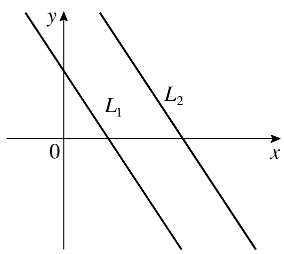
2 SYSTEMS OF LINEAR EQUATIONS AND MATRICES

2.1 Systems of Linear Equations: An Introduction

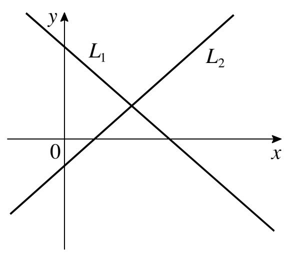
Concept Questions page 79

1. a. There may be no solution, a unique solution, or infinitely many solutions.

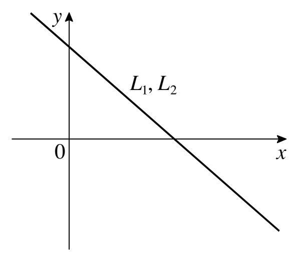
b. There is no solution if the two lines represented by the given system of linear equations are parallel and distinct; there is a unique solution if the two lines intersect at precisely one point; there are infinitely many solutions if the two lines are parallel and coincident.



No solution



A unique solution

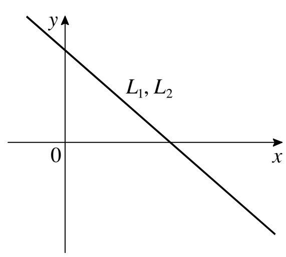


Infinitely many solutions

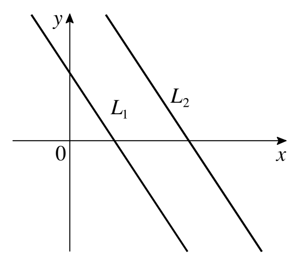
2. a. i. The system is dependent if the two equations in the system describe the same line.

ii. The system is inconsistent if the two equations in the system describe two lines that are parallel and distinct.

b.



Two (coincident) lines in a dependent system



Two lines in an inconsistent system

Exercises page 79

1. Solving the first equation for x, we find $x = 3y - 1$. Substituting this value of x into the second equation yields $4(3y - 1) + 3y = 11$, so $12y - 4 + 3y = 11$ and $y = 1$. Substituting this value of y into the first equation gives $x = 3(1) - 1 = 2$. Therefore, the unique solution of the system is $(2, 1)$.

2. Solving the first equation for x, we have $2x = 4y - 10$, so $x = 2y - 5$. Substituting this value of x into the second equation, we have $3(2y - 5) + 2y = 1$, $6y - 15 + 2y = 1$, $8y = 16$, and $y = 2$. Then $x = 2(2) - 5 = -1$. Therefore, the solution is $(-1, 2)$.

3. Solving the first equation for x, we have $x = 7 - 4y$. Substituting this value of x into the second equation, we have $\frac{1}{2}(7-4y) + 2y = 5$, so $7 - 4y + 4y = 10$, and $7 = 10$. Clearly, this is impossible and we conclude that the system of equations has no solution.

4. Solving the first equation for x, we obtain $3x = 7 + 4y$, so $x = \frac{7}{3} + \frac{4}{3}y$. Substituting this value of x into the second equation, we obtain $9\left(\frac{7}{3} + \frac{4}{3}y\right) - 12y = 14$, so $21 + 12y - 12y = 14$, or $21 = 14$. Since this is impossible, we conclude that the system of equations has no solution.

5. Solving the first equation for x, we obtain $x = 7 - 2y$. Substituting this value of x into the second equation, we have $2(7-2y) - y = 4$, so $14 - 4y - y = 4$, $-5y = -10$, and $y = 2$. Then $x = 7 - 2(2) = 7 - 4 = 3$. We conclude that the solution to the system is $(3, 2)$.

6. Solving the second equation for x, we obtain $x = -\frac{1}{3}y + 2$. Substituting this value of x into the first equation, gives $\frac{3}{2}\left(-\frac{1}{3}y + 2\right) - 2y = 4$, $-\frac{1}{2}y - 2y = 4 - 3$, $-\frac{5}{2}y = 1$, and $y = -\frac{2}{5}$. Then $x = 2 - \frac{1}{3}\left(-\frac{2}{5}\right) = \frac{32}{15}$. Therefore, the solution of the system is $\left(\frac{32}{15}, -\frac{2}{5}\right)$.

7. Solving the first equation for x, we have $2x = 5y + 10$, so $x = \frac{5}{2}y + 5$. Substituting this value of x into the second equation, we have $6\left(\frac{5}{2}y + 5\right) - 15y = 30$, $15y + 30 - 15y = 30$, and $0 = 0$. This result tells us that the second equation is equivalent to the first. Thus, any ordered pair of numbers (x, y) satisfying the equation $2x - 5y = 10$ (or $6x - 15y = 30$) is a solution to the system. In particular, by assigning the value t to x, where t is any real number, we find that $y = -2 + \frac{2}{5}t$ so the ordered pair, $\left(t, \frac{2}{5}t - 2\right)$ is a solution to the system, and we conclude that the system has infinitely many solutions.

8. Solving the first equation for x, we have $5x = 6y + 8$, so $x = \frac{6}{5}y + \frac{8}{5}$. Substituting this value of x into the second equation gives $10\left(\frac{6}{5}y + \frac{8}{5}\right) - 12y = 16$, $12y + 16 - 12y = 16$, and $0 = 0$. This result tells us that the second equation is equivalent to the first. Thus, any ordered pair of numbers (x, y) satisfying the equation $5x - 6y = 8$ (or $10x - 12y = 16$) is a solution to the system. In particular, by assigning the value t to x, where t is any real number, we find that $y = \frac{5}{6}t - \frac{4}{3}$. So the ordered pair,$\left(t, \frac{5}{6}t - \frac{4}{3}\right)$ is a solution to the system, and we conclude that the system has infinitely many solutions.

9. Solving the first equation for x, we obtain $4x - 5y = 14$, so $4x = 14 + 5y$, and $x = \frac{14}{4} + \frac{5}{4}y = \frac{7}{2} + \frac{5}{4}y$. Substituting this value of x into the second equation gives $2\left(\frac{7}{2} + \frac{5}{4}y\right) + 3y = -4$, so $7 + \frac{5}{2}y + 3y = -4$, $\frac{11}{2}y = -11$, and $y = -2$. Thus, $x = \frac{7}{2} + \frac{5}{4}(-2) = 1$. We conclude that the ordered pair $(1, -2)$ satisfies the given system of equations.

10. Solving the first equation for x, we have $\frac{5}{4}x - \frac{2}{3}y = 3$, so $\frac{5}{4}x = \frac{2}{3}y + 3$ and $x = \frac{4}{5}\left(\frac{2}{3}y + 3\right) = \frac{8}{15}y + \frac{12}{5}$. Substituting this value of x into the second equation yields $\frac{1}{4}\left(\frac{8}{15}y + \frac{12}{5}\right) + \frac{5}{3}y = 6$, so $\frac{2}{15}y + \frac{3}{5} + \frac{5}{3}y = 6$, $\frac{27}{15}y = \frac{27}{5}$, and $y = 3$. Then $x = \frac{8}{15}(3) + \frac{12}{5} = \frac{20}{5} = 4$. Thus, the ordered pair $(4, 3)$ satisfies the given equation.

11. Solving the first equation for x, we obtain $2x = 3y + 6$, so $x = \frac{3}{2}y + 3$. Substituting this value of x into the second equation gives $6\left(\frac{3}{2}y + 3\right) - 9y = 12$, so $9y + 18 - 9y = 12$ and $18 = 12$. which is impossible. We conclude that the system of equations has no solution.

12. Solving the first equation for y, we obtain $\frac{2}{3}x + y = 5$, so $y = -\frac{2}{3}x + 5$. Substituting this value of y into the second equation yields $\frac{1}{2}x + \frac{3}{4}\left(-\frac{2}{3}x + 5\right) = \frac{15}{4}$, so $\frac{1}{2}x - \frac{1}{2}x + \frac{15}{4} = \frac{15}{4}$ and $\frac{15}{4} = \frac{15}{4}$. We conclude that the system of equations has infinitely many solutions of the form $\left(t, 5 - \frac{2}{3}t\right)$.

13. Solving the first equation for x, we obtain $-3x = -5y + 1$, so $x = \frac{5}{3}y - \frac{1}{3}$. Substituting this value of y into the second equation yields $2\left(\frac{5}{3}y - \frac{1}{3}\right) - 4y = -1$, $\frac{10}{3}y - \frac{2}{3} - 4y = -1$, $-\frac{2}{3}y = -\frac{1}{3}$, and $y = \frac{1}{2}$. Thus, $x = \frac{5}{3}\left(\frac{1}{2}\right) - \frac{1}{3} = \frac{1}{2}$, and the system has the unique solution $\left(\frac{1}{2}, \frac{1}{2}\right)$.

14. Solving the first equation for x, we obtain $-10x = -15y - 3$, so $x = \frac{3}{2}y + \frac{3}{10}$. Substituting this value of y into the second equation yields $4\left(\frac{3}{2}y + \frac{3}{10}\right) - 6y = -3$, $6y + \frac{6}{5} - 6y = -3$, and $\frac{6}{5} = -3$, which is impossible. We conclude that the system of equations has no solution.

15. Solving the first equation for x, we obtain $3x = 6y + 2$, so $x = 2y + \frac{2}{3}$. Substituting this value of y into the second equation yields $-\frac{3}{2}\left(2y + \frac{2}{3}\right) + 3y = -1$, $-3y - 1 + 3y = -1$, and $0 = 0$. We conclude that the system of equations has infinitely many solutions of the form $\left(2t + \frac{2}{3}, t\right)$, where t is a parameter.

16. Solving the first equation for x, we obtain $\frac{3}{2}x = \frac{1}{2}y + 1$, so $x = \frac{1}{3}y + \frac{2}{3}$. Substituting this value of y into the second equation yields $-\left(\frac{1}{3}y + \frac{2}{3}\right) + \frac{1}{3}y = -\frac{2}{3}$ and $0 = 0$. We conclude that the system of equations has infinitely many solutions of the form $\left(\frac{1}{3}t + \frac{2}{3}, t\right)$, where t is a parameter.

17. Solving the first equation for y, we obtain $y = -0.2x + 1.8$. Substituting this value of y into the second equation gives $0.4x + 0.3(-0.2x + 1.8) = 0.2$, $0.34x = -0.34$, and $x = -1$. Substituting this value of x into the first equation, we have $y = -0.2(-1) + 1.8 = 2$. Therefore, the solution is $(-1, 2)$.

18. Solving the first equation for x, we find $0.3x = 0.4y + 0.2$, $3x = 4y + 2$, and $x = \frac{4}{3}y + \frac{2}{3}$. Substituting this value of x into the second equation, which we rewrite as $-2x + 5y = 1$, we have $-2\left(\frac{4}{3}y + \frac{2}{3}\right) + 5y = 1$, $-\frac{8}{3}y - \frac{4}{3} + 5y = 1$, $\frac{7}{3}y = \frac{7}{3}$, and $y = 1$. Thus, $x = \frac{4}{3}(1) + \frac{2}{3} = 2$ and the unique solution is $(2, 1)$.

19. Solving the first equation for y, we obtain $y = 2x - 3$. Substituting this value of y into the second equation yields $4x + k(2x - 3) = 4$, so $4x + 2xk - 3k = 4$, $2x(2 + k) = 4 + 3k$, and $x = \dfrac{4 + 3k}{2(2 + k)}$. Since x is not defined when the denominator of this last expression is zero, we conclude that the system has no solution when $k = -2$.

20. Solving the second equation for x, we have $x = 4 - ky$. Substituting this value of x into the first equation gives $3(4 - ky) + 4y = 12$, so $12 - 3ky + 4y = 12$ and $y(-3k + 4) = 0$. Since this last equation is always true when $k = \frac{4}{3}$, we see that the system has infinitely many solutions when $k = \frac{4}{3}$. When $k = \frac{4}{3}$, $x = 4 - ky = 4 - \frac{4}{3}y$, so the solutions are the set of all ordered pairs $\left(4 - \frac{4}{3}t, t\right)$, where t is a parameter.

21. Solving the first equation for x in terms of y, we have $ax = by + c$ or $x = \frac{b}{a}y + \frac{c}{a}$ (provided $a \neq 0$). Substituting this value of x into the second equation gives $a\left(\frac{b}{a}y + \frac{c}{a}\right) + by = d$, $by + c + by = d$, $2by = d - c$, and $y = \frac{d-c}{2b}$ (provided $b \neq 0$). Substituting this into the expression for x gives $x = \frac{b}{a}\left(\frac{d-c}{2b}\right) + \frac{c}{a} = \frac{d-c}{2a} + \frac{c}{a} = \frac{c+d}{2a}$. Thus, the system has the unique solution $\left(\frac{c+d}{2a}, \frac{d-c}{2b}\right)$ if $a \neq 0$ and $b \neq 0$.

22. Solving the first equation for x in terms of y, we have $ax = -by + e$ or $x = -\frac{b}{a}y + \frac{e}{a}$ (provided $a \neq 0$). Substituting this value of x into the second equation gives $c\left(-\frac{b}{a}y + \frac{e}{a}\right) + dy = f$, $-\frac{bc}{a}y + \frac{ce}{a} + dy = f$, $\left(\frac{ad-bc}{a}\right)y = f - \frac{ce}{a}$, and $y = \frac{a}{ad-bc}\left(\frac{af-ce}{a}\right) = \frac{af-ce}{ad-bc}$ (provided $ad - bc \neq 0$). Substituting this into the expression for x gives $x = -\frac{b}{a}\left(\frac{af-ce}{ad-bc}\right) + \frac{e}{a} = \frac{-b(af-ce) + e(ad-bc)}{a(ad-bc)} = \frac{ed-bf}{ad-bc}$. If $a \neq 0$, the system reduces to

$$\begin{aligned} by &= e \\ cd + dy &= f \end{aligned}$$

and so $y = \frac{e}{b}$ and $x = \frac{bf-ed}{bc}$, provided $b \neq 0$ and $c \neq 0$. Thus, if $a \neq 0$, $b \neq 0$, $c \neq 0$, and $ad - bc \neq 0$, the system has the unique solution $\left(\frac{ed-bf}{ad-bc}, \frac{af-ce}{ad-bc}\right)$.

23. Let x and y denote the number of acres of corn and wheat planted, respectively. Then $x + y = 500$. Since the cost of cultivating corn is \$42/acre and that of wheat \$30/acre and Mr. Johnson has \$18,600 available for cultivation, we have $42x + 30y = 18{,}600$. Thus, the solution is found by solving the system of equations

$$\begin{aligned} x + y &= 500 \\ 42x + 30y &= 18{,}600 \end{aligned}$$

24. Let x be the amount of money Michael invests in the institution that pays interest at the rate of 3% per year and y the amount of money invested in the institution paying 4% per year. Since his total investment is \$2000, we have $x + y = 2000$. Next, since the interest earned during a one-year period was \$144, we have $0.03x + 0.04y = 144$. Thus, the solution is found by solving the system of equations

$$\begin{aligned} x + y &= 2000 \\ 0.03x + 0.04y &= 144 \end{aligned}$$

25. Let x denote the number of pounds of the \$8.00/lb coffee and y denote the number of pounds of the \$9/lb coffee. Then $x + y = 100$. Since the blended coffee sells for \$8.60/lb, we know that the blended mixture is worth $8.60(100) = \$860$. Therefore, $8x + 9y = 860$. Thus, the solution is found by solving the system of equations

$$\begin{aligned} x + y &= 1000 \\ 8x + 9y &= 860 \end{aligned}$$

26. Let the amount of money invested in the bonds yielding 4% be x dollars and the amount of money invested in the bonds yielding 5% be y dollars. Then $x + y = 30{,}000$. Also, since the yield from both investments totals \$1320, we have $0.04x + 0.05y = 1320$. Thus, the solution to the problem can be found by solving the system of equations

$$\begin{aligned} x + \quad y &= 30{,}000 \\ 0.04x + 0.05y &= \quad 1320 \end{aligned}$$

27. Let x denote the number of children who ride the bus during the morning shift and y the number of adults who ride the bus during the morning shift. Then $x + y = 1000$. Since the total fare collected is \$1300, we have $0.5x + 1.5y = 1300$. Thus, the solution to the problem can be found by solving the system of equations

$$\begin{aligned} x + \quad y &= 1000 \\ 0.5x + 1.5y &= 1300 \end{aligned}$$

28. Let x, y, and z denote the number of one-bedroom units, two-bedroom townhouses, and three-bedroom townhouses, respectively. Since the total number of units is 192, we have $x + y + z = 192$. Next, the number of family units is equal to the number of one-bedroom units, and this implies that $y + z = x$, or $x - y - z = 0$. Finally, the number of one-bedroom units is three times the number of three-bedroom units, and this implies that $x = 3z$, or $x - 3z = 0$. Summarizing, we have the system

$$\begin{array}{rrrrr} x & + \; y & + \; z & = & 192 \\ x & - \; y & - \; z & = & 0 \\ x & & - \; 3z & & 0 \end{array}$$

29. Let x and y denote the costs of the ball and the bat, respectively. Then

$$\begin{aligned} x + y &= 110 \\ y - x &= 100 \end{aligned} \quad \text{or} \quad \begin{array}{rcrcr} x & + & y & = & 110 \\ -x & + & y & = & 100 \end{array}$$

30. Let x and y denote the amounts of money invested in projects A and B, respectively. Then

$$\begin{aligned} x + y &= 70{,}000 \\ x - y &= 20{,}000 \end{aligned}$$

31. Let x be the amount of money invested at 6% in a savings account, y the amount of money invested at 8% in mutual funds, and z the amount of money invested at 12% in bonds. Since the total interest was \$21,600, we have $0.06x + 0.08y + 0.12z = 21{,}600$. Also, since the amount of Sid's investment in bonds is twice the amount of the investment in the savings account, we have $z = 2x$. Finally, the interest earned from his investment in bonds was equal to the interest earned on his money mutual funds, so $0.08y = 0.12z$. Thus, the solution to the problem can be found by solving the system of equations

$$\begin{array}{rrrrr} 0.06x & + \; 0.08y & + \; 0.12z & = & 21{,}600 \\ 2x & & - \quad z & = & 0 \\ & 0.08y & - \; 0.12z & & 0 \end{array}$$

32. Let x, y, and z denote the amount to be invested in high-risk, medium-risk, and low-risk stocks, respectively. Since all of the \$200,000 is to be invested, we have $x + y + z = 200{,}000$. The investment goal of a return of \$20,000/year leads to $0.15x + 0.10y + 0.06z = 20{,}000$. Finally, the decision that the investment in low-risk stocks be equal to the sum of the investments in the stocks of the other two categories leads to $z = x + y$. So, we are led to the problem of solving the system

$$\begin{aligned} x + y + z &= 200{,}000 \\ 0.15x + 0.1y + 0.06z &= 20{,}000 \\ x + y - z &\qquad 0 \end{aligned}$$

33. The percentages must add up to 100%, so

$$\begin{aligned} x + y + z &= 100 \\ x + y \phantom{{}+z} &= 67 \\ x \phantom{{}+y} - z &= 17 \end{aligned}$$

34. Let x, y, and z denote the numbers of respondents who answered “yes,” “no,” and “not sure,” respectively. Then we have

$$\begin{aligned} x + y + z &= 1000 \\ y + z &= 370 \\ x - y \phantom{{}+z} &= 340 \end{aligned}$$

35. Let x, y, and z denote the number of 100-lb. bags of grade A, grade B, and grade C fertilizers to be produced. The amount of nitrogen required is $18x + 20y + 24z$, and this must be equal to 26,400, so we have $18x + 20y + 24z = 26{,}400$. Similarly, the constraints on the use of phosphate and potassium lead to the equations $4x + 4y + 3z = 4900$ and $5x + 4y + 6z = 6200$, respectively. Thus we have the problem of finding the solution to the system

$$\begin{aligned} 18x + 20y + 24z &= 26{,}400 && \text{(nitrogen)} \\ 4x + 4y + 3z &= 4900 && \text{(phosphate)} \\ 5x + 4y + 6z &= 6200 && \text{(potassium).} \end{aligned}$$

36. Let x be the number of tickets sold to children, y the number of tickets sold to students, and z the number of tickets sold to adults at that particular screening. Since there was a full house at that screening, we have $x + y + z = 900$. Next, since the number of adults present was equal to one-half the number of students and children present, we have $z = \frac{1}{2}(x + y)$. Finally, the receipts totaled \$5600, and this implies that $4x + 6y + 8z = 5600$. Summarizing, we have the system

$$\begin{aligned} x + y + z &= 900 \\ x + y - 2z &= 0 \\ 4x + 6y + 8z &= 5600 \end{aligned}$$

37. Let x, y, and z denote the number of compact, intermediate, and full-size cars to be purchased, respectively. The cost incurred in buying the specified number of cars is $18{,}000x + 27{,}000y + 36{,}000z$. Since the budget is \$2.25 million, we have the system

$$\begin{aligned} 18{,}000x + 27{,}000y + 36{,}000z &= 2{,}250{,}000 \\ x - 2y &= 0 \\ x + y + z &= 100 \end{aligned}$$

38. Let x be the amount of money invested in high-risk stocks, y the amount of money invested in medium-risk stocks, and z the amount of money invested in low-risk stocks. Since a total of \$200,000 is to be invested, we have $x+y+z=200{,}000$. Next, since the investment in low-risk stocks is to be twice the sum of the investments in high- and medium-risk stocks, we have $z=2(x+y)$. Finally, the expected return of the three investments is given by $0.15x+0.10y+0.06z$ and the goal of the investment club is that an average return of 9% be realized on the total investment. If this goal is realized, then $0.15x+0.10y+0.06z=0.09(x+y+z)$. Summarizing, we have the system of equations

$$\begin{aligned} x+y+z&=200{,}000\\ 2x+2y-z&=0\\ 6x+y-3z&=0 \end{aligned}$$

39. Let x be the number of ounces of Food I used in the meal, y the number of ounces of Food II used in the meal, and z the number of ounces of Food III used in the meal. Since 100% of the daily requirement of proteins, carbohydrates, and iron is to be met by this meal, we have the system of linear equations

$$\begin{aligned} 10x+6y+8z&=100\\ 10x+12y+6z&=100\\ 5x+4y+12z&=100 \end{aligned}$$

40. Let x, y, and z denote the amounts of money invested in stocks, bonds, and the money market, respectively. Then we have

$$\begin{aligned} x+y+z&=100{,}000 && \text{(the investments total \$100,000)}\\ 0.12x+0.08y+0.04z&=10{,}000 && \text{(the annual income is \$10,000)}\\ z&=0.20x+0.10y && \text{(the investment mix)} \end{aligned}$$

Equivalently,

$$\begin{aligned} x+y+z&=100{,}000\\ 12x+8y+4z&=1{,}000{,}000\\ 20x+10y-100z&=0 \end{aligned}$$

41. Let x, y, and z denote the numbers of front orchestra, rear orchestra, and front balcony seats sold for this performance, respectively. Then we have

$$\begin{aligned} x+y+z&=1000 && \text{(tickets sold total 1000)}\\ 80x+60y+50z&=62{,}800 && \text{(total revenue)}\\ x+y-2z&=400 && \text{(relationship among different types of tickets)} \end{aligned}$$

42. Let x, y, and z denote the numbers of dozens of sleeveless, short-sleeve, and long-sleeve blouses produced per day, respectively. Then we have

$$\begin{aligned} 9x+12y+15z&=4800\\ 22x+24y+28z&=9600\\ 6x+8y+8z&=2880 \end{aligned}$$

43. Let x, y, and z denote the numbers of days spent in London, Paris, and Rome, respectively. Then we have

$$\begin{aligned} 280x+330y+260z&=4060 && \text{(hotel bills)}\\ 130x+140y+110z&=1800 && \text{(meals)}\\ x-y-z&=0 && (\text{since } x=y+z) \end{aligned}$$

44. Let x, y, z, and w denote the numbers of days spent in Boston, New York, Philadelphia, and Washington, D.C., respectively. Then we have

$$\begin{aligned} x + y + z + w &= 14 \\ 240x + 400y + 160z + 200w &= 4040 \\ y &= x + w \\ y &= 3z \end{aligned}$$

Equivalently,

$$\begin{aligned} x + y + z + w &= 14 \\ x - y + w &= 0 \\ y - 3z &= 0 \\ 240x + 400y + 160z + 200w &= 4040 \end{aligned}$$

45. True. In fact it has exactly one solution. Suppose the system is $y = m_1x + b_1$, $y = m_2x + b_2$, with $m_1 \neq m_2$. Then subtracting, we obtain $0 = (m_2 - m_1)x + (b_2 - b_1)$. Therefore, $x = \dfrac{b_1 - b_2}{m_2 - m_1}$ and $y = m_1\left(\dfrac{b_1 - b_2}{m_2 - m_1}\right) + b_1 = \dfrac{m_1b_1 - m_1b_2 + m_2b_1 - m_1b_1}{m_2 - m_1} = \dfrac{m_2b_1 - m_1b_2}{m_2 - m_1}$.

46. True. If the three lines coincide, then the system has infinitely many solutions corresponding to all points on the (common) line. If at least one line is distinct from the others, then the system has no solution.

47. False. If all three lines are parallel and coincide, then the system has infinitely many solutions corresponding to all points on the (common) line.

48. True. The two (or more) parallel lines that are distinct have no point in common. This means that there is no point common to all the lines, and so the system has no solution.

2.2 Systems of Linear Equations: Unique Solutions

Concept Questions

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1. a. The two systems are equivalent to each other if they have precisely the same solutions.

b. **i.** Interchange row i with row j.

ii. Replace row i with c times row i.

iii. Replace row i with the sum of row i and a times row j.

2. a. The coefficient matrix is the $m \times n$ matrix made up of the coefficients of the system of m linear equations in the n variables. The augmented matrix for the system is obtained from the matrix of coefficients by adjoining the column of constants to it. A column in the coefficient matrix is a unit column if one of the entries in the column is 1 and all other entries are 0.

b. To pivot about an element means to transform the column containing that element into a unit column with a 1 in the position previously occupied by that element.

3. a. It lies below any other row having nonzero entries.

b. It is a 1.

c. The leading 1 in the lower row lies to the right of the leading 1 in the upper row.

d. They are all 0.

Exercises

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1. $\left[\begin{array}{cc|c} 2 & -3 & 7 \\ 3 & 1 & 4 \end{array}\right]$

2. $\left[\begin{array}{ccc|c} 3 & 7 & -8 & 5 \\ 1 & 0 & 3 & -2 \\ 4 & -3 & 0 & 7 \end{array}\right]$

3. $\left[\begin{array}{ccc|c} 0 & -1 & 2 & 5 \\ 2 & 2 & -8 & 4 \\ 0 & 3 & 4 & 0 \end{array}\right]$

4. $\left[\begin{array}{ccc|c} 3 & 2 & 0 & 0 \\ 1 & -1 & 2 & 4 \\ 0 & 2 & -3 & 5 \end{array}\right]$

5. $\begin{aligned} 3x + 2y &= -4 \\ x - y &= 5 \end{aligned}$

6. $\begin{aligned} 3x \phantom{{}- y} + 2z &= 4 \\ x - y - 2z &= -3 \\ 4x \phantom{{}- y} + 3z &= 2 \end{aligned}$

7. $\begin{aligned} x + 3y + 2z &= 4 \\ 2x \phantom{{}+ 3y + 2z} &= 5 \\ 3x - 3y + 2z &= 6 \end{aligned}$

8. $\begin{aligned} 2x + 3y + z &= 6 \\ 4x + 3y + 2z &= 5 \end{aligned}$

9. Yes. Conditions 1–4 are satisfied (see page 86 of the text).

10. Yes. Conditions 1–4 are satisfied.

11. No. Condition 3 is violated. The first nonzero entry in the second row does not lie to the right of the first nonzero entry (1) in the first row.

12. Yes. Conditions 1–4 are satisfied.

13. Yes. Conditions 1–4 are satisfied.

14. No. Condition 2 is violated. The first nonzero entry in the third row is not a 1.

15. No. Condition 2 and consequently condition 4 are not satisfied. The first nonzero entry in the last row is not a 1 and the column containing that entry does not have zeros elsewhere.

16. Yes. Conditions 1–4 are satisfied.

17. No. Condition 1 is violated. The first row consists entirely of zeros and it lies above row 2.

18. No. Conditions 2 and 3 are violated. Row 3 should lie above row 3, and the entry in row 3, column 4 should be a 1, not a 4.

19. $\left[\begin{array}{cc|c} 1 & 3 & 4 \\ 2 & 4 & 6 \end{array}\right] \xrightarrow{R_2 - 2R_1} \left[\begin{array}{cc|c} 1 & 3 & 4 \\ 0 & -2 & -2 \end{array}\right]$

20. $\left[\begin{array}{cc|c} 2 & 4 & 8 \\ 3 & 1 & 2 \end{array}\right] \xrightarrow{\frac{1}{2}R_1} \left[\begin{array}{cc|c} 1 & 2 & 4 \\ 3 & 1 & 2 \end{array}\right] \xrightarrow{R_2 - 3R_1} \left[\begin{array}{cc|c} 1 & 2 & 4 \\ 0 & -5 & -10 \end{array}\right]$

21. $\left[\begin{array}{cc|c} -1 & 2 & 3 \\ 6 & 8 & 2 \end{array}\right] \xrightarrow{-R_1} \left[\begin{array}{cc|c} 1 & -2 & -3 \\ 6 & 8 & 2 \end{array}\right] \xrightarrow{R_2 - 6R_1} \left[\begin{array}{cc|c} 1 & -2 & -3 \\ 0 & 20 & 20 \end{array}\right]$

22. $$\left[\begin{array}{cc|c} 3 & 2 & 6 \\ 4 & 2 & 5 \end{array}\right] \xrightarrow{\frac{1}{4}R_2} \left[\begin{array}{cc|c} 3 & 2 & 6 \\ 1 & \frac{1}{2} & \frac{5}{4} \end{array}\right] \xrightarrow{R_1-3R_2} \left[\begin{array}{cc|c} 0 & \frac{1}{2} & \frac{9}{4} \\ 1 & \frac{1}{2} & \frac{5}{4} \end{array}\right]$$

23. $$\left[\begin{array}{ccc|c} 2 & 4 & 6 & 12 \\ 2 & 3 & 1 & 5 \\ 3 & -1 & 2 & 4 \end{array}\right] \xrightarrow{\frac{1}{2}R_1} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 2 & 3 & 1 & 5 \\ 3 & -1 & 2 & 4 \end{array}\right] \xrightarrow[R_3-3R_1]{R_2-2R_1} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 6 \\ 0 & -1 & -5 & -7 \\ 0 & -7 & -7 & -14 \end{array}\right]$$

24. $$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 4 \\ 2 & 4 & 8 & 6 \\ -1 & 2 & 3 & 4 \end{array}\right] \xrightarrow{\frac{1}{2}R_2} \left[\begin{array}{ccc|c} 1 & 3 & 2 & 4 \\ 1 & 2 & 4 & 3 \\ -1 & 2 & 3 & 4 \end{array}\right] \xrightarrow[R_3+R_2]{R_1-R_2} \left[\begin{array}{ccc|c} 0 & 1 & -2 & 1 \\ 1 & 2 & 4 & 3 \\ 0 & 4 & 7 & 7 \end{array}\right]$$

25. $$\left[\begin{array}{ccc|c} 0 & 1 & 3 & 4 \\ 2 & 4 & 1 & 3 \\ 5 & 6 & 2 & -4 \end{array}\right] \xrightarrow[R_3-2R_2]{R_1-3R_2} \left[\begin{array}{ccc|c} -6 & -11 & 0 & -5 \\ 2 & 4 & 1 & 3 \\ 1 & -2 & 0 & -10 \end{array}\right]$$

26. $$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 0 & -3 & 3 & 2 \\ 0 & 4 & -1 & 3 \end{array}\right] \xrightarrow{-\frac{1}{3}R_2} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 5 \\ 0 & 1 & -1 & -\frac{2}{3} \\ 0 & 4 & -1 & 3 \end{array}\right] \xrightarrow[R_3-4R_2]{R_1-2R_2} \left[\begin{array}{ccc|c} 1 & 0 & 5 & \frac{19}{3} \\ 0 & 1 & -1 & -\frac{2}{3} \\ 0 & 0 & 3 & \frac{17}{3} \end{array}\right]$$

27. $$\left[\begin{array}{cc|c} 3 & 9 & 6 \\ 2 & 1 & 4 \end{array}\right] \xrightarrow{\frac{1}{3}R_1} \left[\begin{array}{cc|c} 1 & 3 & 2 \\ 2 & 1 & 4 \end{array}\right] \xrightarrow{R_2-2R_1} \left[\begin{array}{cc|c} 1 & 3 & 2 \\ 0 & -5 & 0 \end{array}\right] \xrightarrow{-\frac{1}{5}R_2} \left[\begin{array}{cc|c} 1 & 3 & 2 \\ 0 & 1 & 0 \end{array}\right] \xrightarrow{R_1-3R_2} \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 0 \end{array}\right]$$

28. $$\left[\begin{array}{cc|c} 1 & 2 & 1 \\ 2 & 3 & -1 \end{array}\right] \xrightarrow{R_2-2R_1} \left[\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & -1 & -3 \end{array}\right] \xrightarrow{-R_2} \left[\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & 1 & 3 \end{array}\right] \xrightarrow{R_1-2R_2} \left[\begin{array}{cc|c} 1 & 0 & -5 \\ 0 & 1 & 3 \end{array}\right]$$

29. $$\left[\begin{array}{ccc|c} 1 & 3 & 1 & 3 \\ 3 & 8 & 3 & 7 \\ 2 & -3 & 1 & -10 \end{array}\right] \xrightarrow[R_3-2R_1]{R_2-3R_1} \left[\begin{array}{ccc|c} 1 & 3 & 1 & 3 \\ 0 & -1 & 0 & -2 \\ 0 & -9 & -1 & -16 \end{array}\right] \xrightarrow{-R_2} \left[\begin{array}{ccc|c} 1 & 3 & 1 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & -9 & -1 & -16 \end{array}\right] \xrightarrow[R_3+9R_2]{R_1-3R_2}$$
$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & -3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & -1 & 2 \end{array}\right] \xrightarrow[-R_3]{R_1+R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

30. $$\left[\begin{array}{ccc|c} 0 & 1 & 3 & -4 \\ 1 & 2 & 1 & 7 \\ 1 & -2 & 0 & 1 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 7 \\ 0 & 1 & 3 & -4 \\ 1 & -2 & 0 & 1 \end{array}\right] \xrightarrow{R_3-R_1} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 7 \\ 0 & 1 & 3 & -4 \\ 0 & -4 & -1 & -6 \end{array}\right] \xrightarrow[R_3+4R_2]{R_1-2R_2} \left[\begin{array}{ccc|c} 1 & 0 & -5 & 15 \\ 0 & 1 & 3 & -4 \\ 0 & 0 & 11 & -22 \end{array}\right] \xrightarrow{\frac{1}{11}R_3}$$
$$\left[\begin{array}{ccc|c} 1 & 0 & -5 & 15 \\ 0 & 1 & 3 & -4 \\ 0 & 0 & 1 & -2 \end{array}\right] \xrightarrow[R_2-3R_3]{R_1+5R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

31. The augmented matrix is equivalent to the system of linear equations $3x + 9y = 6$, $2x + y = 4$. The ordered pair $(2, 0)$ is the solution to the system.

32. The augmented matrix is equivalent to the system of linear equations $x + 2y = 1$, $2x + 3y = -1$. The solution to the system is $x = -5$, $y = 3$.

33. The augmented matrix is equivalent to the system of linear equations $x + 3y + z = 3$, $3x + 8y + 3z = 7$, $2x - 3y + z = -10$. Reading off the solution from the last augmented matrix, $\left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -2 \end{array}\right]$, which is in row-reduced form, we have $x = -1$, $y = 2$, and $z = -2$.

34. The augmented matrix is equivalent to the system of linear equations $y + 3z = -4$, $x + 2y + z = 7$, $x - 2y = 1$. The solution to the system is $x = 5$, $y = 2$, $z = -2$.

35. Using the Gauss-Jordan elimination method, we have

$$\left[\begin{array}{cc|c} 1 & 1 & 3 \\ 2 & -1 & 3 \end{array}\right] \xrightarrow{R_2 - 2R_1} \left[\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & -3 & -3 \end{array}\right] \xrightarrow{\frac{1}{3}R_2} \left[\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & 1 & 1 \end{array}\right] \xrightarrow{R_1 - R_2} \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 1 \end{array}\right].$$ The solution is $(2, 1)$.

36. Using the Gauss-Jordan elimination method, we have

$$\left[\begin{array}{cc|c} 1 & -2 & -3 \\ 2 & 3 & 8 \end{array}\right] \xrightarrow{R_2 - 2R_1} \left[\begin{array}{cc|c} 1 & -2 & -3 \\ 0 & 7 & 14 \end{array}\right] \xrightarrow{\frac{1}{7}R_2} \left[\begin{array}{cc|c} 1 & -2 & -3 \\ 0 & 1 & 2 \end{array}\right] \xrightarrow{R_1 + 2R_2} \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 2 \end{array}\right].$$ The solution is $(1, 2)$.

37. Using the Gauss-Jordan elimination method, we have

$$\left[\begin{array}{cc|c} 1 & -2 & 8 \\ 3 & 4 & 4 \end{array}\right] \xrightarrow{R_2 - 3R_1} \left[\begin{array}{cc|c} 1 & -2 & 8 \\ 0 & 10 & -20 \end{array}\right] \xrightarrow{\frac{1}{10}R_2} \left[\begin{array}{cc|c} 1 & -2 & 8 \\ 0 & 1 & -2 \end{array}\right] \xrightarrow{R_1 + 2R_2} \left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & -2 \end{array}\right].$$

The solution is $(4, -2)$.

38. Using the Gauss-Jordan elimination method, we have

$$\left[\begin{array}{cc|c} 3 & 1 & 1 \\ -7 & -2 & -1 \end{array}\right] \xrightarrow{\frac{1}{3}R_1} \left[\begin{array}{cc|c} 1 & \frac{1}{3} & \frac{1}{3} \\ -7 & -2 & -1 \end{array}\right] \xrightarrow{R_2 + 7R_1} \left[\begin{array}{cc|c} 1 & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{3} & \frac{4}{3} \end{array}\right] \xrightarrow{3R_2} \left[\begin{array}{cc|c} 1 & \frac{1}{3} & \frac{1}{3} \\ 0 & 1 & 4 \end{array}\right] \xrightarrow{R_1 - \frac{1}{3}R_2} \left[\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 1 & 4 \end{array}\right].$$

The solution is $(-1, 4)$.

39. Using the Gauss-Jordan elimination method, we have

$$\left[\begin{array}{cc|c} 2 & -3 & -8 \\ 4 & 1 & -2 \end{array}\right] \xrightarrow{\frac{1}{2}R_1} \left[\begin{array}{cc|c} 1 & -\frac{3}{2} & -4 \\ 4 & 1 & -2 \end{array}\right] \xrightarrow{R_2 - 4R_1} \left[\begin{array}{cc|c} 1 & -\frac{3}{2} & -4 \\ 0 & 7 & 14 \end{array}\right] \xrightarrow{\frac{1}{7}R_2} \left[\begin{array}{cc|c} 1 & -\frac{3}{2} & -4 \\ 0 & 1 & 2 \end{array}\right] \xrightarrow{R_1 + \frac{3}{2}R_2} \left[\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 1 & 2 \end{array}\right].$$

The solution is $(-1, 2)$.

40. Using the Gauss-Jordan elimination method, we have

$$\left[\begin{array}{cc|c} 5 & 3 & 9 \\ -2 & 1 & -8 \end{array}\right] \xrightarrow{\frac{1}{5}R_1} \left[\begin{array}{cc|c} 1 & \frac{3}{5} & \frac{9}{5} \\ -2 & 1 & -8 \end{array}\right] \xrightarrow{R_2 + 2R_1} \left[\begin{array}{cc|c} 1 & \frac{3}{5} & \frac{9}{5} \\ 0 & \frac{11}{5} & -\frac{22}{5} \end{array}\right] \xrightarrow{\frac{5}{11}R_2} \left[\begin{array}{cc|c} 1 & \frac{3}{5} & \frac{9}{5} \\ 0 & 1 & -2 \end{array}\right] \xrightarrow{R_1 - \frac{3}{5}R_2} \left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & -2 \end{array}\right].$$

The solution is $(3, -2)$.

41. Using the Gauss-Jordan elimination method, we have

$$\left[\begin{array}{cc|c} 6 & 8 & 15 \\ 2 & -4 & -5 \end{array}\right] \xrightarrow{\frac{1}{6}R_1} \left[\begin{array}{cc|c} 1 & \frac{4}{3} & \frac{5}{2} \\ 2 & -4 & -5 \end{array}\right] \xrightarrow{R_2-2R_1} \left[\begin{array}{cc|c} 1 & \frac{4}{3} & \frac{5}{2} \\ 0 & -\frac{20}{3} & -10 \end{array}\right] \xrightarrow{-\frac{3}{20}R_2} \left[\begin{array}{cc|c} 1 & \frac{4}{3} & \frac{5}{2} \\ 0 & 1 & \frac{3}{2} \end{array}\right] \xrightarrow{R_1-\frac{4}{3}R_2} \left[\begin{array}{cc|c} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} \end{array}\right].$$

The solution is $\left(\frac{1}{2}, \frac{3}{2}\right)$.

42. Using the Gauss-Jordan elimination method, we have

$$\left[\begin{array}{cc|c} 2 & 10 & 1 \\ -4 & 6 & 11 \end{array}\right] \xrightarrow{\frac{1}{2}R_1} \left[\begin{array}{cc|c} 1 & 5 & \frac{1}{2} \\ -4 & 6 & 11 \end{array}\right] \xrightarrow{R_2+4R_1} \left[\begin{array}{cc|c} 1 & 5 & \frac{1}{2} \\ 0 & 26 & 13 \end{array}\right] \xrightarrow{\frac{1}{26}R_2} \left[\begin{array}{cc|c} 1 & 5 & \frac{1}{2} \\ 0 & 1 & \frac{1}{2} \end{array}\right] \xrightarrow{R_1-5R_2} \left[\begin{array}{cc|c} 1 & 0 & -2 \\ 0 & 1 & \frac{1}{2} \end{array}\right].$$ The solution is $\left(-2, \frac{1}{2}\right)$.

43. Using the Gauss-Jordan elimination method, we have

$$\left[\begin{array}{cc|c} 3 & -2 & 1 \\ 2 & 4 & 2 \end{array}\right] \xrightarrow{\frac{1}{3}R_1} \left[\begin{array}{cc|c} 1 & -\frac{2}{3} & \frac{1}{3} \\ 2 & 4 & 2 \end{array}\right] \xrightarrow{R_2-2R_1} \left[\begin{array}{cc|c} 1 & -\frac{2}{3} & \frac{1}{3} \\ 0 & \frac{16}{3} & \frac{4}{3} \end{array}\right] \xrightarrow{\frac{3}{16}R_2} \left[\begin{array}{cc|c} 1 & -\frac{2}{3} & \frac{1}{3} \\ 0 & 1 & \frac{1}{4} \end{array}\right] \xrightarrow{R_1+\frac{2}{3}R_2} \left[\begin{array}{cc|c} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{1}{4} \end{array}\right].$$ The solution is $\left(\frac{1}{2}, \frac{1}{4}\right)$.

44. Using the Gauss-Jordan elimination method, we have

$$\left[\begin{array}{cc|c} 1 & -\frac{1}{2} & \frac{7}{6} \\ -\frac{1}{2} & 4 & \frac{2}{3} \end{array}\right] \xrightarrow{R_2+\frac{1}{2}R_1} \left[\begin{array}{cc|c} 1 & -\frac{1}{2} & \frac{7}{6} \\ 0 & \frac{15}{4} & \frac{5}{4} \end{array}\right] \xrightarrow{\frac{4}{15}R_2} \left[\begin{array}{cc|c} 1 & -\frac{1}{2} & \frac{7}{6} \\ 0 & 1 & \frac{1}{3} \end{array}\right] \xrightarrow{R_1+\frac{1}{2}R_2} \left[\begin{array}{cc|c} 1 & 0 & \frac{4}{3} \\ 0 & 1 & \frac{1}{3} \end{array}\right].$$ The solution is $\left(\frac{4}{3}, \frac{1}{3}\right)$.

45. $$\left[\begin{array}{ccc|c} 2 & 1 & -2 & 4 \\ 1 & 3 & -1 & -3 \\ 3 & 4 & -1 & 7 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|c} 1 & 3 & -1 & -3 \\ 2 & 1 & -2 & 4 \\ 3 & 4 & -1 & 7 \end{array}\right] \xrightarrow[R_3-3R_1]{R_2-2R_1} \left[\begin{array}{ccc|c} 1 & 3 & -1 & -3 \\ 0 & -5 & 0 & 10 \\ 0 & -5 & 2 & 16 \end{array}\right] \xrightarrow{-\frac{1}{5}R_2}$$

$$\left[\begin{array}{ccc|c} 1 & 3 & -1 & -3 \\ 0 & 1 & 0 & -2 \\ 0 & -5 & 2 & 16 \end{array}\right] \xrightarrow[R_3+5R_2]{R_1-3R_2} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 2 & 6 \end{array}\right] \xrightarrow{\frac{1}{2}R_3} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right] \xrightarrow{R_1+R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right].$$

The solution is $(6, -2, 3)$.

46. Using the Gauss-Jordan elimination method, we have $$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 2 & -1 & 1 & 1 \\ 1 & 1 & -2 & 2 \end{array}\right] \xrightarrow[R_3-R_1]{R_2-2R_1} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -3 & -1 & 1 \\ 0 & 0 & -3 & 2 \end{array}\right] \xrightarrow{-\frac{1}{3}R_2}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & \frac{1}{3} & -\frac{1}{3} \\ 0 & 0 & -3 & 2 \end{array}\right] \xrightarrow{R_1-R_2} \left[\begin{array}{ccc|c} 1 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 1 & \frac{1}{3} & -\frac{1}{3} \\ 0 & 0 & -3 & 2 \end{array}\right] \xrightarrow{-\frac{1}{3}R_3} \left[\begin{array}{ccc|c} 1 & 0 & \frac{2}{3} & \frac{1}{3} \\ 0 & 1 & \frac{1}{3} & -\frac{1}{3} \\ 0 & 0 & 1 & -\frac{2}{3} \end{array}\right] \xrightarrow[R_2-\frac{1}{3}R_3]{R_1-\frac{2}{3}R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{7}{9} \\ 0 & 1 & 0 & -\frac{1}{9} \\ 0 & 0 & 1 & -\frac{2}{3} \end{array}\right].$$

The solution is $\left(\frac{7}{9}, -\frac{1}{9}, -\frac{2}{3}\right)$.

47. $\left[\begin{array}{ccc|c} 2 & 2 & 1 & 9 \\ 1 & 0 & 1 & 4 \\ 0 & 4 & -3 & 17 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 4 \\ 2 & 2 & 1 & 9 \\ 0 & 4 & -3 & 17 \end{array}\right] \xrightarrow{R_2 - 2R_1} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 4 \\ 0 & 2 & -1 & 1 \\ 0 & 4 & -3 & 17 \end{array}\right] \xrightarrow{\frac{1}{2}R_2} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 4 \\ 0 & 1 & -\frac{1}{2} & \frac{1}{2} \\ 0 & 4 & -3 & 17 \end{array}\right] \xrightarrow{R_3 - 4R_2}$

$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 4 \\ 0 & 1 & -\frac{1}{2} & \frac{1}{2} \\ 0 & 0 & -1 & 15 \end{array}\right] \xrightarrow{-R_3} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 4 \\ 0 & 1 & -\frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 & -15 \end{array}\right] \xrightarrow[R_2 + \frac{1}{2}R_3]{R_1 - R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 19 \\ 0 & 1 & 0 & -7 \\ 0 & 0 & 1 & -15 \end{array}\right]$. The solution is $(19, -7, -15)$.

48. $\left[\begin{array}{ccc|c} 2 & 3 & -2 & 10 \\ 3 & -2 & 2 & 0 \\ 4 & -1 & 3 & -1 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|c} 3 & -2 & 2 & 0 \\ 2 & 3 & -2 & 10 \\ 4 & -1 & 3 & -1 \end{array}\right] \xrightarrow{R_1 - R_2} \left[\begin{array}{ccc|c} 1 & -5 & 4 & -10 \\ 2 & 3 & -2 & 10 \\ 4 & -1 & 3 & -1 \end{array}\right] \xrightarrow[R_3 - 4R_1]{R_2 - 2R_1}$

$\left[\begin{array}{ccc|c} 1 & -5 & 4 & -10 \\ 0 & 13 & -10 & 30 \\ 0 & 19 & -13 & 39 \end{array}\right] \xrightarrow{\frac{1}{13}R_2} \left[\begin{array}{ccc|c} 1 & -5 & 4 & -10 \\ 0 & 1 & -\frac{10}{13} & \frac{30}{13} \\ 0 & 19 & -13 & 39 \end{array}\right] \xrightarrow[R_3 - 19R_2]{R_1 + 5R_2} \left[\begin{array}{ccc|c} 1 & 0 & \frac{2}{13} & \frac{20}{13} \\ 0 & 1 & -\frac{10}{13} & \frac{30}{13} \\ 0 & 0 & \frac{21}{13} & -\frac{63}{13} \end{array}\right] \xrightarrow{\frac{13}{21}R_3}$

$\left[\begin{array}{ccc|c} 1 & 0 & \frac{2}{13} & \frac{20}{13} \\ 0 & 1 & -\frac{10}{13} & \frac{30}{13} \\ 0 & 0 & 1 & -3 \end{array}\right] \xrightarrow[R_2 + \frac{10}{13}R_3]{R_1 - \frac{2}{13}R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -3 \end{array}\right]$. The solution is $(2, 0, -3)$.

49. $\left[\begin{array}{ccc|c} 0 & -1 & 1 & 2 \\ 4 & -3 & 2 & 16 \\ 3 & 2 & 1 & 11 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|c} 4 & -3 & 2 & 16 \\ 0 & -1 & 1 & 2 \\ 3 & 2 & 1 & 11 \end{array}\right] \xrightarrow{R_1 - R_3} \left[\begin{array}{ccc|c} 1 & -5 & 1 & 5 \\ 0 & -1 & 1 & 2 \\ 3 & 2 & 1 & 11 \end{array}\right] \xrightarrow[R_3 - 3R_1]{-R_2} \left[\begin{array}{ccc|c} 1 & -5 & 1 & 5 \\ 0 & 1 & -1 & -2 \\ 0 & 17 & -2 & -4 \end{array}\right] \xrightarrow[R_3 - 17R_2]{R_1 + 5R_2}$

$\left[\begin{array}{ccc|c} 1 & 0 & -4 & -5 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 15 & 30 \end{array}\right] \xrightarrow{\frac{1}{15}R_3} \left[\begin{array}{ccc|c} 1 & 0 & -4 & -5 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 1 & 2 \end{array}\right] \xrightarrow[R_2 + R_3]{R_1 + 4R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{array}\right]$. The solution is $(3, 0, 2)$.

50. $\left[\begin{array}{ccc|c} 2 & 4 & -6 & 38 \\ 1 & 2 & 3 & 7 \\ 3 & -4 & 4 & -19 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 7 \\ 2 & 4 & -6 & 38 \\ 3 & -4 & 4 & -19 \end{array}\right] \xrightarrow[R_3 - 3R_1]{R_2 - 2R_1} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 7 \\ 0 & 0 & -12 & 24 \\ 0 & -10 & -5 & -40 \end{array}\right] \xrightarrow{R_2 \leftrightarrow R_3}$

$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 7 \\ 0 & -10 & -5 & -40 \\ 0 & 0 & -12 & 24 \end{array}\right] \xrightarrow{-\frac{1}{10}R_2} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 7 \\ 0 & 1 & \frac{1}{2} & 4 \\ 0 & 0 & -12 & 24 \end{array}\right] \xrightarrow{R_1 - 2R_2} \left[\begin{array}{ccc|c} 1 & 0 & 2 & -1 \\ 0 & 1 & \frac{1}{2} & 4 \\ 0 & 0 & -12 & 24 \end{array}\right] \xrightarrow{-\frac{1}{12}R_3}$

$\left[\begin{array}{ccc|c} 1 & 0 & 2 & -1 \\ 0 & 1 & \frac{1}{2} & 4 \\ 0 & 0 & 1 & -2 \end{array}\right] \xrightarrow[R_2 - \frac{1}{2}R_3]{R_1 - 2R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & -2 \end{array}\right]$. The solution is $(3, 5, -2)$.

51. Using the Gauss-Jordan elimination method, we have $\left[\begin{array}{rrr|r} 1 & -2 & 1 & 6 \\ 2 & 1 & -3 & -3 \\ 1 & -3 & 3 & 10 \end{array}\right] \xrightarrow[R_3 - R_1]{R_2 - 2R_1} \left[\begin{array}{rrr|r} 1 & -2 & 1 & 6 \\ 0 & 5 & -5 & -15 \\ 0 & -1 & 2 & 4 \end{array}\right] \xrightarrow{\frac{1}{5}R_2}$

$\left[\begin{array}{rrr|r} 1 & -2 & 1 & 6 \\ 0 & 1 & -1 & -3 \\ 0 & -1 & 2 & 4 \end{array}\right] \xrightarrow[R_3 + R_2]{R_1 + 2R_2} \left[\begin{array}{rrr|r} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 1 & 1 \end{array}\right] \xrightarrow[R_2 + R_3]{R_1 + R_3} \left[\begin{array}{rrr|r} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{array}\right]$. The solution is $(1, -2, 1)$.

52. Using the Gauss-Jordan elimination method, we have

$\left[\begin{array}{rrr|r} 2 & 3 & -6 & -11 \\ 1 & -2 & 3 & 9 \\ 3 & 1 & 0 & 7 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{rrr|r} 1 & -2 & 3 & 9 \\ 2 & 3 & -6 & -11 \\ 3 & 1 & 0 & 7 \end{array}\right] \xrightarrow[R_3 - 3R_1]{R_2 - 2R_1} \left[\begin{array}{rrr|r} 1 & -2 & 3 & 9 \\ 0 & 7 & -12 & -29 \\ 0 & 7 & -9 & -20 \end{array}\right] \xrightarrow{\frac{1}{7}R_2}$

$\left[\begin{array}{rrr|r} 1 & -2 & 3 & 9 \\ 0 & 1 & -\frac{12}{7} & -\frac{29}{7} \\ 0 & 7 & -9 & -20 \end{array}\right] \xrightarrow[R_3 - 7R_2]{R_1 + 2R_2} \left[\begin{array}{rrr|r} 1 & 0 & -\frac{3}{7} & \frac{5}{7} \\ 0 & 1 & -\frac{12}{7} & -\frac{29}{7} \\ 0 & 0 & 3 & 9 \end{array}\right] \xrightarrow{\frac{1}{3}R_3} \left[\begin{array}{rrr|r} 1 & 0 & -\frac{3}{7} & \frac{5}{7} \\ 0 & 1 & -\frac{12}{7} & -\frac{29}{7} \\ 0 & 0 & 1 & 3 \end{array}\right] \xrightarrow[R_2 + \frac{12}{7}R_3]{R_1 + \frac{3}{7}R_3} \left[\begin{array}{rrr|r} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 3 \end{array}\right]$.

The solution is $(2, 1, 3)$.

53. Using the Gauss-Jordan elimination method, we have

$\left[\begin{array}{rrr|r} 2 & 0 & 3 & -1 \\ 3 & -2 & 1 & 9 \\ 1 & 1 & 4 & 4 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_3} \left[\begin{array}{rrr|r} 1 & 1 & 4 & 4 \\ 3 & -2 & 1 & 9 \\ 2 & 0 & 3 & -1 \end{array}\right] \xrightarrow[R_3 - 2R_1]{R_2 - 3R_1} \left[\begin{array}{rrr|r} 1 & 1 & 4 & 4 \\ 0 & -5 & -11 & -3 \\ 0 & -2 & -5 & -9 \end{array}\right] \xrightarrow{-\frac{1}{5}R_2}$

$\left[\begin{array}{rrr|r} 1 & 1 & 4 & 4 \\ 0 & 1 & \frac{11}{5} & \frac{3}{5} \\ 0 & -2 & -5 & -9 \end{array}\right] \xrightarrow[R_3 + 2R_2]{R_1 - R_2} \left[\begin{array}{rrr|r} 1 & 0 & \frac{9}{5} & \frac{17}{5} \\ 0 & 1 & \frac{11}{5} & \frac{3}{5} \\ 0 & 0 & -\frac{3}{5} & -\frac{39}{5} \end{array}\right] \xrightarrow{-\frac{5}{3}R_3} \left[\begin{array}{rrr|r} 1 & 0 & \frac{9}{5} & \frac{17}{5} \\ 0 & 1 & \frac{11}{5} & \frac{3}{5} \\ 0 & 0 & 1 & 13 \end{array}\right] \xrightarrow[R_2 - \frac{11}{5}R_3]{R_1 - \frac{9}{5}R_3} \left[\begin{array}{rrr|r} 1 & 0 & 0 & -20 \\ 0 & 1 & 0 & -28 \\ 0 & 0 & 1 & 13 \end{array}\right]$.

The solution is $(-20, -28, 13)$.

54. Using the Gauss-Jordan elimination method, we have

$\left[\begin{array}{rrr|r} 2 & -1 & 3 & -4 \\ 1 & -2 & 1 & -1 \\ 1 & -5 & 2 & -3 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{rrr|r} 1 & -2 & 1 & -1 \\ 2 & -1 & 3 & -4 \\ 1 & -5 & 2 & -3 \end{array}\right] \xrightarrow[R_3 - R_1]{R_2 - 2R_1} \left[\begin{array}{rrr|r} 1 & -2 & 1 & -1 \\ 0 & 3 & 1 & -2 \\ 0 & -3 & 1 & -2 \end{array}\right] \xrightarrow{\frac{1}{3}R_2} \left[\begin{array}{rrr|r} 1 & -2 & 1 & -1 \\ 0 & 1 & \frac{1}{3} & -\frac{2}{3} \\ 0 & -3 & 1 & -2 \end{array}\right] \xrightarrow[R_3 + 3R_2]{R_1 + 2R_2}$

$\left[\begin{array}{rrr|r} 1 & 0 & \frac{5}{3} & -\frac{7}{3} \\ 0 & 1 & \frac{1}{3} & -\frac{2}{3} \\ 0 & 0 & 2 & -4 \end{array}\right] \xrightarrow{\frac{1}{2}R_3} \left[\begin{array}{rrr|r} 1 & 0 & \frac{5}{3} & -\frac{7}{3} \\ 0 & 1 & \frac{1}{3} & -\frac{2}{3} \\ 0 & 0 & 1 & -2 \end{array}\right] \xrightarrow[R_2 - \frac{1}{3}R_3]{R_1 - \frac{5}{3}R_3} \left[\begin{array}{rrr|r} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 \end{array}\right]$. The solution is $(1, 0, -2)$.

55. Using the Gauss-Jordan elimination method, we have

$\left[\begin{array}{rrr|r} 1 & -1 & 3 & 14 \\ 1 & 1 & 1 & 6 \\ -2 & -1 & 1 & -4 \end{array}\right] \xrightarrow[R_3 + 2R_1]{R_2 - R_1} \left[\begin{array}{rrr|r} 1 & -1 & 3 & 14 \\ 0 & 2 & -2 & -8 \\ 0 & -3 & 7 & 24 \end{array}\right] \xrightarrow{\frac{1}{2}R_2} \left[\begin{array}{rrr|r} 1 & -1 & 3 & 14 \\ 0 & 1 & -1 & -4 \\ 0 & -3 & 7 & 24 \end{array}\right] \xrightarrow[R_3 + 3R_2]{R_1 + R_2} \left[\begin{array}{rrr|r} 1 & 0 & 2 & 10 \\ 0 & 1 & -1 & -4 \\ 0 & 0 & 4 & 12 \end{array}\right] \xrightarrow{\frac{1}{4}R_3}$

$\left[\begin{array}{rrr|r} 1 & 0 & 2 & 10 \\ 0 & 1 & -1 & -4 \\ 0 & 0 & 1 & 3 \end{array}\right] \xrightarrow[R_2 + R_3]{R_1 - 2R_3} \left[\begin{array}{rrr|r} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right]$. The solution is $(4, -1, 3)$.

56. Using the Gauss-Jordan elimination method, we have

$$\left[\begin{array}{rrr|r} 2 & -1 & -1 & 0 \\ 3 & 2 & 1 & 7 \\ 1 & 2 & 2 & 5 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_3} \left[\begin{array}{rrr|r} 1 & 2 & 2 & 5 \\ 3 & 2 & 1 & 7 \\ 2 & -1 & -1 & 0 \end{array}\right] \xrightarrow[R_3 - 2R_1]{R_2 - 3R_1} \left[\begin{array}{rrr|r} 1 & 2 & 2 & 5 \\ 0 & -4 & -5 & -8 \\ 0 & -5 & -5 & -10 \end{array}\right] \xrightarrow{-\frac{1}{4}R_2}$$

$$\left[\begin{array}{rrr|r} 1 & 2 & 2 & 5 \\ 0 & 1 & \frac{5}{4} & 2 \\ 0 & -5 & -5 & -10 \end{array}\right] \xrightarrow[R_3 + 5R_2]{R_1 - 2R_2} \left[\begin{array}{rrr|r} 1 & 0 & -\frac{1}{2} & 1 \\ 0 & 1 & \frac{5}{4} & 2 \\ 0 & 0 & \frac{5}{4} & 0 \end{array}\right] \xrightarrow{\frac{4}{5}R_3} \left[\begin{array}{rrr|r} 1 & 0 & -\frac{1}{2} & 1 \\ 0 & 1 & \frac{5}{4} & 2 \\ 0 & 0 & 1 & 0 \end{array}\right] \xrightarrow[R_2 - \frac{5}{4}R_3]{R_1 + \frac{1}{2}R_3} \left[\begin{array}{rrr|r} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{array}\right].$$

The solution is $(1, 2, 0)$.

57. Using the Gauss-Jordan elimination method, we have

$$\left[\begin{array}{rr|r} 4 & 5 & 3 \\ 3 & k & 10 \end{array}\right] \xrightarrow{R_1 - R_2} \left[\begin{array}{rr|r} 1 & 5-k & -7 \\ 3 & k & 10 \end{array}\right] \xrightarrow{R_2 - 3R_1} \left[\begin{array}{rr|r} 1 & 5-k & -7 \\ 0 & 4k-15 & 31 \end{array}\right].$$

In order for the system to have a unique solution we must have $4k - 15 \neq 0$, so $k \neq \frac{15}{4}$. With this condition, we have

$$\left[\begin{array}{rr|r} 1 & 5-k & -7 \\ 0 & 4k-15 & 31 \end{array}\right] \xrightarrow{\left(\frac{1}{4k-15}\right)R_2} \left[\begin{array}{rr|r} 1 & 5-k & -7 \\ 0 & 4k-15 & \frac{31}{4k-15} \end{array}\right] \xrightarrow{R_1-(5-k)R_2} \left[\begin{array}{rr|r} 1 & 0 & \frac{3k-50}{4k-15} \\ 0 & 1 & \frac{31}{4k-15} \end{array}\right]$$

Thus, the required solution is $x = \dfrac{3k-50}{4k-15}$, $y = \dfrac{31}{4k-15}$.

58. Using the Gauss-Jordan elimination method, we have

$$\left[\begin{array}{rrr|r} 1 & 3 & 1 & 8 \\ 3 & 2 & -2 & 5 \\ 4 & -3 & k & 0 \end{array}\right] \xrightarrow[R_3 - 4R_1]{R_2 - 3R_1} \left[\begin{array}{rrr|r} 1 & 3 & 1 & 8 \\ 0 & -7 & -5 & -19 \\ 0 & -15 & k-4 & -32 \end{array}\right] \xrightarrow{-\frac{1}{7}R_2} \left[\begin{array}{rrr|r} 1 & 3 & 1 & 8 \\ 0 & 1 & \frac{5}{7} & \frac{19}{7} \\ 0 & -15 & k-4 & -32 \end{array}\right] \xrightarrow[R_3 + 15R_2]{R_1 - 3R_2}$$

$$\left[\begin{array}{rrr|r} 1 & 0 & -\frac{8}{7} & -\frac{1}{7} \\ 0 & 1 & \frac{5}{7} & \frac{19}{7} \\ 0 & 0 & \frac{7k+47}{7} & \frac{61}{7} \end{array}\right]$$

The system has a unique solution if $\dfrac{7k+47}{7} \neq 0$, that is, $7k + 47 \neq 0$, or $k \neq -\frac{47}{7}$. In this case, we have

$$\left[\begin{array}{rrr|r} 1 & 0 & -\frac{8}{7} & -\frac{1}{7} \\ 0 & 1 & \frac{5}{7} & \frac{19}{7} \\ 0 & 0 & \frac{7k+47}{7} & \frac{61}{7} \end{array}\right] \xrightarrow{\left(\frac{1}{7k+47}\right)R_3} \left[\begin{array}{rrr|r} 1 & 0 & -\frac{8}{7} & -\frac{1}{7} \\ 0 & 1 & \frac{5}{7} & \frac{19}{7} \\ 0 & 0 & 1 & \frac{61}{7k+47} \end{array}\right] \xrightarrow[R_2 - \frac{5}{7}R_3]{R_1 + \frac{8}{7}R_3} \left[\begin{array}{rrr|r} 1 & 0 & 0 & -\frac{7k+441}{7(7k+47)} \\ 0 & 1 & 0 & \frac{133k+588}{7(7k+47)} \\ 0 & 0 & 1 & \frac{61}{7k+47} \end{array}\right].$$

Thus, provided $k \neq -\frac{47}{7}$, the unique solution is $x = -\dfrac{7k+441}{7(7k+47)} = -\dfrac{k-63}{7k+47}$, $y = \dfrac{133k+588}{7(7k+47)} = \dfrac{19k+84}{7k+47}$,

$z = \dfrac{61}{7k+47}$.

59. We wish to solve the system of equations

$$\begin{aligned} x + \quad y &= 500 \\ 42x + 30y &= 18{,}600 \end{aligned}$$

where x is the number of acres of corn planted and y is the number of acres of wheat planted. Using the Gauss-Jordan elimination method, we find

$$\left[\begin{array}{cc|c} 1 & 1 & 500 \\ 42 & 30 & 18{,}600 \end{array}\right] \xrightarrow{R_2 - 42R_1} \left[\begin{array}{cc|c} 1 & 1 & 500 \\ 0 & -12 & -2400 \end{array}\right] \xrightarrow{-\frac{1}{12}R_2} \left[\begin{array}{cc|c} 1 & 1 & 500 \\ 0 & 1 & 200 \end{array}\right] \xrightarrow{R_1 - R_2} \left[\begin{array}{cc|c} 1 & 0 & 300 \\ 0 & 1 & 200 \end{array}\right].$$

The solution to this system of equations is $x = 300$, $y = 200$. We conclude that Jacob should plant 300 acres of corn and 200 acres of wheat.

60. We wish to solve the system of equations

$$\begin{aligned} x + \quad\quad y &= 2000 \\ 0.03x + 0.04y &= \quad 72 \end{aligned}$$

where x is the amount invested at 3% and y is the amount invested at 4%. Using the Gauss-Jordan elimination method, we find $\left[\begin{array}{cc|c} 1 & 1 & 2000 \\ 0.03 & 0.04 & 72 \end{array}\right] \xrightarrow{R_2 - 0.03R_1} \left[\begin{array}{cc|c} 1 & 1 & 2000 \\ 0 & 0.01 & 12 \end{array}\right] \xrightarrow{100R_2} \left[\begin{array}{cc|c} 1 & 1 & 2000 \\ 0 & 1 & 1200 \end{array}\right] \xrightarrow{R_1 - R_2}$
$\left[\begin{array}{cc|c} 1 & 0 & 800 \\ 0 & 1 & 1200 \end{array}\right].$

The solution to this system of equations is $x = 800$, $y = 1200$. We conclude that Michael should invest \$800 at 3% per year and \$1200 at 4% per year.

61. Let x denote the number of pounds of \$8/lb coffee and y the number of pounds of \$9/lb coffee. Then we wish to solve the system

$$\begin{aligned} x + \ y &= 100 \\ 8x + 9y &= 860 \end{aligned}$$

Using the Gauss-Jordan elimination method, we have $\left[\begin{array}{cc|c} 1 & 1 & 100 \\ 8 & 9 & 860 \end{array}\right] \xrightarrow{R_2 - 8R_1} \left[\begin{array}{cc|c} 1 & 1 & 100 \\ 0 & 1 & 60 \end{array}\right] \xrightarrow{R_1 - R_2}$

$\left[\begin{array}{cc|c} 1 & 0 & 40 \\ 0 & 1 & 60 \end{array}\right]$. Therefore, 40 pounds of \$8/lb coffee and 60 pounds of \$9/lb coffee should be used in the 100-lb. mixture.

62. Let x be the number of dollars invested in bonds yielding 4% and y the number of dollars invested in the bonds yielding 5%. Then the solution to the problem can be found by solving the system of equations

$$\begin{aligned} x + \quad\quad y &= 30{,}000 \\ 0.04x + 0.05y &= \quad 1320 \end{aligned}$$

Using the Gauss-Jordan elimination method, we have

$$\left[\begin{array}{cc|c} 1 & 1 & 30{,}000 \\ 0.04 & 0.05 & 1320 \end{array}\right] \xrightarrow{R_2 - 0.04R_1} \left[\begin{array}{cc|c} 1 & 1 & 30{,}000 \\ 0 & 0.01 & 120 \end{array}\right] \xrightarrow{100R_2} \left[\begin{array}{cc|c} 1 & 1 & 30{,}000 \\ 0 & 1 & 12{,}000 \end{array}\right] \xrightarrow{R_1 - R_2} \left[\begin{array}{cc|c} 1 & 0 & 18{,}000 \\ 0 & 1 & 12{,}000 \end{array}\right].$$

Thus, she has \$18,000 invested in bonds yielding 4% and \$12,000 invested in bonds yielding 5%.

63. Let x and y denote the numbers of children and adults respectively who rode the bus during the morning shift. Then the solution to the problem can be found by solving the system of equations

$$\begin{aligned} x + y &= 1000 \\ 0.5x + 1.5y &= 1300 \end{aligned}$$

Using the Gauss-Jordan elimination method, we have $\left[\begin{array}{cc|c} 1 & 1 & 1000 \\ 0.5 & 1.5 & 1300 \end{array}\right] \xrightarrow{R_2 - 0.5R_1} \left[\begin{array}{cc|c} 1 & 1 & 1000 \\ 0 & 1 & 800 \end{array}\right] \xrightarrow{R_1 - R_2}$

$\left[\begin{array}{cc|c} 1 & 0 & 200 \\ 0 & 1 & 800 \end{array}\right]$. We conclude that 800 adults and 200 children rode the bus during the morning shift.

64. Let x, y, and z denote the numbers of one-bedroom units, two-bedroom townhouses, and three-bedroom townhouses, respectively. Then we are required to solve the system

$$\begin{aligned} x + y + z &= 192 \\ x - y - z &= 0 \\ x \phantom{{}-y} - 3z &= 0 \end{aligned}$$

Using the Gauss-Jordan elimination method, we find

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 192 \\ 1 & -1 & -1 & 0 \\ 1 & 0 & -3 & 0 \end{array}\right] \xrightarrow[R_3 - R_1]{R_2 - R_1} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 192 \\ 0 & -2 & -2 & -192 \\ 0 & -1 & -4 & -192 \end{array}\right] \xrightarrow{-\frac{1}{2}R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 192 \\ 0 & 1 & 1 & 96 \\ 0 & -1 & -4 & -192 \end{array}\right] \xrightarrow[R_3 + R_2]{R_1 - R_2}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 96 \\ 0 & 1 & 1 & 96 \\ 0 & 0 & -3 & -96 \end{array}\right] \xrightarrow{-\frac{1}{3}R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 96 \\ 0 & 1 & 1 & 96 \\ 0 & 0 & 1 & 32 \end{array}\right] \xrightarrow{R_2 - R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 96 \\ 0 & 1 & 0 & 64 \\ 0 & 0 & 1 & 32 \end{array}\right].$$

Therefore, 96 one-bedroom, 64 two-bedroom, and 32 three-bedroom units should be built.

65. Let x and y denote the costs of the ball and the bat, respectively. Then $x + y = 110$ and $y - x = 100$. Using the Gauss-Jordan elimination method, we have

$$\left[\begin{array}{cc|c} 1 & 1 & 110 \\ -1 & 1 & 100 \end{array}\right] \xrightarrow{R_2 + R_1} \left[\begin{array}{cc|c} 1 & 1 & 110 \\ 0 & 2 & 210 \end{array}\right] \xrightarrow{\frac{1}{2}R_2} \left[\begin{array}{cc|c} 1 & 1 & 110 \\ 0 & 1 & 105 \end{array}\right] \xrightarrow{R_1 - R_2} \left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & 105 \end{array}\right].$$

Thus, $x = 5$ and $y = 105$, so the ball costs \$5 and the bat costs \$105.

66. Let x and y denote the amounts of money invested in projects A and B, respectively. Then $x + y = 70{,}000$ and $x - y = 20{,}000$. Using the Gauss-Jordan elimination method, we have

$$\left[\begin{array}{cc|c} 1 & 1 & 70{,}000 \\ 1 & -1 & 20{,}000 \end{array}\right] \xrightarrow{R_2 - R_1} \left[\begin{array}{cc|c} 1 & 1 & 70{,}000 \\ 0 & -2 & -50{,}000 \end{array}\right] \xrightarrow{-\frac{1}{2}R_2} \left[\begin{array}{cc|c} 1 & 1 & 70{,}000 \\ 0 & 1 & 25{,}000 \end{array}\right] \xrightarrow{R_1 - R_2} \left[\begin{array}{cc|c} 1 & 0 & 45{,}000 \\ 0 & 1 & 25{,}000 \end{array}\right].$$

Thus, $x = 45{,}000$ and $y = 25{,}000$, so Josh invested \$45,000 in project A and \$25,000 in project B.

67. Let x, y, and z, denote the amounts of money he should invest in a savings account, mutual funds, and bonds, respectively. Then we are required to solve the system

$$\begin{aligned} 0.06x + 0.08y + 0.12z &= 21{,}600 \\ 2x \qquad\qquad - \quad z &= 0 \\ 0.08y - 0.12z &= 0 \end{aligned}$$

Using the Gauss-Jordan elimination method, we find

$$\left[\begin{array}{ccc|c} 0.06 & 0.08 & 0.12 & 21{,}600 \\ 2 & 0 & -1 & 0 \\ 0 & 0.08 & -0.12 & 0 \end{array}\right] \xrightarrow[\frac{1}{0.08}R_3]{\frac{1}{0.06}R_1} \left[\begin{array}{ccc|c} 1 & \frac{4}{3} & 2 & 360{,}000 \\ 2 & 0 & -1 & 0 \\ 0 & 1 & -\frac{3}{2} & 0 \end{array}\right] \xrightarrow{R_2-2R_1} \left[\begin{array}{ccc|c} 1 & \frac{4}{3} & 2 & 360{,}000 \\ 0 & -\frac{8}{3} & -5 & -720{,}000 \\ 0 & 1 & -\frac{3}{2} & 0 \end{array}\right] \xrightarrow{-\frac{3}{8}R_2}$$

$$\left[\begin{array}{ccc|c} 1 & \frac{4}{3} & 2 & 360{,}000 \\ 0 & 1 & \frac{15}{8} & 270{,}000 \\ 0 & 1 & -\frac{3}{2} & 0 \end{array}\right] \xrightarrow[R_3-R_2]{R_1-\frac{4}{3}R_2} \left[\begin{array}{ccc|c} 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & \frac{15}{8} & 270{,}000 \\ 0 & 0 & -\frac{27}{8} & -270{,}000 \end{array}\right] \xrightarrow{-\frac{8}{27}R_3} \left[\begin{array}{ccc|c} 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & \frac{15}{8} & 270{,}000 \\ 0 & 0 & 1 & 80{,}000 \end{array}\right] \xrightarrow[R_2-\frac{15}{8}R_3]{R_1+\frac{1}{2}R_3}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 40{,}000 \\ 0 & 1 & 0 & 120{,}000 \\ 0 & 0 & 1 & 80{,}000 \end{array}\right].$$

Therefore, Sid should invest \$40,000 in a savings account, \$120,000 in mutual funds, and \$80,000 in bonds.

68. Refer to Exercise 2.1.32 on page 80 of the text. We obtain the following augmented matrices:

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 200{,}000 \\ 15 & 10 & 6 & 2{,}000{,}000 \\ 1 & 1 & -1 & 0 \end{array}\right] \xrightarrow[R_3-R_1]{R_2-15R_1} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 200{,}000 \\ 0 & -5 & -9 & -1{,}000{,}000 \\ 0 & 0 & -2 & -200{,}000 \end{array}\right] \xrightarrow{-\frac{1}{5}R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 200{,}000 \\ 0 & 1 & \frac{9}{5} & 200{,}000 \\ 0 & 0 & -2 & -200{,}000 \end{array}\right] \xrightarrow[-\frac{1}{2}R_3]{R_1-R_2}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -\frac{4}{5} & 0 \\ 0 & 1 & \frac{9}{5} & 200{,}000 \\ 0 & 0 & 1 & 100{,}000 \end{array}\right] \xrightarrow[R_2-\frac{9}{5}R_3]{R_1+\frac{4}{5}R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 80{,}000 \\ 0 & 1 & 0 & 20{,}000 \\ 0 & 0 & 1 & 100{,}000 \end{array}\right].$$

We see that $x = 80{,}000$, $y = 20{,}000$, and $z = 100{,}000$. Therefore, they should invest \$80,000 in high-risk, \$20,000 in medium-risk, and \$100,000 in low-risk stocks.

69. We need to solve the system

$$\begin{aligned} x + y + z &= 100 \\ x + y \qquad &= 67 \\ x \qquad - z &= 17 \end{aligned}$$

Using the Gauss-Jordan elimination method, we have

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 100 \\ 1 & 1 & 0 & 67 \\ 1 & 0 & -1 & 17 \end{array}\right] \xrightarrow[R_3-R_1]{R_2-R_1} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 100 \\ 0 & 0 & -1 & -33 \\ 0 & -1 & -2 & -83 \end{array}\right] \xrightarrow{R_2\leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 100 \\ 0 & -1 & -2 & -83 \\ 0 & 0 & -1 & -33 \end{array}\right] \xrightarrow[-R_3]{-R_2}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 100 \\ 0 & 1 & 2 & 83 \\ 0 & 0 & 1 & 33 \end{array}\right] \xrightarrow[R_2-2R_3]{R_1-R_2} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 17 \\ 0 & 1 & 0 & 17 \\ 0 & 0 & 1 & 33 \end{array}\right] \xrightarrow{R_1+R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 50 \\ 0 & 1 & 0 & 17 \\ 0 & 0 & 1 & 33 \end{array}\right].$$

Thus, $x = 50$, $y = 17$, and $z = 33$.

70. Let x, y, and z denote the numbers of respondents who answered "yes," "no," and "not sure," respectively. Then we have

$$\begin{aligned} x + y + z &= 1000 \\ y + z &= 370 \\ x - y \quad &= 340 \end{aligned}$$

Using the Gauss-Jordan elimination method, we have

$$\left[\begin{array}{rrr|r} 1 & 1 & 1 & 1000 \\ 0 & 1 & 1 & 370 \\ 1 & -1 & 0 & 340 \end{array}\right] \xrightarrow{R_3 - R_1} \left[\begin{array}{rrr|r} 1 & 1 & 1 & 1000 \\ 0 & 1 & 1 & 370 \\ 0 & -2 & -1 & -660 \end{array}\right] \xrightarrow[R_3+2R_2]{R_1 - R_2} \left[\begin{array}{rrr|r} 1 & 0 & 0 & 630 \\ 0 & 1 & 1 & 370 \\ 0 & 0 & 1 & 80 \end{array}\right] \xrightarrow{R_2 - R_3} \left[\begin{array}{rrr|r} 1 & 0 & 0 & 630 \\ 0 & 1 & 0 & 290 \\ 0 & 0 & 1 & 80 \end{array}\right].$$

Thus, $x = 630$, $y = 290$, and $z = 80$. We conclude that 63% said yes, 29% said no, and 8% said they were not sure.

71. Refer to Exercise 2.1.35 on page 81 of the text. We obtain the following augmented matrices:

$$\left[\begin{array}{rrr|r} 18 & 20 & 24 & 26{,}400 \\ 4 & 4 & 3 & 4900 \\ 5 & 4 & 6 & 6200 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_3} \left[\begin{array}{rrr|r} 5 & 4 & 6 & 6200 \\ 4 & 4 & 3 & 4900 \\ 18 & 20 & 24 & 26{,}400 \end{array}\right] \xrightarrow{R_1 - R_2} \left[\begin{array}{rrr|r} 1 & 0 & 3 & 1300 \\ 4 & 4 & 3 & 4900 \\ 18 & 20 & 24 & 26{,}400 \end{array}\right] \xrightarrow[R_3 - 18R_1]{R_2 - 4R_1}$$

$$\left[\begin{array}{rrr|r} 1 & 0 & 3 & 1300 \\ 0 & 4 & -9 & -300 \\ 0 & 20 & -30 & 3000 \end{array}\right] \xrightarrow{\frac{1}{4}R_2} \left[\begin{array}{rrr|r} 1 & 0 & 3 & 1300 \\ 0 & 1 & -\frac{9}{4} & -75 \\ 0 & 20 & -30 & 3000 \end{array}\right] \xrightarrow{R_3 - 20R_2} \left[\begin{array}{rrr|r} 1 & 0 & 3 & 1300 \\ 0 & 1 & -\frac{9}{4} & -75 \\ 0 & 0 & 15 & 4500 \end{array}\right] \xrightarrow{\frac{1}{15}R_3}$$

$$\left[\begin{array}{rrr|r} 1 & 0 & 3 & 1300 \\ 0 & 1 & -\frac{9}{4} & -75 \\ 0 & 0 & 1 & 300 \end{array}\right] \xrightarrow[R_2 + \frac{9}{4}R_3]{R_1 - 3R_3} \left[\begin{array}{rrr|r} 1 & 0 & 0 & 400 \\ 0 & 1 & 0 & 600 \\ 0 & 0 & 1 & 300 \end{array}\right].$$ We see that $x = 400$, $y = 600$, and $z = 300$. Therefore, Lawnco should produce 400, 600, and 300 bags of grades A, B, and C fertilizer, respectively.

72. Let x, y, and z denote the numbers of tickets sold to children, students and adults, respectively. Then the solution to the problem can be found by solving the system

$$\begin{aligned} x + \ y + \ z &= 900 \\ x + \ y - 2z &= 0 \\ 4x + 6y + 8z &= 5600 \end{aligned}$$

Using the Gauss-Jordan elimination method, we have

$$\left[\begin{array}{rrr|r} 1 & 1 & 1 & 900 \\ 1 & 1 & -2 & 0 \\ 4 & 6 & 8 & 5600 \end{array}\right] \xrightarrow[R_3 - 4R_1]{R_2 - R_1} \left[\begin{array}{rrr|r} 1 & 1 & 1 & 900 \\ 0 & 0 & -3 & -900 \\ 0 & 2 & 4 & 2000 \end{array}\right] \xrightarrow{\frac{1}{2}R_3} \left[\begin{array}{rrr|r} 1 & 1 & 1 & 900 \\ 0 & 0 & -3 & -900 \\ 0 & 1 & 2 & 1000 \end{array}\right] \xrightarrow{R_2 \leftrightarrow R_3}$$

$$\left[\begin{array}{rrr|r} 1 & 1 & 1 & 900 \\ 0 & 1 & 2 & 1000 \\ 0 & 0 & -3 & -900 \end{array}\right] \xrightarrow{R_1 - R_2} \left[\begin{array}{rrr|r} 1 & 0 & -1 & -100 \\ 0 & 1 & 2 & 1000 \\ 0 & 0 & -3 & -900 \end{array}\right] \xrightarrow{-\frac{1}{3}R_3} \left[\begin{array}{rrr|r} 1 & 0 & -1 & -100 \\ 0 & 1 & 2 & 1000 \\ 0 & 0 & 1 & 300 \end{array}\right] \xrightarrow[R_2 - 2R_3]{R_1 + R_3} \left[\begin{array}{rrr|r} 1 & 0 & 0 & 200 \\ 0 & 1 & 0 & 400 \\ 0 & 0 & 1 & 300 \end{array}\right].$$

We conclude that 200 children attended the show.

73. Let x, y, and z denote the numbers of compact, intermediate, and full-size cars, respectively, to be purchased. Then the problem can be solved by solving the system

$$\begin{aligned} 18{,}000x + 27{,}000y + 36{,}000z &= 2{,}250{,}000 \\ x - \quad 2y \qquad\qquad &= 0 \\ x + \quad y + \qquad z &= 100 \end{aligned}$$

Using the Gauss-Jordan elimination method, we have

$$\left[\begin{array}{ccc|c} 18{,}000 & 27{,}000 & 36{,}000 & 2{,}250{,}000 \\ 1 & -2 & 0 & 0 \\ 1 & 1 & 1 & 100 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 100 \\ 1 & -2 & 0 & 0 \\ 18{,}000 & 27{,}000 & 36{,}000 & 2{,}250{,}000 \end{array}\right] \xrightarrow[R_3 - 18{,}000R_1]{R_2 - R_1}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 100 \\ 0 & -3 & -1 & -100 \\ 0 & 9000 & 18{,}000 & 450{,}000 \end{array}\right] \xrightarrow{-\frac{1}{3}R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 100 \\ 0 & 1 & \frac{1}{3} & \frac{100}{3} \\ 0 & 9000 & 18{,}000 & 450{,}000 \end{array}\right] \xrightarrow[R_3 - 9000R_2]{R_1 - R_2}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & \frac{2}{3} & \frac{200}{3} \\ 0 & 1 & \frac{1}{3} & \frac{100}{3} \\ 0 & 0 & 15{,}000 & 150{,}000 \end{array}\right] \xrightarrow{\frac{1}{15{,}000}R_3} \left[\begin{array}{ccc|c} 1 & 0 & \frac{2}{3} & \frac{200}{3} \\ 0 & 1 & \frac{1}{3} & \frac{100}{3} \\ 0 & 0 & 1 & 10 \end{array}\right] \xrightarrow[R_2 - \frac{1}{3}R_3]{R_1 - \frac{2}{3}R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 60 \\ 0 & 1 & 0 & 30 \\ 0 & 0 & 1 & 10 \end{array}\right].$$

We conclude that 60 compact cars, 30 intermediate cars, and 10 full-size cars will be purchased.

74. Let x, y, and z denote the amounts of money invested in high-risk stocks, medium- risk stocks, and low-risk stocks, respectively. Then the problem can be solved by solving the system

$$\begin{aligned} x + y + z &= 200{,}000 \\ 2x + 2y - z &= 0 \\ 6x + y - 3z &= 0 \end{aligned}$$

Using the Gauss-Jordan elimination method, we have

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 200{,}000 \\ 2 & 2 & -1 & 0 \\ 6 & 1 & -3 & 0 \end{array}\right] \xrightarrow[R_3 - 6R_1]{R_2 - 2R_1} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 200{,}000 \\ 0 & 0 & -3 & -400{,}000 \\ 0 & -5 & -9 & -1{,}200{,}000 \end{array}\right] \xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 200{,}000 \\ 0 & -5 & -9 & -1{,}200{,}000 \\ 0 & 0 & -3 & -400{,}000 \end{array}\right] \xrightarrow{-\frac{1}{5}R_2}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 200{,}000 \\ 0 & 1 & \frac{9}{5} & 240{,}000 \\ 0 & 0 & -3 & -400{,}000 \end{array}\right] \xrightarrow{R_1 - R_2} \left[\begin{array}{ccc|c} 1 & 0 & -\frac{4}{5} & -40{,}000 \\ 0 & 1 & \frac{9}{5} & 240{,}000 \\ 0 & 0 & -3 & -400{,}000 \end{array}\right] \xrightarrow{-\frac{1}{3}R_3} \left[\begin{array}{ccc|c} 1 & 0 & -\frac{4}{5} & -40{,}000 \\ 0 & 1 & \frac{9}{5} & 240{,}000 \\ 0 & 0 & 1 & \frac{400{,}000}{3} \end{array}\right] \xrightarrow[R_2 - \frac{9}{5}R_3]{R_1 + \frac{4}{5}R_3}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{200{,}000}{3} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{400{,}000}{3} \end{array}\right].$$

We conclude that the investment club should invest \$66,666.67 in high-risk stocks, nothing in medium-risk stocks, and \$133,333.33 in low-risk stocks.

75. Let x, y, and z, represent the numbers of ounces of Foods I, II, III used in the meal, respectively. Then the problem reduces to solving the following system of linear equations:

$$\begin{aligned} 10x + 6y + 8z &= 100 \\ 10x + 12y + 6z &= 100 \\ 5x + 4y + 12z &= 100 \end{aligned}$$

Using the Gauss-Jordan elimination method, we obtain

$$\left[\begin{array}{rrr|r} 10 & 6 & 8 & 100 \\ 10 & 12 & 6 & 100 \\ 5 & 4 & 12 & 100 \end{array}\right] \xrightarrow{\frac{1}{10}R_1} \left[\begin{array}{rrr|r} 1 & \frac{3}{5} & \frac{4}{5} & 10 \\ 10 & 12 & 6 & 100 \\ 5 & 4 & 12 & 100 \end{array}\right] \xrightarrow[R_3-5R_1]{R_2-10R_1} \left[\begin{array}{rrr|r} 1 & \frac{3}{5} & \frac{4}{5} & 10 \\ 0 & 6 & -2 & 0 \\ 0 & 1 & 8 & 50 \end{array}\right] \xrightarrow{\frac{1}{6}R_2} \left[\begin{array}{rrr|r} 1 & \frac{3}{5} & \frac{4}{5} & 10 \\ 0 & 1 & -\frac{1}{3} & 0 \\ 0 & 1 & 8 & 50 \end{array}\right] \xrightarrow[R_3-R_2]{R_1-\frac{3}{5}R_2}$$

$$\left[\begin{array}{rrr|r} 1 & 0 & 1 & 10 \\ 0 & 1 & -\frac{1}{3} & 0 \\ 0 & 0 & \frac{25}{3} & 50 \end{array}\right] \xrightarrow{\frac{3}{25}R_3} \left[\begin{array}{rrr|r} 1 & 0 & 1 & 10 \\ 0 & 1 & -\frac{1}{3} & 0 \\ 0 & 0 & 1 & 6 \end{array}\right] \xrightarrow[R_2+\frac{1}{3}R_3]{R_1-R_3} \left[\begin{array}{rrr|r} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 6 \end{array}\right].$$

We conclude that 4 ounces of Food I, 2 ounces of Food II, and 6 ounces of Food III should be used to prepare the meal.

76. Let x, y, and z denote the amounts of money invested in stocks, bonds, and a money market account, respectively. Then the problem can be solved by solving the system

$$\begin{aligned} x + y + z &= 100{,}000 \\ 12x + 8y + 4z &= 1{,}000{,}000 \\ 20x + 10y - 100z &= 0 \end{aligned}$$

Using the Gauss-Jordan elimination method, we have

$$\left[\begin{array}{rrr|r} 1 & 1 & 1 & 100{,}000 \\ 12 & 8 & 4 & 1{,}000{,}000 \\ 20 & 10 & -100 & 0 \end{array}\right] \xrightarrow[R_3-20R_1]{R_2-12R_1} \left[\begin{array}{rrr|r} 1 & 1 & 1 & 100{,}000 \\ 0 & -4 & -8 & -200{,}000 \\ 0 & -10 & -120 & -2{,}000{,}000 \end{array}\right] \xrightarrow{-\frac{1}{4}R_2}$$

$$\left[\begin{array}{rrr|r} 1 & 1 & 1 & 100{,}000 \\ 0 & 1 & 2 & 50{,}000 \\ 0 & -10 & -120 & -2{,}000{,}000 \end{array}\right] \xrightarrow[R_3+10R_2]{R_1-R_2} \left[\begin{array}{rrr|r} 1 & 0 & -1 & 50{,}000 \\ 0 & 1 & 2 & 50{,}000 \\ 0 & 0 & -100 & -1{,}500{,}000 \end{array}\right] \xrightarrow{-\frac{1}{100}R_3}$$

$$\left[\begin{array}{rrr|r} 1 & 0 & -1 & 50{,}000 \\ 0 & 1 & 2 & 50{,}000 \\ 0 & 0 & 1 & 15{,}000 \end{array}\right] \xrightarrow[R_2-2R_3]{R_1+R_3} \left[\begin{array}{rrr|r} 1 & 0 & 0 & 65{,}000 \\ 0 & 1 & 0 & 20{,}000 \\ 0 & 0 & 1 & 15{,}000 \end{array}\right].$$

We conclude that the Garcias should invest \$65,000 in stocks,\$20,000 in bonds, and \$15,000 in a money market account.

77. Let x, y, and z denote the numbers of front orchestra, rear orchestra, and front balcony seats sold for this performance. Then we are required to solve the system

$$\begin{aligned} x + y + z &= 1000 \\ 80x + 60y + 50z &= 62{,}800 \\ x + y - 2z &= 400 \end{aligned}$$

Using the Gauss-Jordan elimination method, we find

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1000 \\ 80 & 60 & 50 & 62{,}800 \\ 1 & 1 & -2 & 400 \end{array}\right] \xrightarrow[R_3-R_1]{R_2-80R_1} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1000 \\ 0 & -20 & -30 & -17{,}200 \\ 0 & 0 & -3 & -600 \end{array}\right] \xrightarrow[-\frac{1}{3}R_3]{-\frac{1}{20}R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1000 \\ 0 & 1 & \frac{3}{2} & 860 \\ 0 & 0 & 1 & 200 \end{array}\right] \xrightarrow{R_1-R_2}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -\frac{1}{2} & 140 \\ 0 & 1 & \frac{3}{2} & 860 \\ 0 & 0 & 1 & 200 \end{array}\right] \xrightarrow[R_2-\frac{3}{2}R_3]{R_1+\frac{1}{2}R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 240 \\ 0 & 1 & 0 & 560 \\ 0 & 0 & 1 & 200 \end{array}\right].$$

We conclude that tickets for 240 front orchestra seats, 560 rear orchestra seats, and 200 front balcony seats were sold.

78. Let x, y, and z denote the numbers of dozens of sleeveless, short-sleeve, and long-sleeve blouses produced per day. Then we want to solve the system

$$\begin{aligned} 9x + 12y + 15z &= 4800 \\ 22x + 24y + 28z &= 9600 \\ 6x + 8y + 8z &= 2880 \end{aligned}$$

Using the Gauss-Jordan elimination method, we find

$$\left[\begin{array}{ccc|c} 9 & 12 & 15 & 4800 \\ 22 & 24 & 28 & 9600 \\ 6 & 8 & 8 & 2880 \end{array}\right] \xrightarrow{\frac{1}{9}R_1} \left[\begin{array}{ccc|c} 1 & \frac{4}{3} & \frac{5}{3} & \frac{1600}{3} \\ 22 & 24 & 28 & 9600 \\ 6 & 8 & 8 & 2880 \end{array}\right] \xrightarrow[R_3-6R_1]{R_2-22R_1} \left[\begin{array}{ccc|c} 1 & \frac{4}{3} & \frac{5}{3} & \frac{1600}{3} \\ 0 & -\frac{16}{3} & -\frac{26}{3} & -\frac{6400}{3} \\ 0 & 0 & -2 & -320 \end{array}\right] \xrightarrow{-\frac{3}{16}R_2}$$

$$\left[\begin{array}{ccc|c} 1 & \frac{4}{3} & \frac{5}{3} & \frac{1600}{3} \\ 0 & 1 & \frac{13}{8} & 400 \\ 0 & 0 & -2 & -320 \end{array}\right] \xrightarrow[-\frac{1}{2}R_3]{R_1-\frac{4}{3}R_2} \left[\begin{array}{ccc|c} 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & \frac{13}{8} & 400 \\ 0 & 0 & 1 & 160 \end{array}\right] \xrightarrow[R_2-\frac{13}{8}R_3]{R_1+\frac{1}{2}R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 80 \\ 0 & 1 & 0 & 140 \\ 0 & 0 & 1 & 160 \end{array}\right].$$

Therefore, the manufacturer should produce 80 dozen sleeveless, 140 dozen short-sleeve, and 160 dozen long-sleeve blouses per day.

79. Let x, y, and z denote the number of days spent in London, Paris, and Rome, respectively. We have $280x + 330y + 260z = 4060$, $130x + 140y + 110z = 1800$, and $x - y - z = 0$ (since $x = y + z$). Using the Gauss-Jordan elimination method to solve the system, we have

$$\left[\begin{array}{ccc|c} 280 & 330 & 260 & 4060 \\ 130 & 140 & 110 & 1800 \\ 1 & -1 & -1 & 0 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & -1 & -1 & 0 \\ 130 & 140 & 110 & 1800 \\ 280 & 330 & 260 & 4060 \end{array}\right] \xrightarrow[R_3-280R_1]{R_2-130R_1} \left[\begin{array}{ccc|c} 1 & -1 & -1 & 0 \\ 0 & 270 & 240 & 1800 \\ 0 & 610 & 540 & 4060 \end{array}\right] \xrightarrow{\frac{1}{270}R_2}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & -1 & 0 \\ 0 & 1 & \frac{8}{9} & \frac{20}{3} \\ 0 & 610 & 540 & 4060 \end{array}\right] \xrightarrow[R_3-610R_2]{R_1+R_2} \left[\begin{array}{ccc|c} 1 & 0 & -\frac{1}{9} & \frac{20}{3} \\ 0 & 1 & \frac{8}{9} & \frac{20}{3} \\ 0 & 0 & -\frac{20}{9} & -\frac{20}{3} \end{array}\right] \xrightarrow{-\frac{9}{20}R_3} \left[\begin{array}{ccc|c} 1 & 0 & -\frac{3}{23} & \frac{152}{23} \\ 0 & 1 & \frac{20}{23} & \frac{152}{23} \\ 0 & 0 & 1 & 3 \end{array}\right] \xrightarrow[R_2-\frac{20}{23}R_1]{R_1+\frac{3}{23}R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 3 \end{array}\right].$$

The solution is $x = 7$, $y = 4$, and $z = 3$. Therefore, he spent 7 days in London, 4 days in Paris, and 3 days in Rome.

80. Let x, y, z, and w denote the number of days they spent in Boston, New York, Philadelphia, and Washington, respectively. The given information leads to the system of equations

$$\begin{aligned} x + y + z + w &= 14 \\ 240x + 400y + 160z + 200w &= 4040 \\ y &= x + w \\ y &= 3z \end{aligned} \quad \text{or} \quad \begin{aligned} x + y + z + w &= 14 \\ x - y + w &= 0 \\ y - 3z &= 0 \\ 240x + 400y + 160z + 200w &= 4040 \end{aligned}$$

We obtain the following augmented matrices:

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 14 \\ 1 & -1 & 0 & 1 & 0 \\ 0 & 1 & -3 & 0 & 0 \\ 240 & 400 & 160 & 200 & 4040 \end{array}\right] \xrightarrow[R_3 - 240R_1]{R_2 - R_1} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 14 \\ 0 & -2 & -1 & 0 & -14 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 160 & -80 & -40 & 680 \end{array}\right] \xrightarrow{R_2 \leftrightarrow R_3}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 14 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & -2 & -1 & 0 & -14 \\ 0 & 160 & -80 & -40 & 680 \end{array}\right] \xrightarrow[R_4 - 160R_2]{\substack{R_1 - R_2 \\ R_3 + 2R_2}} \left[\begin{array}{cccc|c} 1 & 0 & 4 & 1 & 14 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & -7 & 0 & -14 \\ 0 & 0 & 400 & -40 & 680 \end{array}\right] \xrightarrow{-\frac{1}{7}R_3} \left[\begin{array}{cccc|c} 1 & 0 & 4 & 1 & 14 \\ 0 & 1 & -3 & 0 & 0 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 400 & -40 & 680 \end{array}\right] \xrightarrow[R_4 - 400R_3]{\substack{R_1 - 4R_3 \\ R_2 + 3R_3}}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 6 \\ 0 & 1 & 0 & 0 & 6 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & -40 & -120 \end{array}\right] \xrightarrow{-\frac{1}{40}R_4} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 6 \\ 0 & 1 & 0 & 0 & 6 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right] \xrightarrow{R_1 - R_4} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 & 6 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right].$$

We see that $x = 3$, $y = 6$, $z = 2$, and $w = 3$, and we conclude that they spent 3 days each in Boston and Washington, 6 days in New York, and 2 days in Philadelphia.

81. False. The constant cannot be zero. The system $\begin{cases} 2x + y = 1 \\ 3x - y = 2 \end{cases}$ is not equivalent to $\begin{cases} 2x + y = 1 \\ 0\,(3x - y) = 0\,(2) \end{cases}$ or $\begin{cases} 2x + y = 1 \\ 0 = 0 \end{cases}$

82. True. The row with the given form says that $0x + 0y + 0z = a$, or $0 = a$. But if $a \neq 0$, we have a contradiction.

Technology Exercises

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1. $(3, 1, -1, 2)$

2. $(1, 0, -2, -1)$

3. $(5, 4, -3, -4)$

4. $(-256, -33, -12, 167)$

5. $(1, -1, 2, 0, 3)$

6. $(1.2, -0.8, 3.6, 4.7, 2.1)$

2.3 Systems of Linear Equations: Underdetermined and Overdetermined Systems

Concept Questions

page 106

1. There may be no solution, a unique solution, or infinitely many solutions.

2. There may be no solution or infinitely many solutions.

3. No

Exercises page 106

1. **a.** The system has one solution. **b.** The solution is $(3, -1, 2)$.

2. **a.** The system has one solution. **b.** The solution is $(3, -2, 1)$.

3. **a.** The system has one solution. **b.** The solution is $(2, 5)$.

4. **a.** The system has one solution. **b.** The solution is $(3, 1)$.

5. **a.** The system has no solution. The last row contains all zeros to the left of the vertical line and a nonzero number (-1) to the right.

6. **a.** The system has no solution. The last row contains all zeros to the left of the vertical line and a nonzero number (1) to the right.

7. **a.** The system has infinitely many solutions.

   b. Letting $x_3 = t$, we see that the solutions are given by $(4 - t, -2, t)$, where t is a parameter.

8. **a.** The system has infinitely many solutions.

   b. Letting $x_3 = t$, we see that the solutions are given by $(3, -1 - t, t, 2)$, where t is a parameter.

9. **a.** The system has no solution.

   b. The last row contains all zeros to the left of the vertical line and a nonzero number (1) to its right.

10. **a.** The system has no solution.

    b. The last row contains all zeros to the left of the vertical line and a nonzero number (1) to its right.

11. **a.** The system has infinitely many solutions.

    b. Letting $x_4 = t$, we see that the solutions are given by $(4, -1, 3 - t, t)$, where t is a parameter.

12. **a.** The system has infinitely many solutions.

    b. Letting $x_1 = s$ and $x_4 = t$, we see that the solutions are given by $(s, 3 - t, 4 + 2t, t)$, where s and t are parameters.

13. **a.** The system has infinitely many solutions.

    b. Letting $x_3 = s$ and $x_4 = t$, the solutions are given by $(2 - 3s, 1 + s, s, t)$, where s and t are parameters.

14. **a.** The system has infinitely many solutions.

    b. Letting $x_3 = s$ and $x_4 = t$, we see that the solutions are given by $(4 - 3s + t, 2 + 2s - 3t, s, t)$, where s and t are parameters.

15. Using the Gauss-Jordan elimination method, we have

$$\left[\begin{array}{rr|r} 2 & -1 & 3 \\ 1 & 2 & 4 \\ 2 & 3 & 7 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{rr|r} 1 & 2 & 4 \\ 2 & -1 & 3 \\ 2 & 3 & 7 \end{array}\right] \xrightarrow[R_3 - 2R_1]{R_2 - 2R_1} \left[\begin{array}{rr|r} 1 & 2 & 4 \\ 0 & -5 & -5 \\ 0 & -1 & -1 \end{array}\right] \xrightarrow{-\frac{1}{5}R_2} \left[\begin{array}{rr|r} 1 & 2 & 4 \\ 0 & 1 & 1 \\ 0 & -1 & -1 \end{array}\right] \xrightarrow[R_3 + R_2]{R_1 - 2R_2} \left[\begin{array}{rr|r} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{array}\right].$$

The solution is $(2, 1)$.

16. Using the Gauss-Jordan elimination method, we have

$$\left[\begin{array}{rr|r} 1 & 2 & 3 \\ 2 & -3 & -8 \\ 1 & -4 & -9 \end{array}\right] \xrightarrow[R_3 - R_1]{R_2 - 2R_1} \left[\begin{array}{rr|r} 1 & 2 & 3 \\ 0 & -7 & -14 \\ 0 & -6 & -12 \end{array}\right] \xrightarrow{-\frac{1}{7}R_2} \left[\begin{array}{rr|r} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -6 & -12 \end{array}\right] \xrightarrow[R_3 + 6R_2]{R_1 - 2R_2} \left[\begin{array}{rr|r} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array}\right].$$

We conclude that the solution is $(-1, 2)$.

17. Using the Gauss-Jordan elimination method, we have

$$\left[\begin{array}{rr|r} 3 & -2 & -3 \\ 2 & 1 & 3 \\ 1 & -2 & -5 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_3} \left[\begin{array}{rr|r} 1 & -2 & -5 \\ 2 & 1 & 3 \\ 3 & -2 & -3 \end{array}\right] \xrightarrow[R_3 - 3R_1]{R_2 - 2R_1} \left[\begin{array}{rr|r} 1 & -2 & -5 \\ 0 & 5 & 13 \\ 0 & 4 & 12 \end{array}\right] \xrightarrow{\frac{1}{5}R_2} \left[\begin{array}{rr|r} 1 & -2 & -5 \\ 0 & 1 & \frac{13}{5} \\ 0 & 4 & 12 \end{array}\right] \xrightarrow[R_3 - 4R_2]{R_1 + 2R_2} \left[\begin{array}{rr|r} 1 & 0 & \frac{1}{5} \\ 0 & 1 & \frac{13}{5} \\ 0 & 0 & \frac{8}{5} \end{array}\right].$$

Since the last row implies the $0 = \frac{8}{5}$, we conclude that the system of equations is inconsistent and has no solution.

18. Using the Gauss-Jordan elimination method, we have

$$\left[\begin{array}{rr|r} 2 & 3 & 2 \\ 1 & 3 & -2 \\ 1 & -1 & 3 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{rr|r} 1 & 3 & -2 \\ 2 & 3 & 2 \\ 1 & -1 & 3 \end{array}\right] \xrightarrow[R_3 - R_1]{R_2 - 2R_1} \left[\begin{array}{rr|r} 1 & 3 & -2 \\ 0 & -3 & 6 \\ 0 & -4 & 5 \end{array}\right] \xrightarrow{-\frac{1}{3}R_2} \left[\begin{array}{rr|r} 1 & 3 & -2 \\ 0 & 1 & -2 \\ 0 & -4 & 5 \end{array}\right] \xrightarrow[R_3 + 4R_2]{R_1 - 3R_2}$$

$$\left[\begin{array}{rr|r} 1 & 0 & 4 \\ 0 & 1 & -2 \\ 0 & 0 & -3 \end{array}\right].$$

The last row implies that $0 = -3$, which is impossible. We conclude that the system of equations is inconsistent and has no solution.

19. $$\left[\begin{array}{rr|r} 3 & -2 & 5 \\ -1 & 3 & -4 \\ 2 & -4 & 6 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{rr|r} -1 & 3 & -4 \\ 3 & -2 & 5 \\ 2 & -4 & 6 \end{array}\right] \xrightarrow{-R_1} \left[\begin{array}{rr|r} 1 & -3 & 4 \\ 3 & -2 & 5 \\ 2 & -4 & 6 \end{array}\right] \xrightarrow[R_3 - 2R_1]{R_2 - 3R_1} \left[\begin{array}{rr|r} 1 & -3 & 4 \\ 0 & 7 & -7 \\ 0 & 2 & -2 \end{array}\right] \xrightarrow{\frac{1}{7}R_2}$$

$$\left[\begin{array}{rr|r} 1 & -3 & 4 \\ 0 & 1 & -1 \\ 0 & 2 & -2 \end{array}\right] \xrightarrow[R_3 - 2R_2]{R_1 + 3R_2} \left[\begin{array}{rr|r} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{array}\right].$$ We conclude that the solution is $(1, -1)$.

20. $$\left[\begin{array}{rr|r} 4 & 6 & 8 \\ 3 & -2 & -7 \\ 1 & 3 & 5 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_3} \left[\begin{array}{rr|r} 1 & 3 & 5 \\ 3 & -2 & -7 \\ 4 & 6 & 8 \end{array}\right] \xrightarrow[R_3 - 4R_1]{R_2 - 3R_1} \left[\begin{array}{rr|r} 1 & 3 & 5 \\ 0 & -11 & -22 \\ 0 & -6 & -12 \end{array}\right] \xrightarrow{-\frac{1}{11}R_2} \left[\begin{array}{rr|r} 1 & 3 & 5 \\ 0 & 1 & 2 \\ 0 & -6 & -12 \end{array}\right] \xrightarrow[R_3 + 6R_2]{R_1 - 3R_2}$$

$$\left[\begin{array}{rr|r} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array}\right].$$ We conclude that the solution is $(-1, 2)$.

21. $\left[\begin{array}{cc|c} 1 & -2 & 2 \\ 7 & -14 & 14 \\ 3 & -6 & 6 \end{array}\right] \xrightarrow[R_3-3R_1]{R_2-7R_1} \left[\begin{array}{cc|c} 1 & -2 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]$.

We conclude that the infinitely many solutions are given by $(2t+2, t)$, where t is a parameter.

22. $\left[\begin{array}{ccc|c} 3 & -1 & 2 & 5 \\ 1 & -1 & 2 & 1 \\ 5 & -2 & 4 & 12 \end{array}\right] \xrightarrow{\frac{1}{3}R_1} \left[\begin{array}{ccc|c} 1 & -\frac{1}{3} & \frac{2}{3} & \frac{5}{3} \\ 1 & -1 & 2 & 1 \\ 5 & -2 & 4 & 12 \end{array}\right] \xrightarrow[R_3-5R_1]{R_2-R_1} \left[\begin{array}{ccc|c} 1 & -\frac{1}{3} & \frac{2}{3} & \frac{5}{3} \\ 0 & -\frac{2}{3} & \frac{4}{3} & -\frac{2}{3} \\ 0 & -\frac{1}{3} & \frac{2}{3} & \frac{11}{3} \end{array}\right] \xrightarrow{-\frac{3}{2}R_2} \left[\begin{array}{ccc|c} 1 & -\frac{1}{3} & \frac{2}{3} & \frac{5}{3} \\ 0 & 1 & -2 & 1 \\ 0 & -\frac{1}{3} & \frac{2}{3} & \frac{11}{3} \end{array}\right] \xrightarrow[R_3+\frac{1}{3}R_2]{R_1+\frac{1}{3}R_2}$
$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 4 \end{array}\right]$.

The last row of the augmented matrix says that $0 = 4$, a contradiction. We conclude that the system has no solution.

23. $\left[\begin{array}{ccc|c} 1 & 2 & 1 & -2 \\ -2 & -3 & -1 & 1 \\ 2 & 4 & 2 & -4 \end{array}\right] \xrightarrow[R_3-2R_1]{R_2+2R_1} \left[\begin{array}{ccc|c} 1 & 2 & 1 & -2 \\ 0 & 1 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{R_1-2R_2} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 4 \\ 0 & 1 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right]$. Let $x_3 = t$ and we find that $x_1 = 4 + t$ and $x_2 = -3 - t$. The infinitely many solutions are given by $(4 + t, -3 - t, t)$.

24. $\left[\begin{array}{ccc|c} 0 & 3 & 2 & 4 \\ 2 & -1 & -3 & 3 \\ 2 & 2 & -1 & 7 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|c} 2 & -1 & -3 & 3 \\ 0 & 3 & 2 & 4 \\ 2 & 2 & -1 & 7 \end{array}\right] \xrightarrow{\frac{1}{2}R_1} \left[\begin{array}{ccc|c} 1 & -\frac{1}{2} & -\frac{3}{2} & \frac{3}{2} \\ 0 & 3 & 2 & 4 \\ 2 & 2 & -1 & 7 \end{array}\right] \xrightarrow{R_3-2R_1} \left[\begin{array}{ccc|c} 1 & -\frac{1}{2} & -\frac{3}{2} & \frac{3}{2} \\ 0 & 3 & 2 & 4 \\ 0 & 3 & 2 & 4 \end{array}\right] \xrightarrow{\frac{1}{3}R_2}$
$\left[\begin{array}{ccc|c} 1 & -\frac{1}{2} & -\frac{3}{2} & \frac{3}{2} \\ 0 & 1 & \frac{2}{3} & \frac{4}{3} \\ 0 & 3 & 2 & 4 \end{array}\right] \xrightarrow[R_3-3R_2]{R_1+\frac{1}{2}R_2} \left[\begin{array}{ccc|c} 1 & 0 & -\frac{7}{6} & \frac{13}{6} \\ 0 & 1 & \frac{2}{3} & \frac{4}{3} \\ 0 & 0 & 0 & 0 \end{array}\right]$.

Letting $z = t$, we see that the infinitely many solutions are given by $\left(\frac{13}{6} + \frac{7t}{6}, \frac{4}{3} - \frac{2t}{3}, t\right)$.

25. $\left[\begin{array}{cc|c} 3 & 2 & 4 \\ -\frac{3}{2} & -1 & -2 \\ 6 & 4 & 8 \end{array}\right] \xrightarrow{\frac{1}{3}R_1} \left[\begin{array}{cc|c} 1 & \frac{2}{3} & \frac{4}{3} \\ -\frac{3}{2} & -1 & -2 \\ 6 & 4 & 8 \end{array}\right] \xrightarrow[R_3-6R_1]{R_2+\frac{3}{2}R_1} \left[\begin{array}{cc|c} 1 & \frac{2}{3} & \frac{4}{3} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]$.

We conclude that the infinitely many solutions are given by $\left(\frac{4}{3} - \frac{2}{3}t, t\right)$, where t is a parameter.

26. $\left[\begin{array}{ccc|c} 2 & -1 & 1 & -4 \\ 3 & -\frac{3}{2} & \frac{3}{2} & -6 \\ -6 & 3 & -3 & 12 \end{array}\right] \xrightarrow{\frac{1}{2}R_1} \left[\begin{array}{ccc|c} 1 & -\frac{1}{2} & \frac{1}{2} & -2 \\ 3 & -\frac{3}{2} & \frac{3}{2} & -6 \\ -6 & 3 & -3 & 12 \end{array}\right] \xrightarrow[R_3+6R_1]{R_2-3R_1} \left[\begin{array}{ccc|c} 1 & -\frac{1}{2} & \frac{1}{2} & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$.

We conclude that the infinitely many solutions are given by $\left(-2 + \frac{1}{2}s - \frac{1}{2}t, s, t\right)$ where s and t are parameters.

27. $\left[\begin{array}{rrr|r} 1 & 1 & -2 & -3 \\ 2 & -1 & 3 & 7 \\ 1 & -2 & 5 & 0 \end{array}\right] \xrightarrow[R_3-R_1]{R_2-2R_1} \left[\begin{array}{rrr|r} 1 & 1 & -2 & -3 \\ 0 & -3 & 7 & 13 \\ 0 & -3 & 7 & 3 \end{array}\right] \xrightarrow{-\frac{1}{3}R_2} \left[\begin{array}{rrr|r} 1 & 1 & -2 & -3 \\ 0 & 1 & -\frac{7}{3} & -\frac{13}{3} \\ 0 & -3 & 7 & 3 \end{array}\right] \xrightarrow[R_3+3R_2]{R_1-R_2} \left[\begin{array}{rrr|r} 1 & 0 & \frac{1}{3} & \frac{4}{3} \\ 0 & 1 & -\frac{7}{3} & -\frac{13}{3} \\ 0 & 0 & 0 & -10 \end{array}\right].$

The last row implies that $0 = -10$, which is impossible. We conclude that the system of equations is inconsistent and has no solution.

28. $\left[\begin{array}{rrrr|r} 2 & 6 & -5 & 0 & 5 \\ 1 & 3 & 1 & 7 & -1 \\ 3 & 9 & -1 & 13 & 1 \end{array}\right] \xrightarrow{\frac{1}{2}R_1} \left[\begin{array}{rrrr|r} 1 & 3 & -\frac{5}{2} & 0 & \frac{5}{2} \\ 1 & 3 & 1 & 7 & -1 \\ 3 & 9 & -1 & 13 & 1 \end{array}\right] \xrightarrow[R_3-3R_1]{R_2-R_1} \left[\begin{array}{rrrr|r} 1 & 3 & -\frac{5}{2} & 0 & \frac{5}{2} \\ 0 & 0 & \frac{7}{2} & 7 & -\frac{7}{2} \\ 0 & 0 & \frac{13}{2} & 13 & -\frac{13}{2} \end{array}\right] \xrightarrow{\frac{2}{7}R_2}$

$\left[\begin{array}{rrrr|r} 1 & 3 & -\frac{5}{2} & 0 & \frac{5}{2} \\ 0 & 0 & 1 & 2 & -1 \\ 0 & 0 & \frac{13}{2} & 13 & -\frac{13}{2} \end{array}\right] \xrightarrow[R_3-\frac{13}{2}R_2]{R_1+\frac{5}{2}R_2} \left[\begin{array}{rrrr|r} 1 & 3 & 0 & 5 & 0 \\ 0 & 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$

Let $x_2 = s$ and $x_4 = t$. Then there are infinitely many solutions given by $(-3s-5t, s, -1-2t, t)$, where s and t are parameters.

29. $\left[\begin{array}{rrr|r} 1 & -2 & 3 & 4 \\ 2 & 3 & -1 & 2 \\ 1 & 2 & -3 & -6 \end{array}\right] \xrightarrow[R_3-R_1]{R_2-2R_1} \left[\begin{array}{rrr|r} 1 & -2 & 3 & 4 \\ 0 & 7 & -7 & -6 \\ 0 & 4 & -6 & -10 \end{array}\right] \xrightarrow{\frac{1}{7}R_2} \left[\begin{array}{rrr|r} 1 & -2 & 3 & 4 \\ 0 & 1 & -1 & -\frac{6}{7} \\ 0 & 4 & -6 & -10 \end{array}\right] \xrightarrow[R_3-4R_2]{R_1+2R_2}$

$\left[\begin{array}{rrr|r} 1 & 0 & 1 & \frac{16}{7} \\ 0 & 1 & -1 & -\frac{6}{7} \\ 0 & 0 & -2 & -\frac{46}{7} \end{array}\right] \xrightarrow{-\frac{1}{2}R_3} \left[\begin{array}{rrr|r} 1 & 0 & 1 & \frac{16}{7} \\ 0 & 1 & -1 & -\frac{6}{7} \\ 0 & 0 & 1 & \frac{23}{7} \end{array}\right] \xrightarrow[R_2+R_3]{R_1-R_3} \left[\begin{array}{rrr|r} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & \frac{17}{7} \\ 0 & 0 & 1 & \frac{23}{7} \end{array}\right].$

We conclude that the solution is $\left(-1, \frac{17}{7}, \frac{23}{7}\right)$.

30. $\left[\begin{array}{rrr|r} 1 & -2 & 1 & -3 \\ 2 & 1 & -2 & 2 \\ 1 & 3 & -3 & 5 \end{array}\right] \xrightarrow[R_3-R_1]{R_2-2R_1} \left[\begin{array}{rrr|r} 1 & -2 & 1 & -3 \\ 0 & 5 & -4 & 8 \\ 0 & 5 & -4 & 8 \end{array}\right] \xrightarrow{\frac{1}{5}R_2} \left[\begin{array}{rrr|r} 1 & -2 & 1 & -3 \\ 0 & 1 & -\frac{4}{5} & \frac{8}{5} \\ 0 & 5 & -4 & 8 \end{array}\right] \xrightarrow[R_3-5R_2]{R_1+2R_2} \left[\begin{array}{rrr|r} 1 & 0 & -\frac{3}{5} & \frac{1}{5} \\ 0 & 1 & -\frac{4}{5} & \frac{8}{5} \\ 0 & 0 & 0 & 0 \end{array}\right].$

We conclude that the infinitely many solutions to this system are $\left(\frac{1}{5}+\frac{3}{5}t, \frac{8}{5}+\frac{4}{5}t, t\right)$.

31. $\left[\begin{array}{rrr|r} 4 & 1 & -1 & 4 \\ 8 & 2 & -2 & 8 \end{array}\right] \xrightarrow{\frac{1}{4}R_1} \left[\begin{array}{rrr|r} 1 & \frac{1}{4} & -\frac{1}{4} & 1 \\ 8 & 2 & -2 & 8 \end{array}\right] \xrightarrow{R_2-8R_1} \left[\begin{array}{rrr|r} 1 & \frac{1}{4} & -\frac{1}{4} & 1 \\ 0 & 0 & 0 & 0 \end{array}\right].$

We conclude that the infinitely many solutions are given by $\left(1-\frac{1}{4}s+\frac{1}{4}t, s, t\right)$, where s and t are parameters.

32. $\left[\begin{array}{rrr|r} 1 & 2 & 4 & 2 \\ 1 & 1 & 2 & 1 \end{array}\right] \xrightarrow{R_2-R_1} \left[\begin{array}{rrr|r} 1 & 2 & 4 & 2 \\ 0 & -1 & -2 & -1 \end{array}\right] \xrightarrow{-R_2} \left[\begin{array}{rrr|r} 1 & 2 & 4 & 2 \\ 0 & 1 & 2 & 1 \end{array}\right] \xrightarrow{R_1-2R_2} \left[\begin{array}{rrr|r} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 \end{array}\right].$

We conclude that the infinitely many solutions are given by $(0, 1-2t, t)$, where t is a parameter.

33. $\left[\begin{array}{rrr|r} 2 & 1 & -3 & 1 \\ 1 & -1 & 2 & 1 \\ 5 & -2 & 3 & 6 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{rrr|r} 1 & -1 & 2 & 1 \\ 2 & 1 & -3 & 1 \\ 5 & -2 & 3 & 6 \end{array}\right] \xrightarrow[R_3-5R_1]{R_2-2R_1} \left[\begin{array}{rrr|r} 1 & -1 & 2 & 1 \\ 0 & 3 & -7 & -1 \\ 0 & 3 & -7 & 1 \end{array}\right] \xrightarrow{\frac{1}{3}R_2}$

$\left[\begin{array}{rrr|r} 1 & -1 & 2 & 1 \\ 0 & 1 & -\frac{7}{3} & -\frac{1}{3} \\ 0 & 3 & -7 & 1 \end{array}\right] \xrightarrow[R_3-3R_2]{R_1+R_2} \left[\begin{array}{rrr|r} 1 & 0 & -\frac{1}{3} & \frac{2}{3} \\ 0 & 1 & -\frac{7}{3} & -\frac{1}{3} \\ 0 & 0 & 0 & 2 \end{array}\right].$

The last row implies that $0 = 2$, which is impossible. We conclude that the system of equations is inconsistent and has no solution.

34. $\left[\begin{array}{rrr|r} 3 & -9 & 6 & -12 \\ 1 & -3 & 2 & -4 \\ 2 & -6 & 4 & 8 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{rrr|r} 1 & -3 & 2 & -4 \\ 3 & -9 & 6 & -12 \\ 2 & -6 & 4 & 8 \end{array}\right] \xrightarrow[R_3-2R_1]{R_2-3R_1} \left[\begin{array}{rrr|r} 1 & -3 & 2 & -4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 16 \end{array}\right].$

The last row implies that $0 = 16$, which is impossible. We conclude that the system of equations is inconsistent and has no solution.

35. $\left[\begin{array}{rrr|r} 1 & 2 & -1 & -4 \\ 2 & 1 & 1 & 7 \\ 1 & 3 & 2 & 7 \\ 1 & -3 & 1 & 9 \end{array}\right] \xrightarrow[R_4-R_1]{\substack{R_2-2R_1 \\ R_3-R_1}} \left[\begin{array}{rrr|r} 1 & 2 & -1 & -4 \\ 0 & -3 & 3 & 15 \\ 0 & 1 & 3 & 11 \\ 0 & -5 & 2 & 13 \end{array}\right] \xrightarrow{-\frac{1}{3}R_2} \left[\begin{array}{rrr|r} 1 & 2 & -1 & -4 \\ 0 & 1 & -1 & -5 \\ 0 & 1 & 3 & 11 \\ 0 & -5 & 2 & 13 \end{array}\right] \xrightarrow[R_4+5R_2]{\substack{R_1-2R_2 \\ R_3-R_2}}$

$\left[\begin{array}{rrr|r} 1 & 0 & 1 & 6 \\ 0 & 1 & -1 & -5 \\ 0 & 0 & 4 & 16 \\ 0 & 0 & -3 & -12 \end{array}\right] \xrightarrow{\frac{1}{4}R_3} \left[\begin{array}{rrr|r} 1 & 0 & 1 & 6 \\ 0 & 1 & -1 & -5 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & -3 & -12 \end{array}\right] \xrightarrow[R_4+3R_3]{\substack{R_1-R_3 \\ R_2+R_3}} \left[\begin{array}{rrr|r} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right].$

We conclude that the solution of the system is $(2, -1, 4)$.

36. $\left[\begin{array}{rrr|r} 3 & -2 & 1 & 4 \\ 1 & 3 & -4 & -3 \\ 2 & -3 & 5 & 7 \\ 1 & -8 & 9 & 10 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{rrr|r} 1 & 3 & -4 & -3 \\ 3 & -2 & 1 & 4 \\ 2 & -3 & 5 & 7 \\ 1 & -8 & 9 & 10 \end{array}\right] \xrightarrow[R_4-R_1]{\substack{R_2-3R_1 \\ R_3-2R_1}} \left[\begin{array}{rrr|r} 1 & 3 & -4 & -3 \\ 0 & -11 & 13 & 13 \\ 0 & -9 & 13 & 13 \\ 0 & -11 & 13 & 13 \end{array}\right] \xrightarrow{-\frac{1}{11}R_2}$

$\left[\begin{array}{rrr|r} 1 & 3 & -4 & -3 \\ 0 & 1 & -\frac{13}{11} & -\frac{13}{11} \\ 0 & -9 & 13 & 13 \\ 0 & -11 & 13 & 13 \end{array}\right] \xrightarrow[R_4+11R_2]{\substack{R_1-3R_2 \\ R_3+9R_2}} \left[\begin{array}{rrr|r} 1 & 0 & -\frac{5}{11} & \frac{6}{11} \\ 0 & 1 & -\frac{13}{11} & -\frac{13}{11} \\ 0 & 0 & \frac{26}{11} & \frac{26}{11} \\ 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{\frac{11}{26}R_3} \left[\begin{array}{rrr|r} 1 & 0 & -\frac{5}{11} & \frac{6}{11} \\ 0 & 1 & -\frac{13}{11} & -\frac{13}{11} \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow[R_2+\frac{13}{11}R_3]{R_1+\frac{5}{11}R_3} \left[\begin{array}{rrr|r} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right].$

We conclude that the solution of the system is $(1, 0, 1)$.

37. Let x, y, and z represent the numbers of compact, mid-sized, and full-size cars, respectively, to be purchased. Then the problem can be solved by solving the system

$$\begin{aligned} x + \quad\quad y + \quad\quad z &= \quad\quad 60 \\ 18{,}000x + 28{,}800y + 39{,}600z &= 1{,}512{,}000 \end{aligned}$$

Using the Gauss-Jordan elimination method, we have

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 60 \\ 18{,}000 & 28{,}800 & 39{,}600 & 1{,}512{,}000 \end{array}\right] \xrightarrow{R_2 - 18{,}000R_1} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 60 \\ 0 & 1080 & 21{,}600 & 432{,}000 \end{array}\right] \xrightarrow{\frac{1}{1080}R_2}$$

$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 60 \\ 0 & 1 & 2 & 40 \end{array}\right] \xrightarrow{R_1 - R_2} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 20 \\ 0 & 1 & 2 & 40 \end{array}\right]$. We conclude that the solution is $(20 + z, 40 - 2z, z)$. Letting $z = 5$, we see that one possible solution is (25, 30, 5); that is Hartman should buy 25 compact, 30 mid-size, and 5 full-size cars. Letting $z = 10$, we see that another possible solution is (30, 20, 10); that is, 30 compact, 20 mid-size, and 10 full-size cars.

38. Let x, y, and z denote the numbers of ounces of Foods I, II, and III, respectively, that the dietician includes in the meal. Then the problem can be solved by solving the system

$$\begin{aligned} 400x + 1200y + 800z &= 8800 \\ 110x + 570y + 340z &= 3380 \\ 90x + 30y + 60z &= 1020 \end{aligned}$$

Using the Gauss-Jordan elimination method, we have

$$\left[\begin{array}{ccc|c} 400 & 1200 & 800 & 8800 \\ 110 & 570 & 340 & 3380 \\ 90 & 30 & 60 & 1020 \end{array}\right] \xrightarrow{\frac{1}{400}R_1} \left[\begin{array}{ccc|c} 1 & 3 & 2 & 22 \\ 110 & 570 & 340 & 3380 \\ 90 & 30 & 60 & 1020 \end{array}\right] \xrightarrow[R_3 - 90R_1]{R_2 - 110R_1} \left[\begin{array}{ccc|c} 1 & 3 & 2 & 22 \\ 0 & 240 & 120 & 960 \\ 0 & -240 & -120 & -960 \end{array}\right] \xrightarrow{\frac{1}{240}R_2}$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 22 \\ 0 & 1 & \frac{1}{2} & 4 \\ 0 & -240 & -120 & -960 \end{array}\right] \xrightarrow[R_3 + 240R_2]{R_1 - 3R_2} \left[\begin{array}{ccc|c} 1 & 0 & \frac{1}{2} & 10 \\ 0 & 1 & \frac{1}{2} & 4 \\ 0 & 0 & 0 & 0 \end{array}\right].$$

We conclude that the solution is $\left(10 - \frac{1}{2}z, 4 - \frac{1}{2}z, z\right)$. Letting $z = 2$, we see that one possible solution is a meal prepared with 9 ounces of Food I, 3 ounces of Food II, and 2 ounces of Food III. Another possible solution is obtained by letting $z = 4$. In this case, 8 ounces of Food I, 2 ounces of Food II, and 4 ounces of Food III would be used.

39. Let x, y, and z denote the numbers of ounces of Foods I, II, and III, respectively, that the dietician includes in the meal. Then the problem can be solved by solving the system

$$\begin{aligned} 400x + 1200y + 800z &= 8800 \\ 110x + 570y + 340z &= 2160 \\ 90x + 30y + 60z &= 1020 \end{aligned}$$

Using the Gauss-Jordan elimination method, we have

$$\left[\begin{array}{ccc|c} 400 & 1200 & 800 & 8800 \\ 110 & 570 & 340 & 2160 \\ 90 & 30 & 60 & 1020 \end{array}\right] \xrightarrow{\frac{1}{400}R_1} \left[\begin{array}{ccc|c} 1 & 3 & 2 & 22 \\ 110 & 570 & 340 & 2160 \\ 90 & 30 & 60 & 1020 \end{array}\right] \xrightarrow[R_3 - 90R_1]{R_2 - 110R_1} \left[\begin{array}{ccc|c} 1 & 3 & 2 & 22 \\ 0 & 240 & 120 & -260 \\ 0 & -240 & -120 & -960 \end{array}\right] \xrightarrow{\frac{1}{240}R_2}$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 22 \\ 0 & 1 & \frac{1}{2} & -\frac{13}{12} \\ 0 & -240 & -120 & -960 \end{array}\right] \xrightarrow[R_3 + 240R_2]{R_1 - 3R_2} \left[\begin{array}{ccc|c} 1 & 0 & \frac{1}{2} & \frac{101}{4} \\ 0 & 1 & \frac{1}{2} & -\frac{13}{12} \\ 0 & 0 & 0 & -1220 \end{array}\right].$$

The last row implies that $0 = -1220$, which is impossible. We conclude that the system of equations is inconsistent and has no solution—that is, the dietician cannot prepare a meal from these foods and meet the given requirements.

40. Let x, y, and z denote the numbers of Pandas, Saint Bernards, and Big Birds produced, respectively. Then we have the system of equations

$$\begin{aligned} 1.5x + 2y + 2.5z &= 4700 \\ 30x + 35y + 25z &= 65{,}000 \\ 5x + 8y + 15z &= 23{,}400 \end{aligned}$$

Using the Gauss-Jordan elimination method, we have $\left[\begin{array}{ccc|c} \frac{3}{2} & 2 & \frac{5}{2} & 4700 \\ 6 & 7 & 5 & 13{,}000 \\ 5 & 8 & 15 & 23{,}200 \end{array}\right] \xrightarrow{\frac{2}{3}R_1} \left[\begin{array}{ccc|c} 1 & \frac{4}{3} & \frac{5}{3} & \frac{9400}{3} \\ 6 & 7 & 5 & 13{,}000 \\ 5 & 8 & 15 & 23{,}200 \end{array}\right] \xrightarrow[R_3 - 5R_1]{R_2 - 6R_1}$

$$\left[\begin{array}{ccc|c} 1 & \frac{4}{3} & \frac{5}{3} & \frac{9400}{3} \\ 0 & -1 & -5 & -5800 \\ 0 & \frac{4}{3} & \frac{20}{3} & 23{,}200 \end{array}\right] \xrightarrow{-R_2} \left[\begin{array}{ccc|c} 1 & \frac{4}{3} & \frac{5}{3} & \frac{9400}{3} \\ 0 & 1 & 5 & 5800 \\ 0 & \frac{4}{3} & \frac{20}{3} & \frac{23200}{3} \end{array}\right] \xrightarrow[R_3 - \frac{4}{3}R_2]{R_1 - \frac{4}{3}R_2} \left[\begin{array}{ccc|c} 1 & 0 & -5 & -4600 \\ 0 & 1 & 5 & 5800 \\ 0 & 0 & 0 & 0 \end{array}\right].$$

Thus, the system has infinitely many solutions of the form $(5z - 4600, -5z + 5800, z)$. Observe that $5z - 4600 \geq 0$ and $-5z + 5800 \geq 0$, that is, $920 \leq z \leq 1160$. One possible solution is to make 1000 Big Birds, 800 Saint Bernards, and 400 Giant Pandas. Another solution is to make 110 Big Birds, 300 Saint Bernards, and 900 Giant Pandas.

41. Let x, y, and z denote the amounts of money invested in stocks, bonds, and a money-market account, respectively. Then the problem can be solved by solving the system

$$\begin{aligned} x + y + z &= 100{,}000 \\ 6x + 4y + 2z &= 500{,}000 \\ x - y - 3z &= 0 \end{aligned}$$

Using the Gauss-Jordan elimination method, we have

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 100{,}000 \\ 6 & 4 & 2 & 500{,}000 \\ 1 & -1 & -3 & 0 \end{array}\right] \xrightarrow[R_3 - R_1]{R_2 - 6R_1} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 100{,}000 \\ 0 & -2 & -4 & -100{,}000 \\ 0 & -2 & -4 & -100{,}000 \end{array}\right] \xrightarrow{-\frac{1}{2}R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 100{,}000 \\ 0 & 1 & 2 & 50{,}000 \\ 0 & -2 & -4 & -100{,}000 \end{array}\right] \xrightarrow[R_3 + 2R_2]{R_1 - R_2}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 50{,}000 \\ 0 & 1 & 2 & 50{,}000 \\ 0 & 0 & 0 & 0 \end{array}\right].$$

We conclude that the solution is $(50000 + z, 50000 - 2z, z)$. Therefore, one possible solution for the Garcias is to invest \$10,000 in a money-market account, \$60,000 in stocks, and \$30,000 in bonds. Another possible solution is for the Garcias to invest \$20,000 in a money-market account, \$70,000 in stocks, and \$10,000 in bonds.

42. a.

$$\begin{aligned} x_1 - x_2 \qquad\qquad\qquad &= 200 \\ x_1 \qquad\qquad - x_5 \qquad &= 100 \\ -x_2 + x_3 \qquad\qquad + x_6 &= 600 \\ -x_3 + x_4 \qquad\qquad &= 200 \\ x_4 - x_5 + x_6 &= 700 \end{aligned}$$

b.

$$\left[\begin{array}{cccccc|c} 1 & -1 & 0 & 0 & 0 & 0 & 200 \\ 1 & 0 & 0 & 0 & -1 & 0 & 100 \\ 0 & -1 & 1 & 0 & 0 & 1 & 600 \\ 0 & 0 & -1 & 1 & 0 & 0 & 200 \\ 0 & 0 & 0 & 1 & -1 & 1 & 700 \end{array}\right] \xrightarrow{R_2 - R_1} \left[\begin{array}{cccccc|c} 1 & -1 & 0 & 0 & 0 & 0 & 200 \\ 0 & 1 & 0 & 0 & -1 & 0 & -100 \\ 0 & -1 & 1 & 0 & 0 & 1 & 600 \\ 0 & 0 & -1 & 1 & 0 & 0 & 200 \\ 0 & 0 & 0 & 1 & -1 & 1 & 700 \end{array}\right] \xrightarrow[R_1 + R_2]{R_3 + R_2}$$

$$\left[\begin{array}{cccccc|c} 1 & 0 & 0 & 0 & -1 & 0 & 100 \\ 0 & 1 & 0 & 0 & -1 & 0 & -100 \\ 0 & 0 & 1 & 0 & -1 & 1 & 500 \\ 0 & 0 & -1 & 1 & 0 & 0 & 200 \\ 0 & 0 & 0 & 1 & -1 & 1 & 700 \end{array}\right] \xrightarrow{R_4 + R_3} \left[\begin{array}{cccccc|c} 1 & 0 & 0 & 0 & -1 & 0 & 100 \\ 0 & 1 & 0 & 0 & -1 & 0 & -100 \\ 0 & 0 & 1 & 0 & -1 & 1 & 500 \\ 0 & 0 & 0 & 1 & -1 & 1 & 700 \\ 0 & 0 & 0 & 1 & -1 & 1 & 700 \end{array}\right] \xrightarrow{R_5 - R_4} \left[\begin{array}{cccccc|c} 1 & 0 & 0 & 0 & -1 & 0 & 100 \\ 0 & 1 & 0 & 0 & 1 & 0 & -100 \\ 0 & 0 & 1 & 0 & -1 & 1 & 500 \\ 0 & 0 & 0 & 1 & -1 & 1 & 700 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right].$$

We conclude that the solution is $(s+100, s-100, s-t+500, s-t+700, s, t)$. Taking $s=150$ and $t=50$, we see that one possible traffic pattern is $(250, 50, 600, 800, 150, 50)$. Similarly, taking $s=200$, and $t=100$, we see that another possible traffic pattern is $(300, 100, 600, 800, 200, 100)$.

c. Taking $t=0$ and $s=200$, we see that another possible traffic pattern is $(300, 100, 700, 900, 200, 0)$.

43. a.

$$\begin{aligned} x_1 \qquad\qquad\qquad\qquad + x_6 \qquad &= 1700 \\ x_1 - x_2 \qquad\qquad\qquad\qquad + x_7 &= 700 \\ x_2 - x_3 \qquad\qquad\qquad\qquad &= 300 \\ -x_3 + x_4 \qquad\qquad\quad &= 400 \\ -x_4 + x_5 \qquad + x_7 &= 700 \\ x_5 + x_6 \qquad &= 1800 \end{aligned}$$

b.

$$\left[\begin{array}{ccccccc|c} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1700 \\ 1 & -1 & 0 & 0 & 0 & 0 & 1 & 700 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 300 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 400 \\ 0 & 0 & 0 & -1 & 1 & 0 & 1 & 700 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1800 \end{array}\right] \xrightarrow{R_2 - R_1} \left[\begin{array}{ccccccc|c} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1700 \\ 0 & -1 & 0 & 0 & 0 & -1 & 1 & -1000 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 300 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 400 \\ 0 & 0 & 0 & -1 & 1 & 0 & 1 & 700 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1800 \end{array}\right] \xrightarrow{-R_2}$$

$$\left[\begin{array}{ccccccc|c} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1700 \\ 0 & 1 & 0 & 0 & 0 & 1 & -1 & 1000 \\ 0 & 1 & -1 & 0 & 0 & 0 & 0 & 300 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 400 \\ 0 & 0 & 0 & -1 & 1 & 0 & 1 & 700 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1800 \end{array}\right] \xrightarrow{R_3 - R_2} \left[\begin{array}{ccccccc|c} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1700 \\ 0 & 1 & 0 & 0 & 0 & 1 & -1 & 1000 \\ 0 & 0 & -1 & 0 & 0 & -1 & 1 & -700 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 400 \\ 0 & 0 & 0 & -1 & 1 & 0 & 1 & 700 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1800 \end{array}\right] \xrightarrow{-R_3}$$

$$\left[\begin{array}{ccccccc|c} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1700 \\ 0 & 1 & 0 & 0 & 0 & 1 & -1 & 1000 \\ 0 & 0 & 1 & 0 & 0 & 1 & -1 & 700 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 400 \\ 0 & 0 & 0 & -1 & 1 & 0 & 1 & 700 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1800 \end{array}\right] \xrightarrow{R_4 + R_3} \left[\begin{array}{ccccccc|c} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1700 \\ 0 & 1 & 0 & 0 & 0 & 1 & -1 & 1000 \\ 0 & 0 & 1 & 0 & 0 & 1 & -1 & 700 \\ 0 & 0 & 0 & 1 & 0 & 1 & -1 & 1100 \\ 0 & 0 & 0 & -1 & 1 & 0 & 1 & 700 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1800 \end{array}\right] \xrightarrow{R_5 + R_4}$$