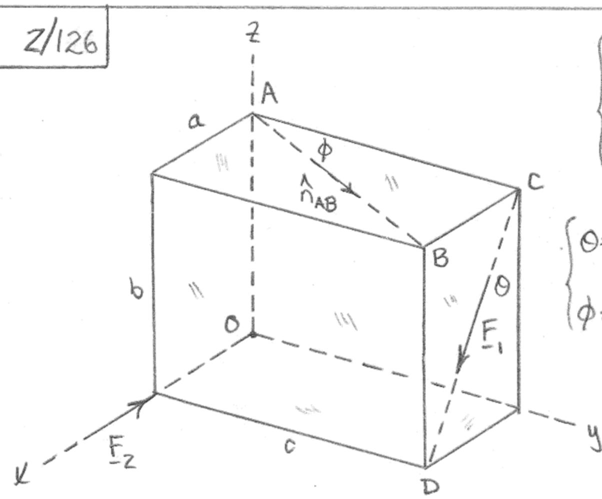
$$M_{O_x} = (0.710)(143.4)(0.45\cos 20°) - 0.542(143.4)(0.45\sin 20°) = \underline{31.1 \text{ N}\cdot\text{m}}$$

The moment of the weight W of the 15-kg plate about the x-axis is

$$(M_{O_x})_W = -15(9.81)\,\frac{0.45\cos 20°}{2} = \underline{-31.1 \text{ N}\cdot\text{m}}$$

The moment of $\underline{T}$ about the line OB is <u>zero</u>, because $\underline{T}$ intersects OB.

2/126



$$\begin{cases} a = 200 \text{ mm} \\ b = 400 \text{ mm} \\ c = 500 \text{ mm} \end{cases} \quad \begin{cases} F_1 = 450 \text{ N} \\ M_{AB} = 30 \text{ N·m} \end{cases}$$

$$\begin{cases} \theta = \tan^{-1}\left(\frac{a}{b}\right) = \tan^{-1}\left(\frac{200}{400}\right) = 26.6° \\ \phi = \tan^{-1}\left(\frac{a}{c}\right) = \tan^{-1}\left(\frac{200}{500}\right) = 21.8° \end{cases}$$

$$\underline{F}_1 = 450(\sin\theta\,\underline{i} - \cos\theta\,\underline{k}) \longrightarrow \underline{F}_1 = 201\,\underline{i} - 402\,\underline{k} \text{ N}$$

$$\underline{n}_{AB} = \sin\phi\,\underline{i} + \cos\phi\,\underline{j} \longrightarrow \underline{n}_{AB} = 0.371\,\underline{i} + 0.928\,\underline{j}$$

$$\underline{M}_A = bF_2\,\underline{j} - cF_{1_z}\,\underline{i} - cF_{1_x}\,\underline{k} = 0.4F_2\,\underline{j} - 0.5(402)\,\underline{i} - 0.5(201)\,\underline{k}$$

$$\underline{M}_A = -201\,\underline{i} + 0.4F_2\,\underline{j} - 100.6\,\underline{k} \text{ N·m}$$

$$M_{AB} = \underline{M}_A \cdot \underline{n}_{AB} \longrightarrow 30 = (-201\,\underline{i} + 0.4F_2\,\underline{j} - 100.6\,\underline{k}) \cdot (0.371\,\underline{i} + 0.928\,\underline{j})$$

$$30 = 0.371F_2 - 74.7 \longrightarrow \underline{F_2 = 282 \text{ N}}$$

2/127 Using the coordinates of the figure:

$\underline{M}_A = \underline{r} \times \underline{F}$, $\underline{F} = -1.8\,\underline{k}$ lb

$\underline{r} = [(2+1)\cos 30°]\underline{i} + 3\underline{j} + [(2+1)\sin 30°]\underline{k}$ in.

$\therefore$ $\underline{M}_A = -5.40\underline{i} + 4.68\underline{j}$ lb-in.

$\underline{M}_{AB} = (\underline{M}_A \cdot \underline{n}_{AB})\,\underline{n}_{AB}$, $\underline{n}_{AB} = \cos 30°\,\underline{i} + \sin 30°\,\underline{k}$

$\therefore$ $\underline{M}_{AB} = -4.05\underline{i} - 2.34\underline{k}$ lb-in.

2/128 Moment of couple is $240(\underline{j}\cos 30° - \underline{k}\sin 30°)$

$= 207.8\underline{j} - 120\underline{k}$ N·m

Moment of force is

$1200\cos 30°(-0.250\underline{i} + 0.200\underline{k}) + 1200\sin 30°(0.200\underline{j})$

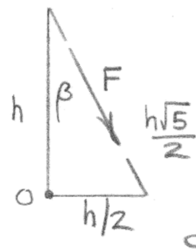
$= -259.8\underline{i} + 120\underline{j} + 207.8\underline{k}$ N·m

Thus total moment is

$\underline{M}_O = -259.8\underline{i} + 327.8\underline{j} + 87.8\underline{k}$ N·m

or $\underline{\underline{M}_O = -260\underline{i} + 328\underline{j} + 88\underline{k}}$ N·m

2/129



$$F_z = -F\cos\beta = -F\frac{1}{\sqrt{5}/2} = -\frac{2F}{\sqrt{5}}$$

$$F_{hor} = F\sin\beta = F/\sqrt{5}$$

$$F_x = F_{hor}\cos\theta = \frac{F}{\sqrt{5}}\cos\theta$$

$$F_y = F_{hor}\sin\theta = \frac{F}{\sqrt{5}}\sin\theta$$

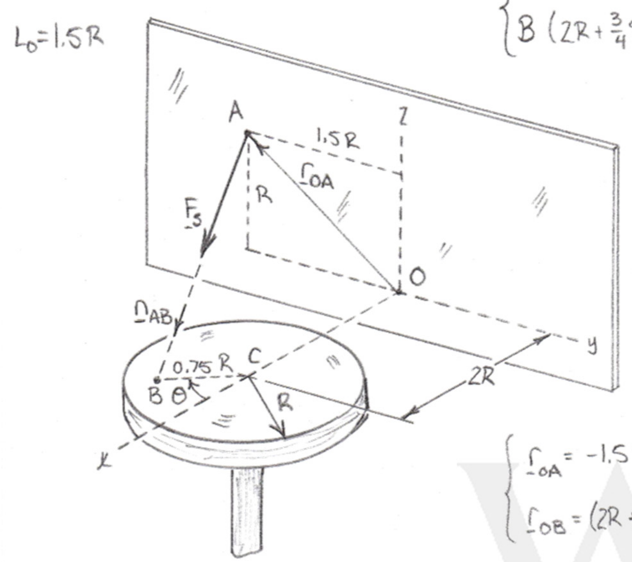
So $\underline{F} = \frac{F}{\sqrt{5}}\left[\cos\theta\,\underline{i} + \sin\theta\,\underline{j} - 2\underline{k}\right]$

$$\underline{M}_O = \underline{r} \times \underline{F} = h\underline{k} \times \frac{F}{\sqrt{5}}\left[\cos\theta\,\underline{i} + \sin\theta\,\underline{j} - 2\underline{k}\right]$$

$$= \frac{Fh}{\sqrt{5}}\left(\cos\theta\,\underline{j} - \sin\theta\,\underline{i}\right)$$

*2/130

$L_0 = 1.5R$



$$\begin{cases} A\ (0, -1.5R, R) \\ B\ (2R + \tfrac{3}{4}R\cos\theta, -\tfrac{3}{4}R\sin\theta, 0) \end{cases}$$

$$\begin{cases} F_s = k\delta \\ \text{AND} \quad \delta = d - L_0 \end{cases}$$

$$\begin{cases} \underline{r}_{OA} = -1.5R\underline{j} + R\underline{k} \\ \underline{r}_{OB} = (2R + \tfrac{3}{4}R\cos\theta)\underline{i} - \tfrac{3}{4}R\sin\theta\,\underline{j} \end{cases}$$

$$\underline{n}_{AB} = \frac{(8+3\cos\theta)\underline{i} + (6-3\sin\theta)\underline{j} - 4\underline{k}}{\sqrt{125 + 48\cos\theta - 36\sin\theta}}$$

$$d = |\underline{r}_{OB} - \underline{r}_{OA}| = \frac{R}{4}\sqrt{125 + 48\cos\theta - 36\sin\theta}$$

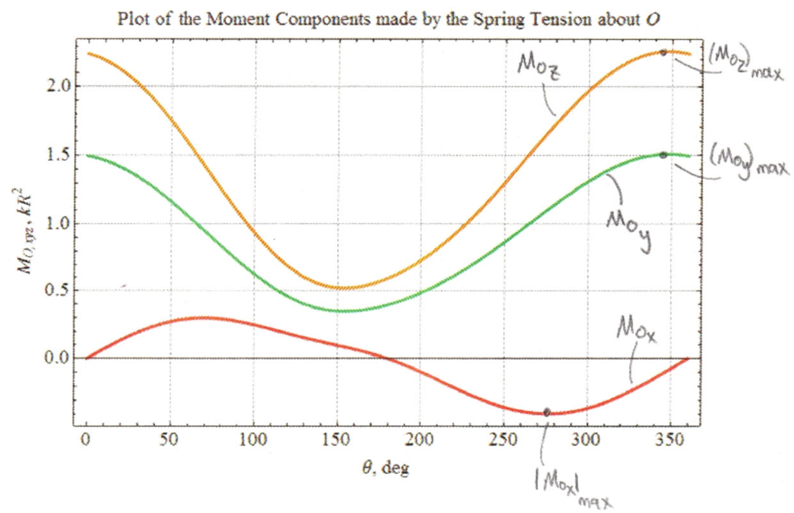
$$\delta = d - L_0 = \frac{R}{4}\left(\sqrt{125 + 48\cos\theta - 36\sin\theta} - 6\right)$$

$$\underline{F}_s = k\delta\,\underline{n}_{AB}$$

$$\underline{M}_O = \underline{r}_{OA} \times \underline{F}_s \quad \text{AND} \quad \underline{r}_{OA} = -1.5R\hat{j} + R\hat{k}$$

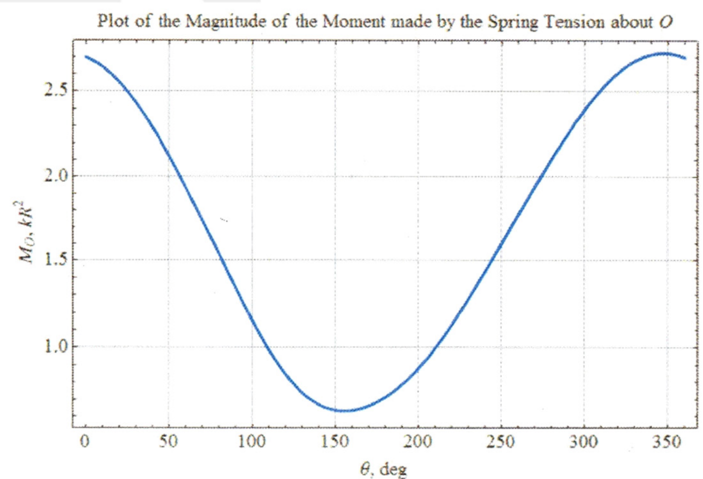
CARRY OUT THE CROSS PRODUCT AND PLOT THE MOMENT COMPONENTS.

Plot of the Moment Components made by the Spring Tension about O



$$\begin{cases} |M_{O_x}|_{max} = 0.398\ kR^2 \quad \text{AT} \quad \theta = 277^\circ \\ |M_{O_y}|_{max} = 1.509\ kR^2 \quad \text{AT} \quad \theta = 348^\circ \\ |M_{O_z}|_{max} = 2.26\ kR^2 \quad \text{AT} \quad \theta = 348^\circ \end{cases}$$

Plot of the Magnitude of the Moment made by the Spring Tension about O



$$|M_O|_{max} = 2.72\ kR^2 \quad \text{AT} \quad \theta = 347^\circ$$

2/131

$$\begin{cases} R_x = \Sigma F_x = -7 \text{ kN} \\ R_y = \Sigma F_y = 4 - F_3 \cos\theta = -5 \text{ kN} & (1) \\ R_z = \Sigma F_z = F_3 \sin\theta = 6 \text{ kN} & (2) \end{cases}$$

(1): $F_3 \cos\theta = 9$

(2): $F_3 \sin\theta = 6$

Divide Eq. (2) by Eq. 1: $\tan\theta = \frac{2}{3}$

$\underline{\theta = 33.7°}$

Then $\underline{F_3 = 10.82 \text{ kN}}$

$R = \sqrt{7^2 + 5^2 + 6^2} = \underline{10.49 \text{ kN}}$

2/132 $R = \Sigma F = 160 + 200 + 80 + 160 = \underline{600 \text{ N} \downarrow}$

$\underline{M}_O = (-200 - 160)(0.6)\underline{i} + (80 + 160)(0.9)\underline{j}$

$= \underline{-216\underline{i} + 216\underline{j} \text{ N}\cdot\text{m}}$

$\underline{R} \cdot \underline{M}_O = -600\underline{k} \cdot (-216\underline{i} + 216\underline{j}) = 0, \quad \underline{\underline{R} \perp \underline{M}_O}$

2/133

$$\underline{R} = -2F\underline{k} + F\underline{k} + F(\cos 30^\circ \underline{k} + \sin 30^\circ \underline{j})$$

$$= \frac{F}{2}\underline{j} + F\left(\frac{\sqrt{3}}{2} - 1\right)\underline{k} = \underline{F\left(\frac{1}{2}\underline{j} + \left(\frac{\sqrt{3}}{2} - 1\right)\underline{k}\right)}$$

$$\underline{M}_o = 2Fb\underline{j} + Fb\underline{i} + \frac{F}{2}(2b)\underline{k} + \frac{\sqrt{3}}{2}Fb\underline{i} - \frac{\sqrt{3}}{2}F(2b)\underline{j}$$

$$= \underline{Fb\left[\left(1 + \frac{\sqrt{3}}{2}\right)\underline{i} + (2 - \sqrt{3})\underline{j} + \underline{k}\right]}$$

$$\underline{R} \cdot \underline{M}_o = \left[\frac{1}{2}(2 - \sqrt{3}) + \left(\frac{\sqrt{3}}{2} - 1\right)(1)\right]F^2 b = 0, \text{ so}$$

$\underline{\underline{R} \perp \underline{M}_o}$.

2/134 $\underline{R} = \Sigma \underline{F} = \underline{-8\underline{i} \text{ kN}}$

$\underline{M}_G = 50(10)\underline{k} + 8(6)\underline{j} + 8(40)\underline{k}$

$= \underline{48\underline{j} + 820\underline{k} \text{ kN}\cdot\text{m}}$

2/135 $R = \Sigma F_z = 70 + 30 - 80 - 60 - 50 = -90 \text{ lb}$

$-|R|y = \Sigma M_x: \; -90y = 30(12) + 70(12) - 60(6) - 50(12)$

$\underline{y = -2.67 \text{ in.}}$

$|R|x = \Sigma M_y: \; 90x = 80(10) - 30(10) - 50(8)$

$\underline{x = 1.111 \text{ in.}}$

2/136 The two 160-N forces constitute a couple $160(0.250)\mathbf{j} = 40\mathbf{j}$ N·m

$\mathbf{R} = \Sigma \mathbf{F} = \underline{120\mathbf{i} - 180\mathbf{j} - 100\mathbf{k}}$ N

$\mathbf{M} = \Sigma \mathbf{M}_A = [120(0.25) + 100(0.3) + 40]\mathbf{j} + 50\mathbf{k}$ N·m

$= \underline{100\mathbf{j} + 50\mathbf{k} \text{ N·m}}$

2/137

$$\underline{R} = \Sigma \underline{F} = 600(\sin 30^\circ \underline{j} + \cos 30^\circ \underline{k}) + 800(-\sin 45^\circ \underline{j} + \cos 45^\circ \underline{k})$$

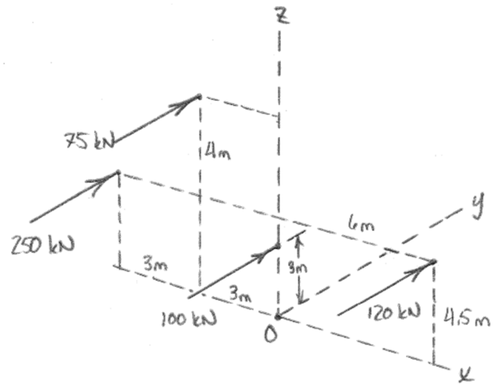
$$= -266\underline{j} + 1085\underline{k} \text{ N}$$

$$\underline{M}_O = -0.080\underline{i} \times 600(\sin 30^\circ \underline{j} + \cos 30^\circ \underline{k}) + 0.160\underline{i} \times 800(-\sin 45^\circ \underline{j} + \cos 45^\circ \underline{k})$$

$$= -48.9\underline{j} - 114.5\underline{k} \text{ N·m}$$

$\underline{R}$ is not perpendicular to $\underline{M}_O$, because $\underline{R} \cdot \underline{M}_O \neq 0$.

2/138



$\underline{r} = x\underline{i} + z\underline{k}$

$\underline{R} = (75 + 250 + 100 + 120)\underline{j} \longrightarrow \underline{R} = 545\,\underline{j}$ kN

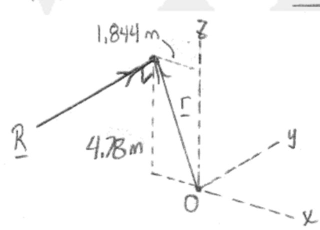
$\underline{M}_O = -[120(4.5) + 100(3) + 250(4.5) + 75(8.5)]\underline{i} + [120(6) - 75(3) - 250(6)]\underline{k}$

$\underline{M}_O = -2600\,\underline{i} - 1010\,\underline{k}$ kN·m

$\underline{r} \times \underline{R} = \underline{M}_O \rightarrow (x\underline{i} + z\underline{k}) \times 545\,\underline{j} = -2600\,\underline{i} - 1010\,\underline{k}$

$\begin{cases} \underline{i}: -545\,z = -2600 \\ \underline{k}: 545\,x = -1010 \end{cases}$ Solving... $x = -1.844$ m $\quad z = 4.78$ m

$P = (-1.844, 0, 4.78)$ m



2/139

$$\underline{R} = (200 + 800)\underline{i} + 1200(\cos 10^\circ \underline{j} - \sin 10^\circ \underline{i})$$

$$= \underline{792\,\underline{i} + 1182\,\underline{j}\ \text{N}}$$

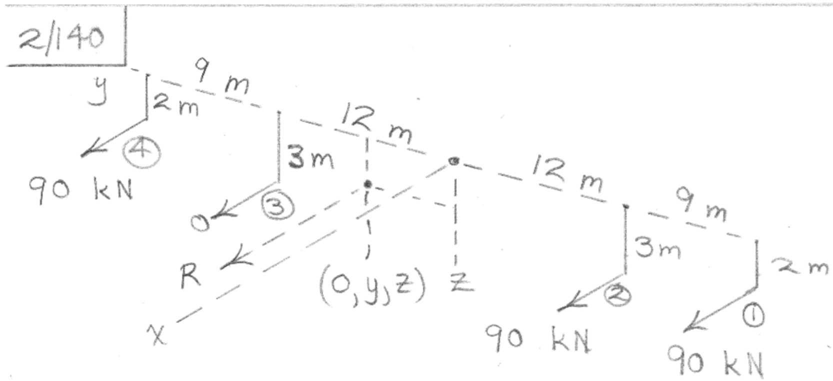
$$\underline{M}_O = \left[(200-800)(0.1) + (1200\cos 10^\circ)(0.075)\right]\underline{k}$$

$$+\left[-(200+800)(0.220+0.330) + 1200\sin 10^\circ(0.220)\right]\underline{j}$$

$$+\left[1200\cos 10^\circ(0.220)\right]\underline{i}$$

$$= \underline{260\,\underline{i} - 504\,\underline{j} + 28.6\,\underline{k}\ \text{N}\cdot\text{m}}$$

2/140



$R = \Sigma F = 3(90) = 270 \text{ kN}$

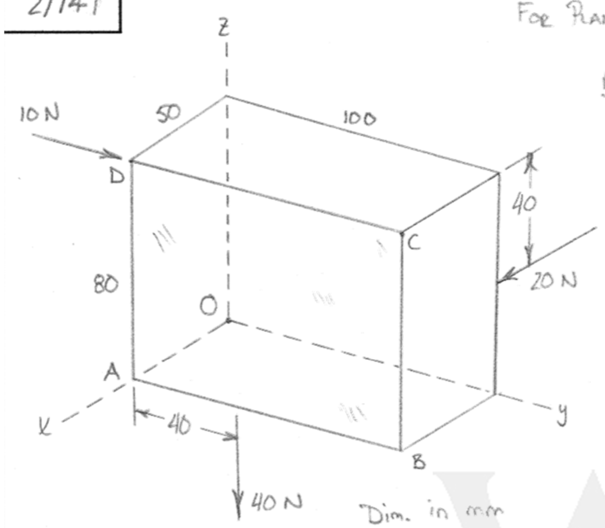
$\Sigma M_z = -Ry : 90(21) + 90(12) - 90(21) = -270y$

$\underline{y = -4 \text{ m}}$

$\Sigma M_y = Rz : 2(90)(2) + 90(3) = 270z$

$\underline{z = 2.33 \text{ m}}$

2/141



For Plane ABCD

$$\underline{r} = 50\underline{i} + y\underline{j} + z\underline{k} \text{ mm}$$

$$\underline{R} = 20\underline{i} + 10\underline{j} - 40\underline{k} \qquad \underline{n} = \frac{\underline{R}}{R} = \frac{1}{\sqrt{21}}(2\underline{i} + \underline{j} - 4\underline{k})$$

$$\underline{M}_O = [-10(0.08) - 40(0.04)]\,\underline{i} + [20(0.04) + 40(0.05)]\,\underline{j} + [10(0.05) - 20(0.1)]\,\underline{k}$$

$$\underline{M}_O = -2.4\underline{i} + 2.8\underline{j} - 1.5\underline{k} \text{ N·m}$$

$$M = \underline{M}_O \cdot \underline{n} = (-2.4\underline{i} + 2.8\underline{j} - 1.5\underline{k}) \cdot \left[\frac{1}{\sqrt{21}}(2\underline{i} + \underline{j} - 4\underline{k})\right] \longrightarrow \underline{M = 0.873 \text{ N·m } (+)}$$

$$\underline{M} = M\underline{n} = 0.873\left(\frac{2}{\sqrt{21}}\underline{i} + \frac{1}{\sqrt{21}}\underline{j} - \frac{4}{\sqrt{21}}\underline{k}\right) \longrightarrow \underline{M} = 0.381\underline{i} + 0.1905\underline{j} - 0.762\underline{k} \text{ N·m}$$

$$\underline{r} \times \underline{R} + \underline{M} = \underline{M}_O \longrightarrow \left(\frac{50}{1000}\underline{i} + \frac{y}{1000}\underline{j} + \frac{z}{1000}\underline{k}\right) \times (20\underline{i} + 10\underline{j} - 40\underline{k}) + 0.381\underline{i} + 0.1905\underline{j} -$$

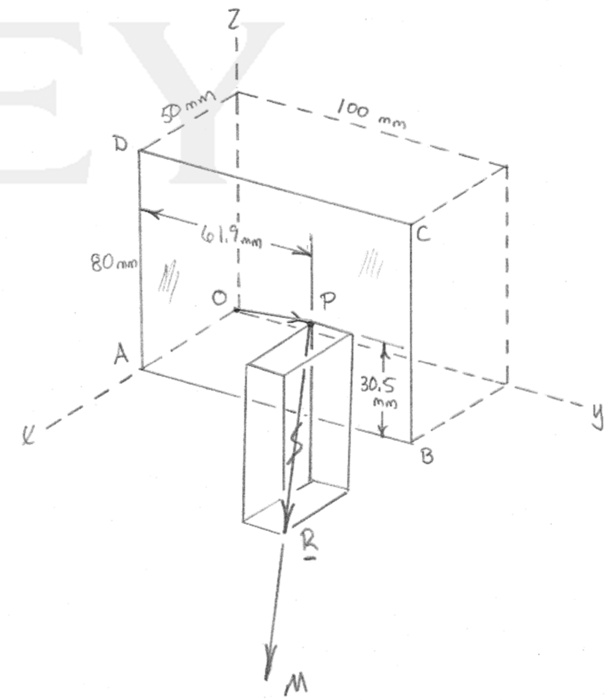
$$\ldots\ 0.762\underline{k} = -2.4\underline{i} + 2.8\underline{j} - 1.5\underline{k}$$

$$\begin{cases} \underline{i}: & -0.04y - 0.01z + 0.381 = -2.4 \\ \underline{j}: & 0.02z + 2.19 = 2.8 \\ \underline{k}: & -0.02y - 0.262 = -1.5 \end{cases}$$

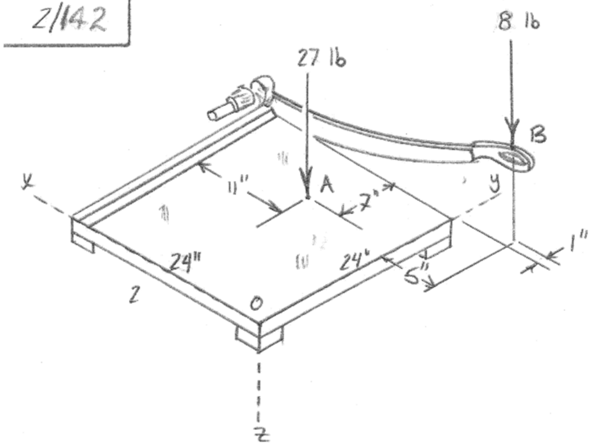
Solving...

$$\begin{cases} y = 61.9 \text{ mm} \\ z = 30.5 \text{ mm} \end{cases}$$

$$\underline{P = (50, 61.9, 30.5) \text{ mm}}$$



2/142



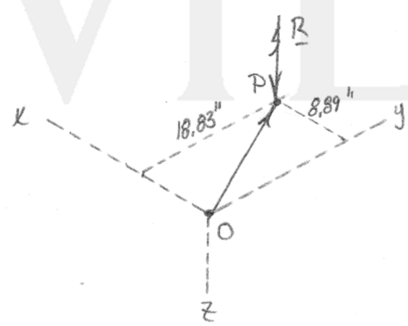
$\underline{R} = 35\,\underline{k}$ lb

$\underline{r} = x\underline{i} + y\underline{j}$

$\underline{M}_O = [27(24-7) + 8(24+1)]\underline{i} + [8(5) - 27(24-11)]\underline{j} \rightarrow \underline{M}_O = 659\hat{i} - 311\hat{j}$ lb-in.

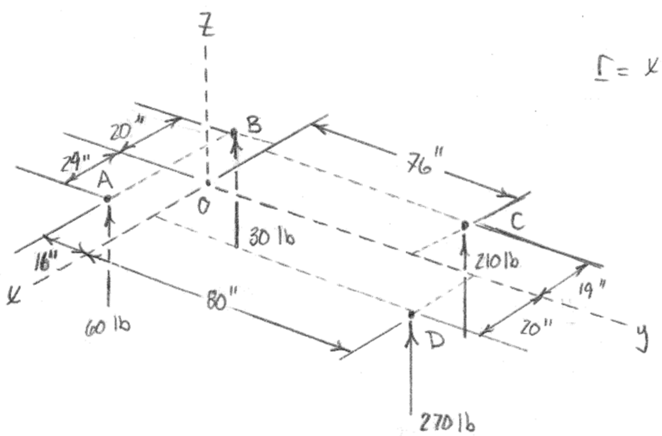
$\underline{r} \times \underline{R} = \underline{M}_O \rightarrow (x\underline{i} + y\underline{j}) \times 35\underline{k} = 659\underline{i} - 311\underline{j}$

$\begin{cases} \underline{i}: & 35y = 659 \\ \underline{j}: & -35x = -311 \end{cases} \longrightarrow \underline{x = 8.89 \text{ in.}}$ & $\underline{y = 18.83 \text{ in.}}$



2/143

$\underline{R} = (60 + 30 + 210 + 270)\underline{k} \longrightarrow \underline{R} = 570\underline{k}$ lb



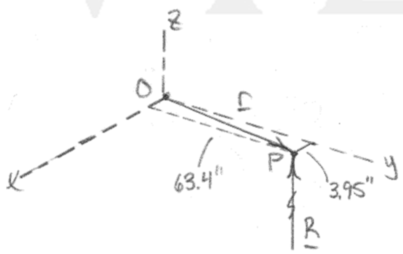
$\underline{r} = x\underline{i} + y\underline{j}$

$\underline{M}_O = [270(80) + 210(76) - 60(16) - 30(16)]\underline{i} + [210(19) + 30(20) - 270(20) - 60(24)]\underline{j}$

$\underline{M}_O = 36{,}120\underline{i} - 2250\underline{j}$ lb-in.

$\underline{r} \times \underline{R} = \underline{M}_O \longrightarrow (x\underline{i} + y\underline{j}) \times 570\underline{k} = 36{,}120\underline{i} - 2250\underline{j}$

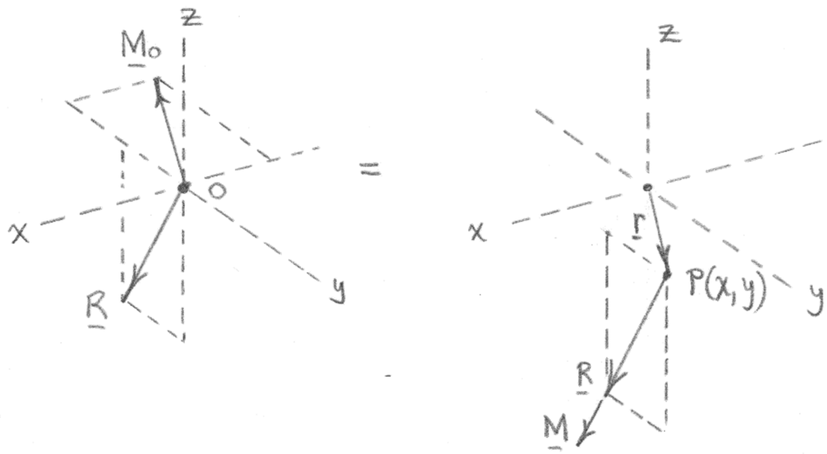
$\begin{cases} \underline{i}: 570y = 36{,}120 \\ \underline{j}: -570x = -2250 \end{cases} \longrightarrow \underline{x = 3.95 \text{ in.}} \ \& \ \underline{y = 63.4 \text{ in.}}$



2/144

$$\mathbf{R} = -20\mathbf{j} - 40\mathbf{k} \text{ lb} \quad (= 44.7(-0.447\mathbf{j} - 0.894\mathbf{k}))$$

$$\mathbf{M}_O = -40(1.4)\mathbf{i} - 40(8)\mathbf{j}$$
$$= -56\mathbf{i} - 320\mathbf{j} \text{ lb-in.}$$



$$\mathbf{M}_O: -56\mathbf{i} - 320\mathbf{j} = \mathbf{r} \times \mathbf{R} + \mathbf{M} = (x\mathbf{i} + y\mathbf{j}) \times (-20\mathbf{j} - 40\mathbf{k}) + M(-0.447\mathbf{j} - 0.894\mathbf{k})$$

Equate coefficients:

$\mathbf{i}: -56 = -40y$

$\mathbf{j}: -320 = 40x - 0.447M$

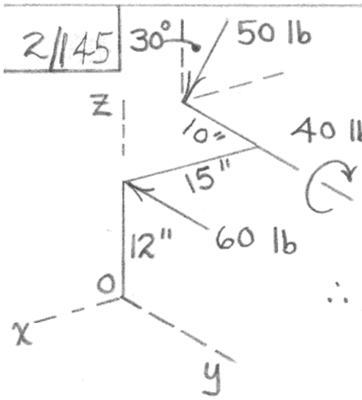
$\mathbf{k}: 0 = -20x - 0.894M$

Solution:

$x = -6.4$ in.

$y = 1.4$ in.

2/145



$R_x = \Sigma F_x = 50 \sin 30° = 25$ lb

$R_y = \Sigma F_y = -60$ lb

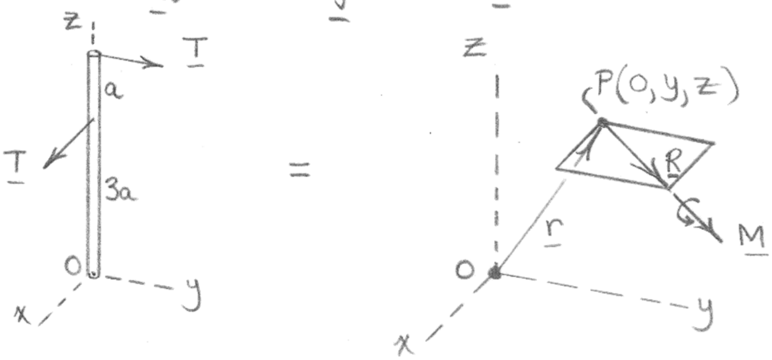
$R_z = \Sigma F_z = -50 \cos 30° = -43.3$ lb

$\therefore \mathbf{R} = 25\mathbf{i} - 60\mathbf{j} - 43.3\mathbf{k}$ lb

$$\mathbf{M}_O = 12(60)\mathbf{i} + 50 \sin 30° (12)\mathbf{j} + 50 \sin 30° (10)\mathbf{k}$$
$$+ 50 \cos 30° (10)\mathbf{i} - 50 \cos 30° (15)\mathbf{j} - 40(12)\mathbf{j}$$
$$= 1153\mathbf{i} - 830\mathbf{j} + 250\mathbf{k} \text{ lb-in.}$$

2/146 $\underline{R} = \Sigma \underline{F} = T\underline{i} + T\underline{j} = \sqrt{2}T\left[\frac{1}{\sqrt{2}}\underline{i} + \frac{1}{\sqrt{2}}\underline{j}\right]$

$\Sigma \underline{M}_O = 3aT\underline{j} - 4aT\underline{i}$



$\Sigma \underline{M}_O = \underline{r} \times \underline{R} + \underline{M}$

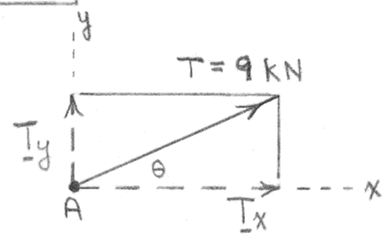
$3aT\underline{j} - 4aT\underline{i} = (y\underline{j} + z\underline{k}) \times (T\underline{i} + T\underline{j}) + M\left(\frac{1}{\sqrt{2}}\underline{i} + \frac{1}{\sqrt{2}}\underline{j}\right)$

$$\Rightarrow \begin{cases} -4aT = -zT + \frac{M}{\sqrt{2}} \\ 3aT = zT + \frac{M}{\sqrt{2}} \\ 0 = -yT \end{cases} \quad \text{So} \quad \begin{cases} y = 0 \\ z = \frac{7}{2}a \\ M = -\frac{\sqrt{2}}{2}aT \end{cases}$$

So $\underline{M} = -\frac{\sqrt{2}}{2}aT\left(\frac{1}{\sqrt{2}}\underline{i} + \frac{1}{\sqrt{2}}\underline{j}\right)$

$= \underline{\frac{-aT}{2}(\underline{i} + \underline{j})}$ (a negative wrench)

2/147



$\theta = \tan^{-1} \frac{6}{10} = 31.0°$

$T_x = T \cos\theta = 9 \cos 31.0°$

$= 7.72 \text{ kN}$

$T_y = T \sin\theta = 9 \sin 31.0°$

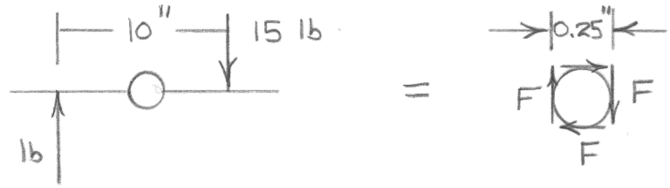
$= 4.63 \text{ kN}$

$\underline{T} = 7.72\underline{i} + 4.63\underline{j} \text{ kN}$

2/148

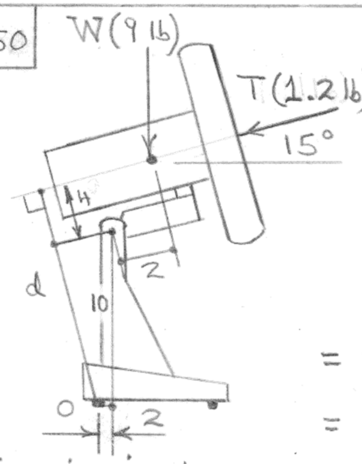
$$\begin{cases} \underline{M}_1 = \underline{-cF_1\,\underline{i}} \\ \underline{M}_2 = cF_2\,\underline{i} - aF_2\,\underline{k} = \underline{F_2(c\underline{i} - a\underline{k})} \\ \underline{M}_3 = \underline{-aF_3\,\underline{k}} \end{cases}$$

2/149



$M = Fd = 15(10) = 2F(0.25)$, $\underline{F = 300 \text{ lb}}$

2/150



Dim. in inches

$$M_O = Fd$$

$$= 1.2(4 + 10\cos 15° - 2\sin 15°)$$

$$= 15.77 \text{ lb-in.}$$

$$M_{O_W} = W d_W$$

$$= 9(2 + 2\cos 15° - 4\sin 15°)$$

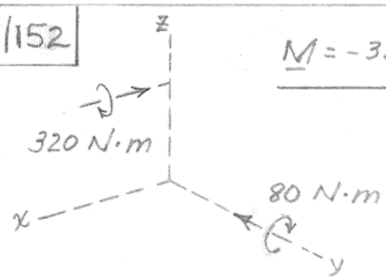
$$= 26.1 \text{ lb-in.}$$

2/151 $\underline{P} = P\left(\frac{4}{5}\underline{i} + \frac{3}{5}\underline{j}\right)$; $\underline{r}_{AB} = b(-\underline{i} + \underline{j} + \underline{k})$

Carry out $\underline{M}_A = \underline{r}_{AB} \times \underline{P}$ to obtain

$$\underline{M}_A = \frac{Pb}{5}\left(-3\underline{i} + 4\underline{j} - 7\underline{k}\right)$$

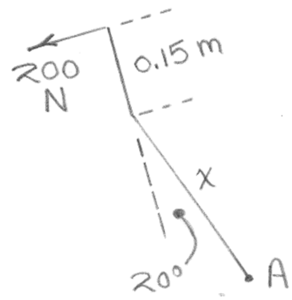
2/152



$\mathbf{M} = -320\mathbf{i} - 80\mathbf{j}$ N·m

$$\cos\theta_x = \frac{M_x}{|\mathbf{M}|} = \frac{-320}{\sqrt{320^2 + 80^2}} = \underline{-0.970}$$

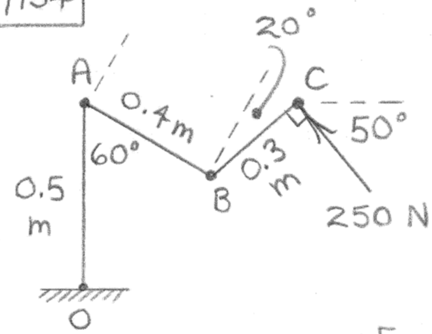
2/153



$M_A = Fd$: $80 = 200(0.15 + x \cos 20°)$

$x = 0.266$ m or $\underline{266 \text{ mm}}$

2/154



$$\circlearrowleft + M_O = 250 \cos 50° \left[0.5 - 0.4 \cos 60° + 0.3 \sin 40°\right]$$
$$+ 250 \sin 50° \left[0.4 \sin 60° + 0.3 \cos 40°\right]$$
$$= \underline{189.6 \text{ N}\cdot\text{m CCW}}$$

2/155

$$\underline{R} = 800\left[-\sin 30^\circ \cos 20^\circ \underline{i} + \sin 30^\circ \sin 20^\circ \underline{j} + \cos 30^\circ \underline{k}\right]$$

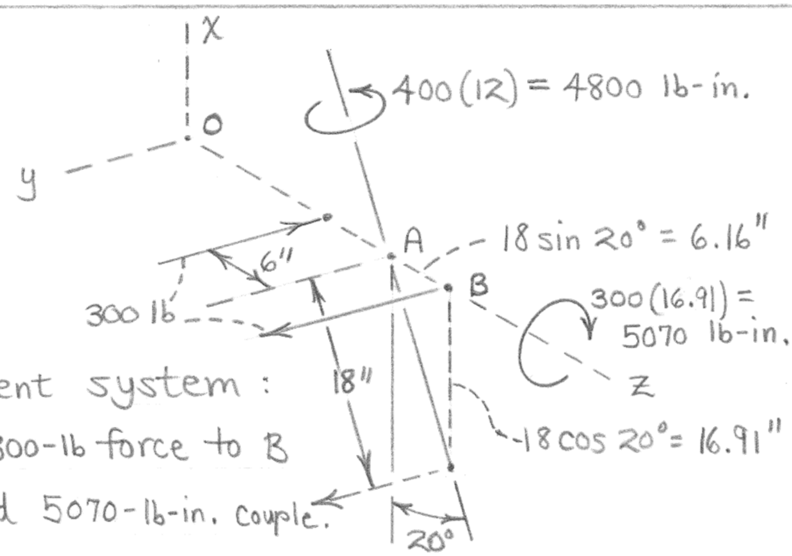
$$= \underline{-376\underline{i} + 136.8\underline{j} + 693\underline{k} \quad \text{N}}$$

$$\underline{M}_O = \underline{r}_{OB} \times \underline{F}$$

$$\underline{r}_{OB} = \left[300 \sin 20^\circ \underline{i} + 300 \cos 20^\circ \underline{j} + 250\underline{k}\right] \text{mm}$$

$$\underline{\underline{M}_O = 161.1\underline{i} - 165.1\underline{j} + 120\underline{k} \quad \text{N}\cdot\text{m}}$$

2/156



Equivalent system:
Move 300-lb force to B
and add 5070-lb-in. couple.

The components of the resultant couple are

$M_x = -300(6+6.16) + 4800 \cos 20° = 864$ lb-in.

$M_y = 0$

$M_z = -4800 \sin 20° - 5070 = -6720$ lb-in.

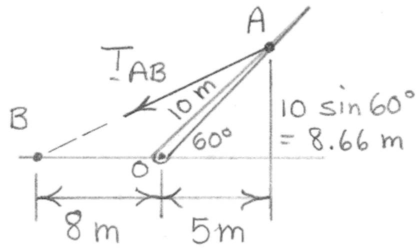
$\underline{M} = 864\underline{i} - 6720\underline{k}$ lb-in.

$M = \sqrt{864^2 + 6720^2} = \underline{6770 \text{ lb-in.}}$

2/157

$$\overline{AB}^2 = 8^2 + 10^2 - 2(8)(10)\cos 120^\circ$$

$$\overline{AB} = 15.62 \text{ m}$$



(a) $\underline{T}_{AB} = 3\left[-\frac{13}{15.62}\cos 35^\circ \underline{i} - \frac{13}{15.62}\sin 35^\circ \underline{j} - \frac{8.66}{15.62}\underline{k}\right]$

$= -2.05\underline{i} - 1.432\underline{j} - 1.663\underline{k}$ kN

(b) Carry out $\underline{r}_{OB} \times \underline{T}_{AB}$, where $\underline{r}_{OB} = 8(-\cos 35^\circ \underline{i} - \sin 35^\circ \underline{j})$ m to obtain

$\underline{M}_O = 7.63\underline{i} - 10.90\underline{j}$ kN·m

$\therefore M_{O_x} = 7.63$ kN·m, $M_{O_y} = -10.90$ kN·m, $M_{O_z} = 0$

(c) $T_{AO} = \underline{T}_{AB} \cdot \underline{n}_{AO}$

With $\underline{n}_{AO} = -\cos 60^\circ \cos 35^\circ \underline{i} - \cos 60^\circ \sin 35^\circ \underline{j} - \sin 60^\circ \underline{k}$,

we obtain $T_{AO} = 2.69$ kN

2/158

$$\underline{R} = \Sigma \underline{F} = 500\cos 45°(\underline{i}) + 400\underline{j} - (600 + 500\sin 45°)\underline{k}$$

$$= 354\underline{i} + 400\underline{j} - 954\underline{k} \text{ lb}$$

$$R = \sqrt{\overline{354}^2 + \overline{400}^2 + \overline{954}^2} = \underline{1093 \text{ lb}}$$

$$\underline{M} = [500\cos 45°(3) - 600(3) - 400(10)]\underline{i}$$
$$+ [500\cos 45°(6) + 500\sin 45°(7) + 600(6)]\underline{j}$$
$$+ [500\sin 45°(3) + 400(3)]\underline{k}$$

$$= -4739\underline{i} + 8196\underline{j} + 2261\underline{k} \text{ lb-ft}$$

$$M = \sqrt{\overline{4739}^2 + \overline{8196}^2 + \overline{2261}^2} = \underline{9730 \text{ lb-ft}}$$

*2/159

$\Sigma F_x = 0: -360 - 240 \sin\theta + T \sin 30° + 400 \cos 30° = 0$ (1)

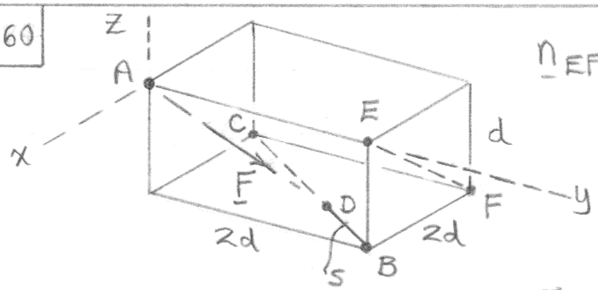
$\Sigma F_y = 600: 240 \cos\theta + T \cos 30° + 400 \sin 30° = 600$ (2)

Numerical solution of Eqs. (1) & (2):

$\underline{\theta = 21.7°, \quad T = 204 \text{ lb}}$

(We could eliminate T between Eqs. (1) & (2), but the resulting equation is still transcendental.)

*2/160



$$\underline{n}_{EF} = \frac{-2d\underline{i} - d\underline{k}}{\sqrt{5}\,d} = \frac{-2\underline{i} - \underline{k}}{\sqrt{5}}$$

$$\underline{AD} = \underline{AB} + \underline{BD} = 2d\underline{j} - d\underline{k} + s\left[\frac{-2d\underline{i} - 2d\underline{j}}{\sqrt{8}\,d}\right]$$

$$= -\frac{s}{\sqrt{2}}\underline{i} + \left(2d - \frac{s}{\sqrt{2}}\right)\underline{j} - d\underline{k}$$

$$|\underline{AD}| = \sqrt{\frac{s^2}{2} + \left(2d - \frac{s}{\sqrt{2}}\right)^2 + d^2} = \sqrt{s^2 + 5d^2 - 2\sqrt{2}\,ds}$$

$$\underline{F} = F\frac{\underline{AD}}{|\underline{AD}|} = F\,\frac{-\frac{s}{\sqrt{2}}\underline{i} + \left(2d - \frac{s}{\sqrt{2}}\right)\underline{j} - d\underline{k}}{\sqrt{s^2 + 5d^2 - 2\sqrt{2}\,ds}}$$

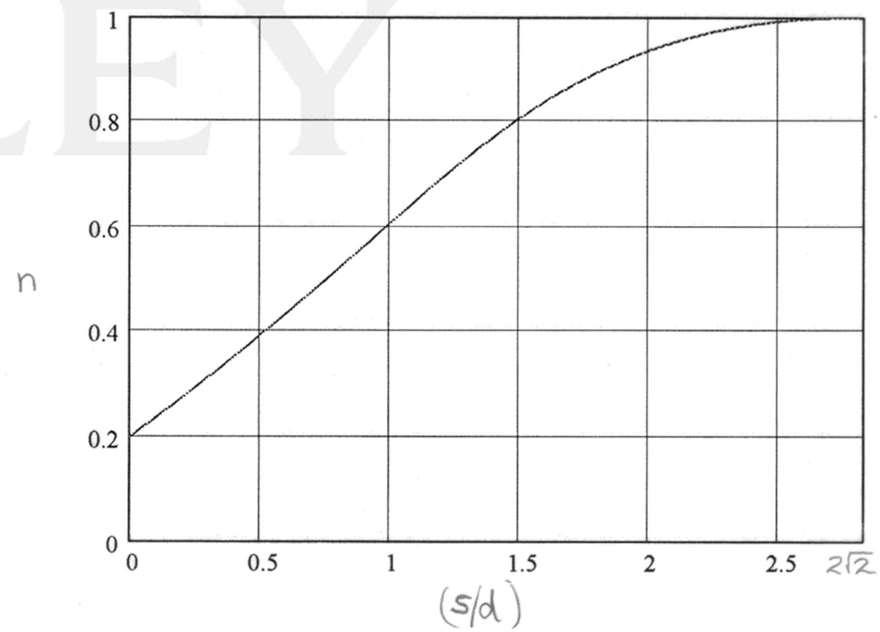
Carry out $\underline{F} \cdot \underline{n}_{EF}$ to obtain the projection

$$\underline{F} \cdot \underline{n}_{EF} = \frac{F(s\sqrt{2} + d)}{\sqrt{5}\sqrt{s^2 + 5d^2 - 2\sqrt{2}\,ds}}$$

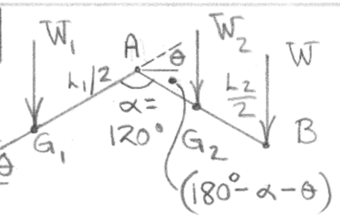
The nondimensionalized fraction n of the magnitude F projected is then

$$n = \frac{\underline{F} \cdot \underline{n}_{EF}}{F} = \underline{\frac{\sqrt{2}\,\frac{s}{d} + 1}{\sqrt{5}\sqrt{\left(\frac{s}{d}\right)^2 + 5 - 2\sqrt{2}\,\frac{s}{d}}}}$$

We let $\frac{s}{d}$ vary from 0 to $2\sqrt{2}$ as D moves from B to C. Resulting plot:



*2/161



$$\begin{cases} W_1 = 120 \text{ lb} & L_1 = 3 \text{ ft} \\ W_2 = 50 \text{ lb} & L_2 = 2 \text{ ft} \\ W = 300 \text{ lb} & \alpha = 120° \end{cases}$$

$$+\circlearrowleft M_O = W_1 \frac{L_1}{2} \cos\theta + W_2\left(L_1 \cos\theta + \frac{L_2}{2} \cos\left(180° - \alpha - \theta\right)\right)$$
$$+ W\left(L_1 \cos\theta + L_2 \cos\left(180° - \alpha - \theta\right)\right)$$

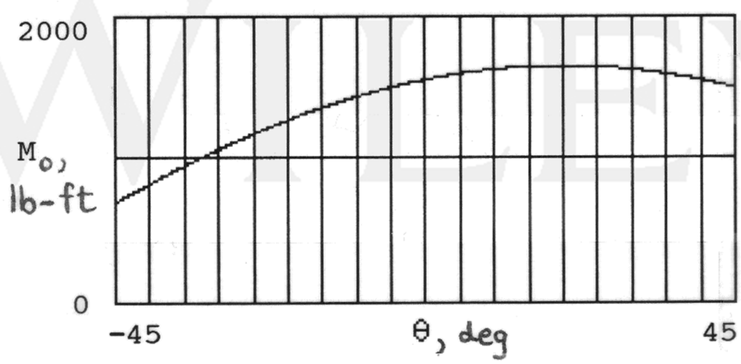
With the above numbers:

$\underline{M_O = 1230 \cos\theta + 650 \cos(60° - \theta)}$ (in lb-ft)

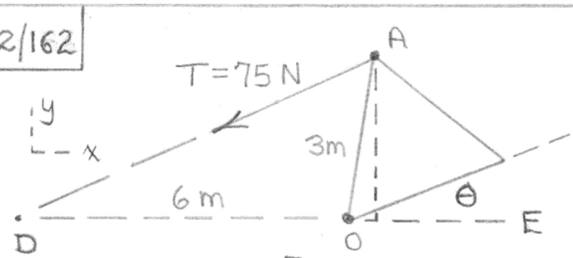
(see plot below)

For $(M_O)_{max}$: $\frac{dM_O}{d\theta} = -1230 \sin\theta + 650 \sin(60° - \theta) = 0$

Numerical solution : $\underline{\theta = 19.90°}$, $\underline{(M_O)_{max} = 1654 \text{ lb-ft}}$



*2/162



Angle AOE = $\theta + 60°$

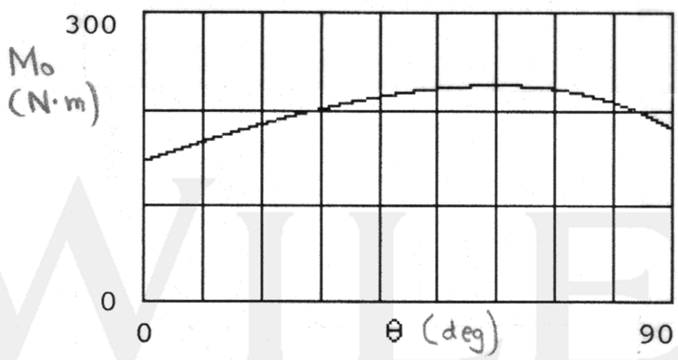
Use $\underline{M}_O = \underline{r}_{OD} \times \underline{T}$

$\underline{r}_{OD} = -6\underline{i}$ m

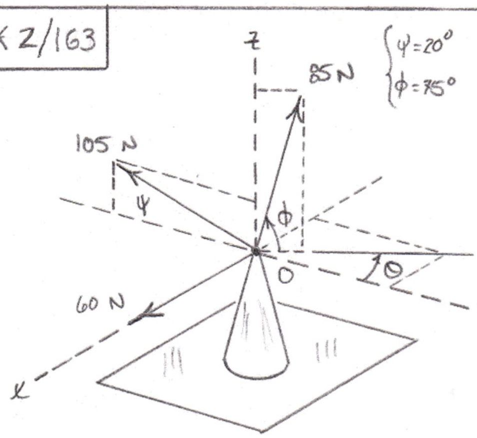
$$\underline{T} = T\underline{n}_{AD} = 75\left[\frac{-(6+3\cos(\theta+60°))\underline{i} - 3\sin(\theta+60°)\underline{j}}{\sqrt{[6+3\cos(\theta+60°)]^2 + [3\sin(\theta+60°)]^2}}\right] \text{N}$$

$$\underline{M}_O = \underline{r}_{OD} \times \underline{T} = \frac{1350\sin(\theta+60°)\,\underline{k}}{\sqrt{45+36\cos(\theta+60°)}} \text{ N·m}$$

M_O is a max @ $\underline{\theta = 60°}$: $\underline{M_O = 225 \text{ N·m}}$



*2/163



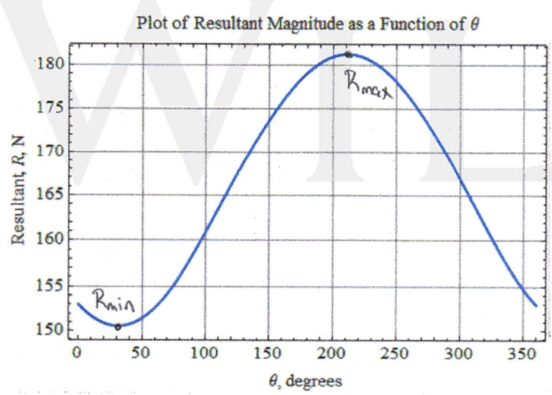
$\psi = 20°$, $\phi = 75°$

$$\mathbf{R} = (60 - 85\cos 75° \sin\theta)\hat{i} + (85\cos 75° \cos\theta - 105\cos 20°)\hat{j} + (85\sin 75° + 105\sin 20°)\hat{k}$$

$$\mathbf{R} = (60 - 22.0\sin\theta)\mathbf{i} + (22.0\cos\theta - 98.7)\mathbf{j} + 118.0\mathbf{k} \text{ N}$$

$$R = \sqrt{\mathbf{R}\cdot\mathbf{R}} \longrightarrow R = \left[(60 - 22.0\sin\theta)^2 + (22.0\cos\theta - 98.7)^2 + 118.0^2\right]^{1/2}$$

PLOTTING R... FOR θ_{max}, SET $\dfrac{dR}{d\theta} = 0$



$R_{min} = 150.6$ N

$\theta_{min} = 31.3°$

$R_{max} = 181.2$ N

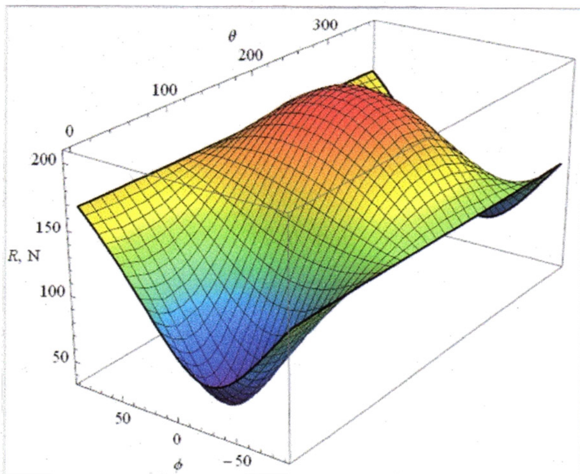
$\theta_{max} = 211°$

$\boxed{*2/164}$

FROM 2/163...

$$\underline{R} = (60 - 85\cos\phi\sin\theta)\underline{i} + (85\cos\phi\cos\theta - 105\cos 20)\underline{j} + (85\sin\phi + 105\sin 20)\underline{k}$$

$R = \sqrt{\underline{R}\cdot\underline{R}}$ AND WE PLOT FOR $0 \le \theta \le 360°$ AND $-90° \le \phi \le 90°$



$R_{min} = 35.9$ N

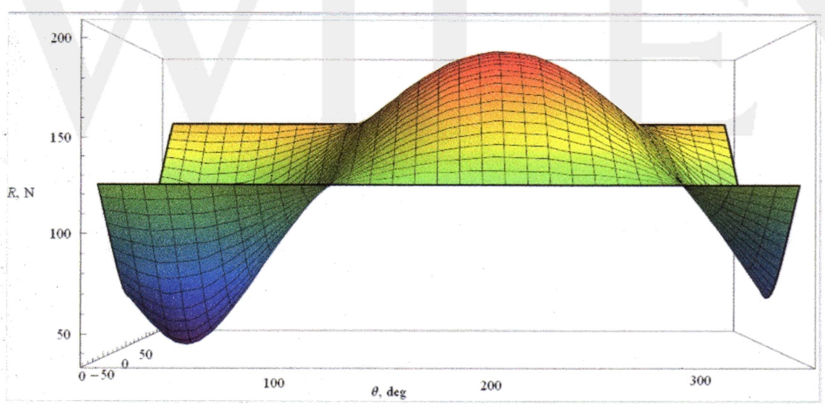
$\theta_{min} = 31.3°$

$\phi_{min} = -17.27°$

$R_{max} = 206$ N

$\theta_{max} = 211°$

$\phi_{max} = 17.27°$



*2/165 $\underline{T} = T\underline{n}_{AB}$

$$\underline{T} = T\left[\frac{(d+40\cos\beta)\underline{i} + 40(1-\sin\beta)\underline{j}}{\sqrt{(d+40\cos\beta)^2 + 40^2(1-\sin\beta)^2}}\right]$$

$$\left(\beta = \theta + \frac{\pi}{4}\right)$$

$\underline{r}_{OB} = (d\underline{i} + 40\underline{j})$

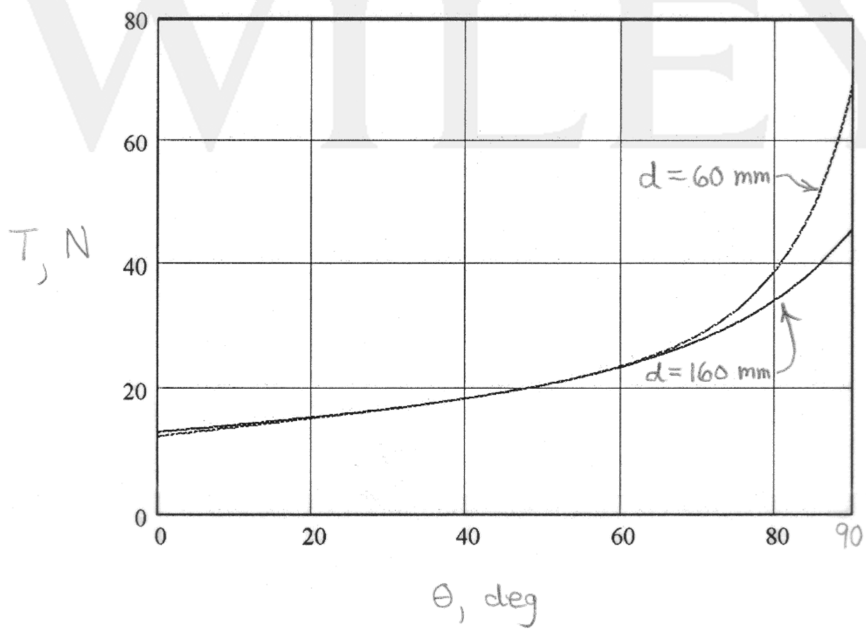
$\Sigma \underline{M}_O = \underline{0}$: $\underline{r}_{OB} \times \underline{T} + K\beta\underline{k} = \underline{0}$

Carry out the cross product, consider the z-component, and solve for T to obtain

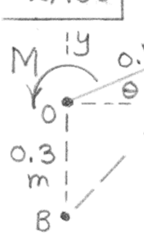
$$T = \frac{12.5\left(\theta+\frac{\pi}{4}\right)\sqrt{d^2 + 80d\cos\left(\theta+\frac{\pi}{4}\right) - 3200\sin\left(\theta+\frac{\pi}{4}\right) + 3200}}{\left[d\sin\left(\theta+\frac{\pi}{4}\right) + 40\cos\left(\theta+\frac{\pi}{4}\right)\right]}$$

(T in N, θ in radians)

Plot of T versus θ for d = 60 mm and for d = 160 mm:



*2/166



$$\underline{F} = F\underline{n}_{AB} = F\frac{\underline{AB}}{|\underline{AB}|}$$

$$\underline{AB} = -0.5\cos\theta\,\underline{i} - (0.5\sin\theta + 0.3)\underline{j} \text{ m}$$

$$|\underline{AB}| = \sqrt{(0.5\cos\theta)^2 + (0.5\sin\theta + 0.3)^2}$$

$$= \sqrt{0.34 + 0.3\sin\theta} \text{ m}$$

$$F = k\delta = 600\left[\sqrt{0.34 + 0.3\sin\theta} - 0.65\right] \text{ N}$$

So

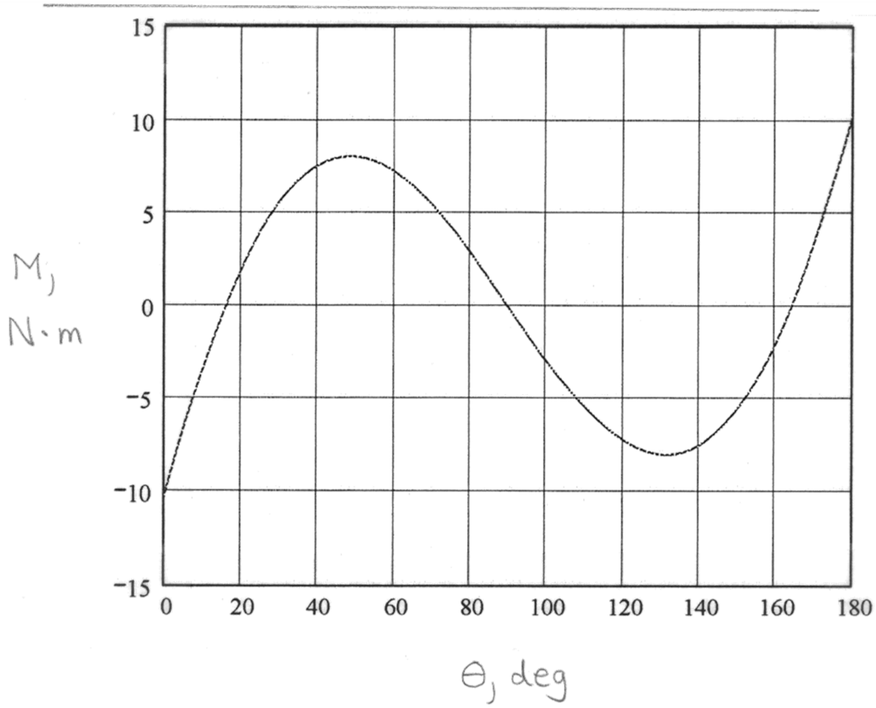
$$\underline{F} = \frac{600\left[\sqrt{0.34 + 0.3\sin\theta} - 0.65\right]}{\sqrt{0.34 + 0.3\sin\theta}}\left[-0.5\cos\theta\,\underline{i} - (0.5\sin\theta + 0.3)\underline{j}\right]$$

Now form $\underline{r}_{OB} \times \underline{F}$, where $\underline{r}_{OB} = -0.3\underline{j}$ m, to obtain

$$-\frac{90\cos\theta\left(\sqrt{0.34 + 0.3\sin\theta} - 0.65\right)}{\sqrt{0.34 + 0.3\sin\theta}}\underline{k}$$

With the above moment plus $M\underline{k}$ summing to zero, we obtain the scalar

$$M = \frac{90\cos\theta\left[\sqrt{0.34 + 0.3\sin\theta} - 0.65\right]}{\sqrt{0.34 + 0.3\sin\theta}} \text{ (N·m)}$$



(Note: $M(0) = -10.33$ N·m

$M(180°) = 10.33$ N·m)