



# Chapter 3

## Applying the Supply-and-Demand Model

### ■ *Solution to Textbook Questions*

1. The effect of a demand shock on quantity exchanged and the price depends on the shape of the supply curve. The graphs in Figure 3.1 illustrate the different possibilities for a positive demand shock.

- Horizontal supply curve:
  - No change in price
  - Quantity exchanged increases

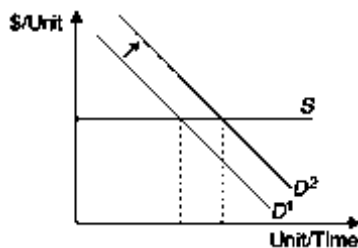


Figure 3.1a

- Linear upward-sloping supply curve:
  - Price increases
  - Quantity exchanged increases

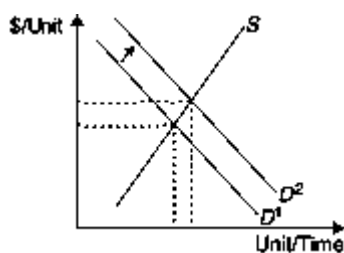


Figure 3.1b

- Linear downward-sloping supply curve:
  - Price decreases
  - Quantity exchanged increases

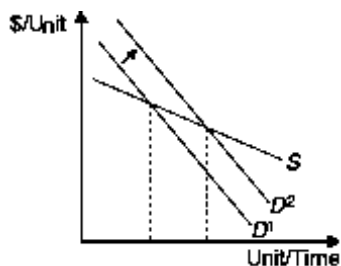


Figure 3.1c

- Vertical supply curve:
  - Price increases
  - No change in quantity exchanged

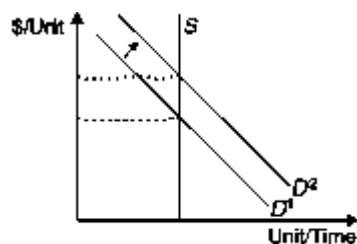


Figure 3.1d

- The supply of inexpensive watches is almost horizontal, so a decrease in demand causes the quantity supplied to drop a lot, but it does not cause a significant decline in price. The situation is depicted in Figure 3.2.

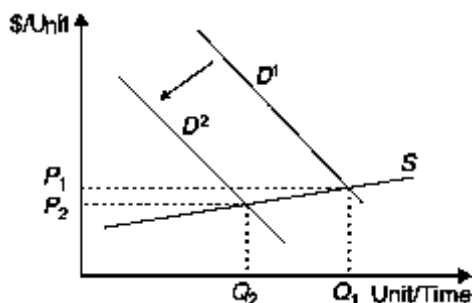


Figure 3.2

- According to Equation 3.1, the elasticity of demand is  $\epsilon = (\text{percentage change in quantity demanded}) \div (\text{percentage change in price}) = -3.8\% \div 10\% = -0.38$ , which is inelastic.
- Suppose there are  $n$  countries in the world. Assume the choke prices for countries are  $P_1^* < P_2^* < P_3^* < \dots < P_n^*$  and the corresponding quantities are  $Q_1^*, Q_2^*, Q_3^*, \dots, Q_n^*$ . The demand function,  $Q_i$ , for each country is as follows:

$$Q_i = Q_i^* \quad \text{if } P \leq P_i^*$$

$$= 0 \quad \text{if } P > P_i^*$$

The world demand function is the horizontal sum of demand in each country. The world demand function,  $Q$  in this case, will be a step function:

$$\begin{aligned}
 Q &= Q_1^* + Q_2^* + Q_3^* + \dots + Q_n^*, & \text{if } P < P_1^* \\
 &= Q_2^* + Q_3^* + \dots + Q_n^*, & \text{if } P_1^* < P \leq P_2^* \\
 &= Q_3^* + \dots + Q_n^*, & \text{if } P_2^* < P \leq P_3^* \\
 &\dots & \dots \\
 &= Q_n^*, & \text{if } P_{n-1}^* < P \leq P_n^* \\
 &= 0 & \text{if } P > P_n^*
 \end{aligned}$$

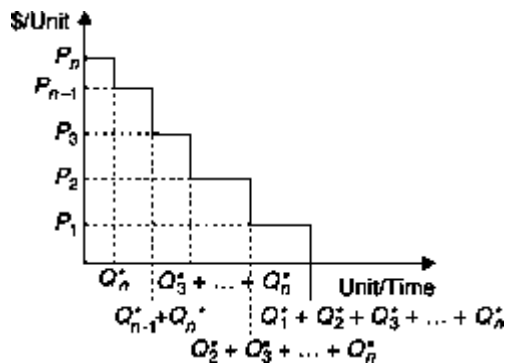


Figure 3.3

When price is exactly equal to any  $P_i^*$ , the world's demand curve is perfectly elastic from  $(Q_n^* + Q_{n-1}^* \dots + Q_{i+1}^*)$  to  $(Q_n^* + Q_{n-1}^* \dots + Q_i^*)$ . For all prices between  $P_i^*$  and  $P_{i-1}^*$ , the world's demand curve is perfectly inelastic.

5. The income elasticity for Pay TV appears to be positive, since the quantity demanded of Pay TV increases as income increases.
6. The innovation in freshwater pearls makes them closer substitutes for saltwater pearls. The similarity in prices indicates that both buyers and sellers regard them as very close substitutes. This will increase the cross-elasticity of demand.
7. When a minimum wage is enforced, the number of workers employed is determined by demand. The tax revenue for the government is  $R = \underline{aw}L^D(\underline{w})$ . Taking the derivative with respect to  $\underline{w}$ , we get:

$$\begin{aligned}
 \frac{dR}{d\underline{w}} &= \underline{a}L^D(\underline{w}) + \underline{aw} \frac{dL^D}{d\underline{w}} \\
 &= \underline{a}L^D(\underline{w}) \left( 1 + \frac{\underline{w}}{L^D(\underline{w})} \frac{dL^D}{d\underline{w}} \right) \\
 &= \underline{a}L^D(\underline{w})(1 + \varepsilon)
 \end{aligned}$$

where  $\varepsilon$  is the price elasticity of demand for labour.

If  $-1 < \varepsilon < 0$ , then  $\frac{dR}{d\underline{w}} > 0$ , but if  $\varepsilon < -1$ , then  $\frac{dR}{d\underline{w}} < 0$ .

Thus, if the elasticity of demand for labour is between  $-1$  and  $0$ , then an increase in the minimum wage increases the amount of tax revenue collected;

if the elasticity of demand for labour is less than  $-1$ , then an increase in the minimum wage decreases the amount of tax revenue collected.

## ■ **Solution to Textbook Problems**

8. Differentiating the demand function as  $Q = AP^\varepsilon$  with respect to  $P$ , we find that  $dQ/dP = \varepsilon AP^{\varepsilon-1}$ . To get the elasticity, we multiply  $dQ/dP$  by  $P/Q = P/AP^\varepsilon = 1/AP^{\varepsilon-1}$ . That is, the elasticity is  $\varepsilon AP^{\varepsilon-1} \times 1/AP^{\varepsilon-1} = \varepsilon$ . Because this result holds for any  $P$ , the elasticity is the same,  $\varepsilon$ , at every point along the demand curve.
9. From Problem 8, we know that the price elasticity of demand for small firms is constant along the demand curve, including the crossing point, and is equal to  $-0.563$ . The price elasticity of demand for large firms is also constant along the demand curve and is equal to  $-0.296$ , including the crossing point.

Is it possible to have different elasticities for one point? Yes. Examine the formula for price elasticity:

$$\varepsilon = (\Delta Q/Q)/(\Delta P/P) = (P/Q)(\Delta Q/\Delta P)$$

At the point of intersection,  $(P/Q)$  is the same for both curves. However,  $(\Delta Q/\Delta P)$ , the slope of the curves at the point of intersection, is different for each curve.

10. Because the linear supply function is  $Q = g + hP$ , a change in price of  $\Delta P$  causes a  $\Delta Q = h\Delta P$  change in quantity. Thus,  $\Delta Q/\Delta P = h$ , and the elasticity of supply is  $\eta = (\Delta Q/\Delta P)(P/Q) = hP/Q$ . By substituting for  $Q$  using the supply function, we find that  $\eta = hP/(g + hP)$ . By using the supply function to substitute for  $P$ , we learn that  $\eta = (Q - g)/Q$ .
11. If  $\varepsilon = \% \Delta Q / \% \Delta P$ , then  $\% \Delta P = \% \Delta Q / \varepsilon$ . Thus, if  $\varepsilon = -1.6$  and  $\% \Delta Q = 10.4\%$ , then  $\% \Delta P = -6.5\%$ .

This result is less than half the percentage price change reported in Solved Problem 2.4.

12. Tax incidence for almonds:

$$12/(12 + 0.47) = 0.96$$

Tax incidence for cotton:

$$0.73/(0.73 + 0.68) = 0.52$$

Tax incidence for processing tomatoes:

$$0.64/(0.64 + 0.26) = 0.71$$

13. In each of the constant elasticity functions shown, the exponent on the price variable is the elasticity. Following Equation 3.7, the incidence that falls on consumers is  $\eta/(\eta - \varepsilon)$ . The incidence that falls on producers is  $1 - \eta/(\eta - \varepsilon)$ . Because the elasticities are constant, this proportion is not dependent on where the supply curve intersects the demand curve.
14. Differentiating quantity,  $Q(P(\tau))$ , with respect to the specific tax  $\tau$ , we learn that the change in quantity as the tax changes is  $dQ/d\tau = (dQ/dP)(dP/d\tau)$ . Multiplying this expression by  $P/Q$ , we find that the change in quantity as the tax changes is  $\varepsilon(Q/P)(dP/d\tau)$ . Thus, the closer  $\varepsilon$  is to zero, the less the quantity falls, all else being the same. The tax causes revenue to change by:

$$\frac{dR}{d\tau} = \left( Q + P \frac{dQ}{dP} \right) \frac{dP}{d\tau} = 1(1 + \varepsilon)Q \frac{dP}{d\tau}$$

The closer  $\varepsilon$  is to zero, the larger the tax revenue effect.

15. Without the tax, the equilibrium price and quantity are determined by the intersection of the supply and demand curves:

$$\begin{aligned} a - bQ^* &= c + dQ^* \\ Q^* &= (a - c)/(b + d) \\ P^* &= a - bQ^* = a - b[(a - c)/(b + d)] = (ab + ad - ab + bc)/(b + d) \\ P^* &= (ad + bc)/(b + d) \end{aligned}$$

A specific tax of  $\tau$  per unit shifts the pre-tax supply curve up by  $\tau$ , thereby changing the inverse supply function to  $P = (c + \tau) + dQ$ . The equilibrium price faced by consumers with the tax is determined by the intersection of the new supply curve and the demand curve:

$$\begin{aligned} a - bQ &= (c + \tau) + dQ \\ Q &= [a - (c + \tau)]/(b + d) = Q^* - \tau/(b + d) \\ P &= [ad + b(c + \tau)]/(b + d) = P^* + b\tau/(b + d) \end{aligned}$$

Thus, the incidence of a specific tax of  $\tau$  per unit falling on consumers is  $b/(b + d)$ .

Note that the slope of the supply curve is  $dQ/dP = 1/d$  and the slope of the demand curve is  $dQ/dP = -1/b$ . At equilibrium with the tax,

$$\begin{aligned} \eta &= (P/Q)(dQ/dP) = (P/Q)(1/d) = P/dQ \\ \varepsilon &= (P/Q)(dQ/dP) = (P/Q)(-1/b) = -P/bQ \end{aligned}$$

Therefore:

$$\begin{aligned} \eta/(\eta - \varepsilon) &= (P/dQ)/[(P/dQ) + (P/bQ)] \\ &= (1/d)/[(b + d)/bd] \\ &= bd/[d(b + d)] \\ &= b/(b + d) \end{aligned}$$

Therefore, the incidence of a specific tax of  $\tau$  per unit falling on consumers is  $b/(b + d) = \eta/(\eta - \varepsilon)$ .